A NEW MODEL OF NONLINEAR DYNAMIC CREEP

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1. Introduction

The creep of structures is a nonlinear mathematical problem and the adequate simulation of creep processes involves several difficulties. In order to overcome them, computer numerical methods involving three simulation steps which are all equally important have been used: accurate simulation of the kinematic (possibly nonlinear) and the quasi static behaviour; appropriate constitutive simulation of the rheological properties of the material; mechanical representation of the creep process, which implies choosing to analyse a steady-state or secondary creep process, creep damage and any possible similar processes, each with greater or lesser degree of mechanical idealization. Each category of simulation directly influences the overall mechanical relevance of the results. Various attempts to include these effects in the analysis have been made, but due to the complexity of the creep problem, most investigators have concentrated their efforts on one specific aspect. Solution of the general problem was not possible until high-speed computers became available and theoretical formulations suitable for numerical analysis had been developed. General computer programs capable of handling all of the above-mentioned effects are in use now.

Another problem is to obtain an effective numerical method for solving nonlinear equations containing some additional differential operators in comparison with the elastic case. The second difference between those problems is connected with the time process of creep. Also, it can proceed during the time loading until the system flows or the creep is stopped by creep rupture. The creep effect leads to nonlinear stress-strain relations containing time as a variable and different kinetical parameters. Special numerical techniques for solving creep nonlinear equations have to incorporate mixed schemes of discretization in space and time.

The aim of the paper is the presentation of a suitable strategy which enables us to analyse complex problems such as creep of structural members and combines both the important aspects, efficiency and accuracy, in numerical computations.

No general method will be given here. The question of the general procedure is yet to be settled. Our aim is to survey the work in which different boundary problems (bodies of revolution and plane bodies, axisymmetrical shells and rectangular plates) have been solved; creep constitutive models have taken into account the whole history

of the deformation, elastic strain and dynamic stress, and accumulation of damage for long time loading while advanced computational models have been used, formulated as a problem-oriented combination of a stable numerical method for time integration and a similar method for solving the linearized equations.

2. Basic Assumptions

The creep laws of metallic specimens loaded by fixed and cyclic loads under elevated temperatures were established by the set of experiments (Lasan [1], Taira et al. [2], Rabotnov [3]). From experiment, the arbitrary functions in the constitutive equations were established. It is well known from experiments (Taira et al. [2]), that if the character of loading is harmonic in time with a small period and with cyclic load component smaller than the static load the global creep process of the metallic material is not just a repeated "microcreep" process. The influence of the varying stress part in each cycle on the creep process rate and on the life-time is a physical phenomenon, which is referred to as dynamic creep.

On the other hand, creep analysis, using a class of implicit single step algorithms, is not possible, because the large number of small time steps usually required for accurate solution could not be successfully extended to dynamic creep problems. This consideration implies a large consumption of computer time. Therefore it is quite interesting to seek solutions by other methods.

Using the usual definition of stress σ_{ij} , strain ε_{ij} and displacement u_i tensors, let us present the systems obtained by use of two time scales method and by considering an inelastic body which occupies domain V with S_1 as the clamped part of the surface and loaded by surface loads $p = p_1 + p_1^1\Phi(\xi)$ over another surface part S_2 and volume forces f_i

$$\begin{aligned} \sigma_{ij,j} + f_i &= \rho \dot{u}_i; \quad \varepsilon_{ij} = C_{ijkl} \sigma_{kl} + c_{ij}, \quad \forall x \in V \\ u_i &= u_i^1, \quad \forall x \in S_1, \quad \sigma_{ij} n_j = p_i, \quad \forall x \in S_2 \\ u_i(0) &= u_i^1(0) = 0, \quad c_{ij} = 0, \end{aligned} \quad (1)$$

where any functions of the basic motion we have to determine change slowly on the scale of the 'slow' time t . For 'fast' oscillatory motion with time scale $\xi: \langle \sigma_{ij}^1 \rangle = \langle u_i^1 \rangle = \langle c_{ij}^1 \rangle = 0$, the equations which can be obtained in this case are:

$$\begin{aligned} \sigma_{ij,j}^1 &= \rho \dot{u}_i^1, \quad \varepsilon_{ij}^1 = C_{ijkl} \sigma_{kl}^1, \quad \forall x \in V \\ u_i^1 &= 0 \quad \forall x \in S_1, \quad \sigma_{ij}^1 n_j = p_i^1 \Phi(\xi), \quad \forall x \in S_2 \\ u_i^1(0) &= \dot{u}_i^1(0) = c_{ij}^1(0) = 0, \quad 0 \leq \xi \leq 1, \\ \langle (\cdot) \rangle &= \int_0^1 (\cdot) d\xi, \quad \xi = t/\mu. \end{aligned} \quad (2)$$

