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NONLINEAR CREEP PROBLEMS OF BODIES UNDER THE ACTION OF FAST FIELD OSCILLATIONS

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Problems of high-temperature creep are essential for metallic materials and alloys used in machine construction. The simultaneous action of high temperatures, static and cyclic loads corresponds to the process of "dynamic creep" in aircraft, leading to intense development of irreversible residual deformations and breakdown. For elastomer construction elements problems of nonlinear thermomechanical deformations and cyclic loads, resonance oscillations and vibrational heating, accompanied by high-frequency oscillations, have been investigated in the monograph [7] within the general statement. Similar results have traditionally referred to resonance regimes of the operating machine for metallic materials. Known results in this direction make it possible to practically solve tuning problems from resonances of both separate details and the machine as a whole. At the same time there is substantial interest in problems related to the determination of conditions of intense development of cyclic creep in such structures in non-resonance or near-resonance operating regimes. The corresponding problems correspond to energy requirements, chemical machine construction, aviation and rocket technology, and contemporary product formation technologies in creep regimes.

Known solutions of creep problems have been obtained for quasistationary loads. Along with the quasistatic loads, however, nonstationary physicomachanical fields (such as aerohydrodynamic flows with fluctuations, and so on), varying quickly with time, act on many structural elements.

In the present study we state these problems, and suggest an algorithm of solving them on the basis of asymptotic expansions of the multiple scaling method [1, 2]. We consider the problem of the effect on body creep due to a harmonic external force, whose time variation frequency Ω is substantially higher than the characteristic frequency ω of the fundamental action.

1. Statement of the Problem. Consider an arbitrary body of volume V , attached to part of the surface S_1 , and loaded by bulk forces f and surface forces p on part of the surface S_2 . In the coordinate system $x = \{x_k\}_{k=1}^3$ the motion of continuum of material points will be described within the Lagrange approach by the displacement vector $u = \{u_k\}_{k=1}^3$, their velocities $v = \{v_k\}_{k=1}^3$ [$v_k = u_k$, $d(\ast)/dt = (\ast)'$], and the stress $\sigma_{ij} = \sigma_{ji}$ and deformation $\epsilon_{ij} = \epsilon_{ji}$ tensors ($i, j = 1, 2, 3$), being functions of coordinates of the reference system x and of time t . The irreversible creep deformations without including

their effect on the compressibility are presented by the tensor $c_{ij} = c_{ji}$. For the latter it is necessary to utilize the tensor relation with the stress tensor and time, adequately reflecting the creep properties of the medium material under the conditions considered of thermal loading. The known relations in the form of equations of state refer to quasistationary loading and, consequently, cannot be carried over directly to the case under consideration, which is also verified by experimental data [5, 6].

We dwell on the refined oscillatory field of external action, considered in the present study. The external forces acting on the body are divided into two shapes – the fundamental and oscillatory action. We primarily refer to the bulk forces f and to that portion of the surface pressure $p^0(x, t)$, $x \in S_2$, which is slowly varying or remains constant in time. An oscillating field is applied to the surface S_2 , and depends harmonically on time $p^1 = p^1_*(x)\Phi(\Omega t)$. The function $\Phi(\Omega t)$ varies with time according to the law of 1-period harmonic function with frequency $\Omega = \text{const}$, substantially higher than the characteristic frequency of the system with motion due to action of the fundamental load.

Under the assumptions formulated above the mathematical statement of the initial-boundary value problem is obtained by the following system of equations:

$$\begin{aligned} \sigma_{ij,j} + f_i &= \rho \dot{v}_i, \quad \forall x \in V; \\ \sigma_{ij} n_j &= p_i^0 + p_i^1 \Phi_i(\Omega t), \quad \forall x \in S_2; \\ \varepsilon_{ij} &= c_{ijkl} \sigma_{kl} + c_{ij} = 1/2 (u_{i,j} + u_{j,i}), \quad \forall x \in V; \\ u_i &= u_i, \quad \forall x \in S_1, \quad u_i(x, 0) = v_i(x, 0) = c_{ij}(x, 0) = 0, \end{aligned} \quad (1.1)$$

where ρ is the mass density, c_{ijkl} is the tensor of elastic properties of the material, $n\{n_i\}_{i=1}^3$ is the unit external normal to the surface of the body, and u_i^* are known values of displacement points of the fixed part of the surface, nonvarying with time. We note the nonlinearity of the system of equations (1.1), related to the fact that the components of the creep deformation tensor c_{ij} are generally nonlinearly related to the components of the stress tensor and, possibly, to other parameters, determined by the solution of system (1.1). This relation is supplemented in the following.

2. Asymptotic Expansions. The suggested property of inertia of motion of body points makes it possible, by the noncommensurability of the frequencies of fundamental and oscillatory actions, to write down for the required unknowns of system (1.1) asymptotic expansions (AE) with a coefficient depending both on time t (slow or macroscopic) and on the variables $\xi = t/\mu$ (fast or microscopic)

$$z \sim \sum_{i=0}^{\infty} \mu^i z^i(x; t, \xi), \quad (2.1)$$

where by z we denote any of the unknowns of system (1.1), and x , t , and ξ are assumed to be formally independent variables. As a small parameter μ we take the quantity $2\pi/\Omega$ and, consequently, z^1 is assumed to be a 1-period function of the variable ξ , $0 \leq \xi \leq 1$

$$\langle z^i(x; t, \xi) \rangle = \int_0^1 z^i d\xi = 0. \quad (2.2)$$

Besides, one usually has

$$\frac{d}{dt} z^i = \frac{\partial}{\partial t} z^i + \frac{1}{\mu} \frac{\partial}{\partial \xi} z^i, \quad \xi = t/\mu. \quad (2.3)$$

In what follows we retain two terms in each of the asymptotic expansions in the form (2.1):

$$\begin{aligned} u_i &\sim u_i^0(t) + \mu u_i^1(\xi) + \dots, \\ \sigma_{ij} &\sim \sigma_{ij}^0(t) + \mu \sigma_{ij}^1(\xi) + \dots, \end{aligned} \quad (2.4)$$

making it possible to write with account of (2.3)

$$\begin{aligned} v_i &\sim v_i^0(t) + \mu v_i^1(\xi) + \dots, \\ \varepsilon_{ij} &\sim \varepsilon_{ij}^0(t) + \mu \varepsilon_{ij}^1(\xi) + \dots, \end{aligned} \quad (2.5)$$

where $\varepsilon_{ij}^0(t) = 1/2(u_{i,j}^0 + u_{j,i}^0)$, $\varepsilon_{ij}^1(\xi) = 1/2(u_{i,j}^1 + u_{j,i}^1)$, $v_i^1 = (1/\mu)u_{i,\xi}^1$, $v_{i,\xi}^1 = (1/\mu^2)u_{i,\xi\xi}^1$.

We note that in equalities (2.4) and (2.5), besides the separated arguments one must also keep in mind the dependence of slow and fast AE coefficients on spatial coordinates x .

The representations (2.4) make it possible to separate the original system (1.1) into two systems, equating to zero terms with identical powers of μ in (1.1), with following substitution of (2.4) into them. In the sense of asymptotic equalities these new equalities are the following:

$$\begin{aligned}\sigma_{ij,j}^0 + f_i &= \rho v_i^0, \quad \varepsilon_{ij}^0 = c_{ijkl}\sigma_{kl}^0 + c_{ij}^0, \quad \forall x \in V; \\ u_i^0 &= u_i^*, \quad \forall x \in S_1; \quad \sigma_{ij}^0 n_j = p_i^0, \quad \forall x \in S_2; \\ u_i^0(0) &= v_i^0 = 0, \quad c_{ij}^0(0) = 0,\end{aligned}\quad (2.6)$$

where all unknown functions of the fundamental motion are slowly varying in the scale of macroscopic time t , and

$$\begin{aligned}\sigma_{ij,j}^1 &= \rho v_{i,\xi}^1, \quad \varepsilon_{ij}^1 = c_{ijkl}\sigma_{kl}^1, \quad \forall x \in V; \\ u_i^1 &= 0, \quad \forall x \in S_1; \quad \sigma_{ij}^1 n_j = p_{i*}^1 \Phi(\xi), \quad \forall x \in S_2, \\ u_i^1(0) &= v_i^1(0) = 0; \quad c_{ij}^1 \equiv 0, \quad 0 \leq \xi \leq 1,\end{aligned}\quad (2.7)$$

where all unknown functions of fast oscillatory motion are varying within the fast time scale ξ : $\langle \sigma_{ij}^1 \rangle = \langle u_i^1 \rangle = \langle c_{ij}^1 \rangle = 0$, as are their derivatives with respect to coordinate variables.