Equations(1) and (2) cannot be considered as independent relations, because the tensor of creep irreversible strains is determined only after solution of both (1) and (2) are

included in (1). In order to complete these systems generalized constitutive equations of dynamic creep are assumed as follows:

$$\begin{aligned}\dot{\epsilon}_{ij} &= \frac{3}{2} \frac{\dot{\epsilon}}{\sigma_i} \left(\sigma_{ij} - \frac{1}{3} \sigma_{mm} \delta_{ij} \right), \\ \dot{\epsilon}_i &= \Lambda(\epsilon_i) \langle F(\sigma_i) H(A) \rangle, \\ A(t) &= \sigma_i^j / \sigma_i, \\ H(A) &= \int_0^1 F(1 + A(t) \Phi(\xi)) d\xi.\end{aligned}\quad (3)$$

σ_i, σ_i^j here are equivalent stresses, for example of von Mises type, determined by solutions of systems (1) and (2). The functions F and Λ are evaluated from the creep curves.

This representation enables us to handle two approaches to models for evaluating the stress-strain state under creep conditions. In the second approach "fast time" models give an adequate mathematical description of the behaviour of an elastic body under fast cyclic loading. In the first one, the "slow time" model gives the mathematical description of the creeping body as well as damage evolution under fixed loading. On the other hand, this model takes into account the distribution of amplitudes of the effective stress corresponding to results previously obtained on the basis of the first approach. The mathematical technique of the theory of two time scales leads to some "homogenisation" of the creep process. Generally speaking, homogenisation is the approach for simulating global homogeneous creep behaviour with local inhomogeneities periodically growing in time. If the character of loading is time-harmonic with a small period and with amplitude smaller than the fixed load, it is well known from experimental investigations that the global creep process of a metallic material does not depend on repeated "micro-creep" process. Creep analysis using a class of explicit or implicit single step algorithms are usually required for accurate solution. The method facilitates the calculation procedure.

3. Numerical Methods

Utilization of specific numerical methods to solve the nonlinear equations by the first "slow time" model is much more complicated than solution by the second one. Both models have to faithfully represent the real creep situation.

Dynamic simulation, which is needed in this way, may be performed by various means. In the case of linear elasticity, this dynamic problem is completely solved. If the character of loading is harmonic in time, we can conclude from the dynamic field equations that the displacement components are harmonic in time and the dynamic problem reduces to a boundary value problem. After integration, we can find the distribution of amplitudes of the effective stress corresponding to forced oscillation of a structural member.

The dynamic creep boundary-initial problem for two-dimensional structural members (plane bodies, bodies of revolution) and shells and plates with complex shape

is simulated by a Finite Element Method (FEM). By neglecting inertial forces in 'slow' motion the solution of the problem described as (1-3) can be obtained. The system of first order differential equations is solved by multistep predictor and corrector schemes. Prediction and correction are carried out both for nodal displacements and their rates and for values of stresses and damage parameters in order to ensure improved integration accuracy. In calculations of 'fast' forced and 'slow' motions co-ordination of FEM schemes is necessary. Such approach allows components of the incremental creep load vector to be easily obtained for an arbitrary time step. Values of the function $A(t)$ in any finite element, determined at each time step through σ_i which is adjusted during the solution process, are used in simulating the dynamic creep process. The harmonic forced oscillation problem is solved only once, before integrating the creep equations. Determined values of σ_i permit calculation of the function $H(A)$ at any time instant. Details of this algorithm can be found in [4].