At the same time $\langle \varphi(\xi) \rangle = (1/\mu)\langle \Phi(\xi) \rangle = 0$, and one does not succeed in partitioning the creep deformation within the fast time scale ξ . This makes it possible to assume system (2.7) for the fast AE coefficients (2.4), corresponding to elastic nonstationary motion under the action of a harmonic oscillatory load $p_{in}^1 = p_{i*}^1 \varphi(\xi)$ on the surface S_2 , where $p_{i*}^1(x)$, $\varphi(\xi) = \Phi(\xi)/\mu$, $\varphi_{,\xi\xi} = -\mu^2 \Omega^2 \varphi(\xi)$ (as a harmonic function). As is well-known, one can then separate variables in (2.7) into coordinates and time

$$\sigma_{ij}^1 = \sigma_{ij*}^1(x) \varphi(\xi), \quad u_i^1 = u_{i*}^1(x) \varphi(\xi). \quad (2.8)$$

The boundary value problem for the amplitude values of the fast unknowns is

$$\begin{aligned}\sigma_{ij*,j}^1 &= -\Omega^2 \rho u_{i*}^1, \quad 1/2(u_{i*,j*}^1 + u_{j*,i*}^1) = C_{ijkl}\sigma_{kl*}^1; \\ \forall x \in v; \quad \sigma_{ij*}^1 n_j &= p_{i*}^1, \quad \forall x \in S_2, \quad u_{i*}^1 = 0, \quad \forall x \in S_1.\end{aligned}\quad (2.9)$$

The inertial term in (2.9) formally follows from the derivation of the equations, but can be neglected in the calculations.

At the same time one must not assume that the systems (2.6) and (2.7) or (2.8), (2.9) are independent, since in (2.6) appears the tensor of irreversible creep deformations c_{ij}^0 , which, as shown below, is determined by the solution of system (2.7) or (2.8), (2.9), i.e., it depends on fast variables.

3. Equations of State of Creep with Applied Oscillations. These equations are required to complete system (1.1) and (2.6). In creep theory the equations of state are usually specified from creep data with a homogeneous stress state of a single-axis loaded sample. In what follows this equation is generalized for a complex stress state.

Let a stress be applied to the sample

$$\sigma = p^0 + \mu p_{*}^1 \varphi(\xi), \quad \varphi(\xi) = \Phi(\xi)/\mu, \quad (3.1)$$

where p^0 and p_{*}^1 are the quasistationary load and the amplitude portion of the oscillatory load, and $\varphi(\xi)$ is a harmonic function of "fast" time. Comparing this stress with the AE (2.4), for a single-axis loading we obtain $\sigma^0 = p^0$, $\sigma^1 = p_{*}^1 \varphi(\xi)$, and with account of (2.8) we have $\sigma_{*}^1 = p_{*}^1$.

Further, we use one of the versions of creep theory, such as, for definiteness, the version of hardening theory

$$\dot{c} = \Lambda(c) F(\sigma), \quad (3.2)$$

where the functions $\Lambda(c)$ and $F(\sigma)$ usually determine for each material the corresponding creep curve. Using (2.3) and the second of the asymptotic expansions (2.4), in retaining the first two terms we obtain from (3.2)

$$\frac{\partial c}{\partial t} + \frac{1}{\mu} \frac{\partial c}{\partial \xi} = \Lambda(c) F(\sigma^0 + \sigma^1 \Phi(\xi)). \quad (3.3)$$

Adopting the following condition $(1/\mu)c, \xi \approx 0$, from (3.3) we find the following asymptotic equality

$$c = c^0(t), \quad \dot{c}^0 = \Lambda(c^0) \langle F(\sigma^0 + \sigma^1 \Phi(\xi)) \rangle.$$

For the following we transform the argument of the function F , introducing the following notation: $A(t) = \sigma^{*1}/\sigma^0$, where t inside the operator $\langle \cdot \rangle$ plays the role of a parameter which can be replaced again by the $A(t)$ introduced. We then have

$$\dot{c}^0 = \Lambda(c^0) \langle F(\sigma^0 Q(A(t), \xi)) \rangle, \quad (3.4)$$

where $Q(A(t), \xi) = 1 + A(t)\Phi(\xi)$.

We note that in the case of homogeneous F (3.4) can be written in the form

$$\dot{c}^0 = \Lambda(c^0) F(\sigma^0) H(A); \quad H(A) = \int_0^1 F(1 + A\Phi(\xi)) d\xi. \quad (3.5)$$

The calculation of $H(A)$ presents no difficulty, and can be carried out analytically for $A(t) \ll 1$ or numerically otherwise. The parameter $A(t)$ can be called the asymmetry parameter of loading cycle, since it is determined by the ratio of the amplitude of the oscillatory loading portion to the quasistatic portion. The equation of state (3.4) corresponds to the well-known Taira-Lazan-Rabotnov hypothesis [5, 6], well-investigated experimentally in many studies devoted to dynamic creep with single-axis loading of samples. Below we suggest a generalization of (3.4), (3.5) for a complex stress state, described by the system of equations (2.6), i.e., for fundamental (slow) motion of a continuum during creep in an oscillatory field.

With this purpose consider the intensity of the stress tensors: $\sigma_i^0, \sigma_i^{*1}$ (usually for any tensor a_{ij} : $a_i = (3/2 a'_{ij} a'_{ij})^{1/2}$, $a'_{ij} = a_{ij} - 1/3 (a_{ij} \delta_{ij}) \delta_{ij}$), which in the single-axis case are equated, respectively, to σ^0 and σ^{*1} . The generalization of (3.4), (3.5) to a complex stress state can then be represented as:

$$\dot{c}_i^0 = \Lambda(c_i^0) \langle F(\sigma_i^0 Q(A, \xi)) \rangle; \quad (3.6)$$

$$\dot{c}_i^0 = \Lambda(c_i^0) F(\sigma_i^0) H(A), \quad (3.7)$$

where $A(t) = \sigma_i^{*1}/\sigma_i^0$, $Q = 1 + A(t)\Phi(\xi)$, $H(A) = \int_0^1 F(1 + A(t)\Phi(\xi)) d\xi$.

The tensor relation between c_{ij}^0 and the components of the tensor σ_{ij}^0 is determined by the known relations of flow theory

$$\dot{c}_{ij}^0 = \frac{3}{2} \frac{c_i^0}{\sigma_i^0} (\sigma_{ij}^0 - 1/3 (\sigma_{ij}^0 \delta_{ij}) \delta_{ij}), \quad (3.8)$$

where $\dot{c}_i^0$ is a completely defined function of σ_i^0 , established by relations of the form (3.6), (3.7).

The system of equations (2.6), (3.6), and (3.8) completely determine the initial-boundary value creep problem of "single-axis slow" body motion with imposed oscillations.

Consider several examples of equations of state for creep of metals under conditions of action of a cyclic oscillatory field.

If the creep process occurs without hardening, and the rate of creep is a power function of the stress, then for harmonic oscillatory loading

$$\dot{c}^0 = B(\sigma^0)^m \int_0^1 (1 + A \sin 2\pi\xi)^m d\xi, \quad (3.9)$$

where $A = \sigma_a/\sigma^0$, σ_a is the amplitude, σ^0 is the static stress component of a loading cycle, and B and m are material constants. For integer m one can calculate the integral (3.9) approximately, following preliminary expansion of the integrand function in a binomial series ($A \ll 1$)

$$H(A, m) = \int_0^1 (1 + A \sin 2\pi\xi)^m d\xi \cong 1 + \frac{m(m-1)}{4} A^2 \left(1 + \frac{(m-2)(m-3)}{16} A^2 \right).$$

With account of the calculations performed we rewrite (3.9)

$$\dot{c}^0 = B(\sigma^0)^m H(A, m). \quad (3.10)$$

We note that for noninteger m one must recommend linear interpolation of H from calculation data for its nearest integer values. The experimental results on creep of samples with harmonic oscillatory loads are described quite satisfactorily by Eq. (3.10), as shown in [5, 6]. Within the discussion above we obtain for hardening media

$$\dot{c}^0 = B(c^0)^n (\sigma^0)^m H(A, m), \quad (3.11)$$

which is also verified by experimental data [5, 6].