For solving another nonlinear boundary problem in order to find the solution of system (1) in the case of thin shell or plate bending a numerical approach is used too. A general issue for nonlinear creep problems of shells is how to reduce both the frequency of solving linearized equations and the outlay (the numerical cost) in solving the latter problem. Large computational effort required for the nonlinear analysis obliges us to seek a form of the linearized equations which is simple and useful for fast numerical calculation. The variational formulation of the equations of shell creep based on the principle of virtual displacements has been applied. The differential equations were used in the generalized forces form. For the elastic creeping material, the expressions for membrane forces and bending moments require numerical integration over the thickness co-ordinate, since the stresses are distributed non-linearly. The variational formulation of the elastic creep problem is derived from the basic field equations by multiplying the equilibrium equations by an arbitrary displacement vector which satisfies both the strain-displacement equations and the homogeneous boundary condition and can be integrated over the domain occupied by the shell (shell middle surface with boundary contour). Then the problem takes the so-called variational form: to find the displacement vector satisfying at any time the variational equality combined with the well-known residual type of the internal virtual work in linear elasticity and the complementary inner product (more strictly duality pairing) defined as an external virtual work by both the pseudo-forces and the external fixed forces. The vector of nonlinear terms in the expressions for the membrane forces and the bending moments is treated as a pseudo force vector-function, with absolute values integrable in the Sobolev space $W(0,t;X)$, where t is the time step, X is the Banach space.

There are different ways to solve the variational problem. The classical Newton-Kantorovich method with a mixed Petrov-Galerkin, or some modified, approximation for the elastic-creep problem is suggested. The detailed numerical procedures required to implement this approximation are presented by Morachkovsky et al. [7]. The major advantage of this technique is the fact, that the effective matrix needs to be decomposed just once. Only the right part of the equations has to be modified in order to take into account the nonlinear terms (the stress-state dependent nonlinear pseudo-force, the quadratic part of the strain-displacement relations).

4. Numerical Results

Creep under tension of a plate having two circular notches of radius 10^{-3} m symmetrically placed about its centre line is considered as an example. The stress-strain state and creep damage accumulation are investigated. The plate material is titanium alloy Ti-6Al-2Cr-2Mo at $T=673\text{K}$ [5], its thickness 10^{-3} m. Such a plate was assumed in tension under stress 400 MPa in the direction perpendicular to the line of notch symmetry. Due to the plate symmetry, the areas of equivalent stresses at time $t=0$ are shown in the top left quarter of Figure 1. The picture in the top right quarter demonstrates the distribution of such stresses at the moment of completion of the hidden damage process. The regions of each damage parameter level at such a moment are shown in the right lower quarter of Figure 1.

Figure 1

It was calculated that there is significant stress redistribution during the creep process in the plate: for example, the value of $\sigma_i(0)=1113$ MPa at the point 10^{-4} m along the plate symmetry axis from the notch boundary decreases to $\sigma_i(t_c)=32$ MPa. The level of maximum stress decreases more than twofold and, appreciably displaced in area, is distant from the notch boundary. Damage distribution is very inhomogeneous and its peak values are located near the notch, where $\omega_{\max}=0.95$. This problem was studied earlier by Walczak et al.[5]. The hidden damage time determined by this paper's authors is $t=524$ h, while $t=572$ h determined by Walczak et al. [5] differs insignificantly.

The second example is the dynamic creep of a rectangular plate with length and width equal to 0.08m and thickness 0.001 m, made from duraluminium alloy D16AT at $T=573K$. The plate was loaded at its central point by a static $P_s=100$ N and cyclic $P_a=20$ N force. The plate edges were held rigidly, and the frequency of vibration was 85 Hz. The experimental investigations were done by the authors of [6]. The mathematical statement and solution of the boundary initial problem based on the formulation (1-3) were made by Morachkovsky et al. [7].

Experimental results were compared with numerical simulation data. The plate normal displacement at a point 0.01m from the central point along a perpendicular to a plate side are presented in Figure 2. The experimental data is scatter presented, the numerical one is drawn by lines. The solid curve and circles demonstrate the creep of a plate loaded by only a static force, while the dashed curve and triangles correspond to dynamic creep.

Figure 2

The results show that our approach is a suitable strategy which enables analysis of elastic creeping structures as an integrated problem and combines such important aspects, as efficiency and accuracy, in numerical computations.

5. References

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