4. Numerical Investigations. As follows from the discussion above, the initial-boundary value creep problem of bodies under conditions of imposed fast oscillating fields (surface pressure) with accurate asymptotic expansions reduces to a coupled system of two shapes: the initial-boundary value problem for slow (fundamental) motion of a continuum (2.6) with equations of state (3.8), (3.10), or (3.11), and the initial-boundary value problem for fast induced motions (vibrations) of the original elastic body (2.7). Initially one solves the system of equations (2.7) and the static boundary value problem, following from (2.6) in the absence of creep deformations, $c_{ij}^0 = 0$, at the initial moment of time $t = 0$. To integrate the initial-boundary value problem (2.6) with equations of state (3.8), (3.10), or (3.11), as well as in creep calculations with quasistationary loads, we transform the basic equations to a solvable form in the unknown velocities [3].

Carrying out the solution by the finite element method, the resolvent system of equations is (neglecting inertial forces for slow motions)

$$[K] \dot{q}^0 = \dot{P} + \dot{R}(\dot{c}^0), \quad (4.1)$$

where $[K]$ is the stiffness matrix, constructed for an acceptable triangularization and shape functions by the FEM for an elastic body, $\dot{P}$ is the velocity vector of nodal loads, and $\dot{R}(\dot{c}^0)$ is the vector of supplementary loads, calculated from the components of the creep deformation rate tensor (at the initial moment of time $t = 0$, $\dot{R} = 0$). The integration of (4.1) in time is implemented by the explicit Runge-Kutta difference scheme or a predictor-corrector [3]. The vector of generalized nodal displacements q^0 , including values of slow functions u_i^0 in triangulation nodes of a spatial body, determines the stress-deformation state at an arbitrary moment of time t and the body distortion due to creep. In the calculations of induced fast and slow motions one usually discretizes the region occupied by the body. This approach makes it possible to calculate the components of the supplementary load vector in (4.1) algorithmically simply at an arbitrary step of time. For the supplementary components, reduced to the node γ of the finite element β , we have

$$\dot{R}_{\gamma i} = \int_{v_\beta} \Psi_{k\gamma i}^\beta D_{knrs}^\beta \dot{c}_{rs}^{0\beta} dv,$$

Fig. 1

which, following the application of quadratic equations, is rewritten as:

$$\dot{R}_{\gamma i} = G_{\beta \chi} \Psi_{k n \gamma i}^{\beta \chi} D_{k n r s}^{\beta \chi} \dot{c}_{r s}^{\beta \chi}, \quad (4.2)$$

where χ is the number of weight points, $G_{\beta \chi}$ is the coefficient of quadrature equations used in the calculations, and Ψ , D are matrices determined by the shape function used and by the elastic properties of the element β . In this case the components of the creep deformation rate, calculated from the equation of state, are defined as:

$$\dot{c}_{r s}^{\beta \chi} = \frac{3}{2} \varphi(\sigma_i^{\beta \chi}, c_i^{\beta \chi}) H(A^{\beta \chi}, m) S_{r s}^{\beta \chi}, \quad (4.3)$$

where φ and H correspond to the dependences chosen in the calculations for the description of creep curves, such as those following from (3.10), (3.11) $A^{\beta \chi} = \sigma_{i*}^{\beta \chi} / \sigma_i^{\beta \chi}$, the initial asymmetry parameter of the loading cycle at the point χ of element β , reaching in intensity the amplitude values of stresses with induced fast ($\sigma_{i*}^{\beta f}$) and slow ($\sigma_i^{\beta f}$) motions, and $S_{r s}^0$ is the deviatorix of the stress tensor $\sigma_{r s}^0$.

The amplitude stress values for induced fast motions are calculated by solving a system of equations of the form

$$[K - \Omega^2 M] q_*^1 = p_*^1, \quad (4.4)$$

where q_*^1 is the amplitude vector of nodal displacements for induced harmonic body oscillations with a given frequency Ω , K and M are the stiffness and mass matrices of the body, and p_*^1 is the amplitude vector of nodal loads. Equations (4.4) correspond to the boundary value problem (2.9) transformed by the FEM scheme.

An example of the calculation (see Fig. 1) refers to creep of steel 20 cylinder at $T = 700$ K under internal pressure. The equation of state (3.10) describes quite satisfactorily the material creep in a wide range of values $A \leq 0.75$ and $\Omega \leq 10^3$ Hz for $B = 6.34 \cdot 10^{-21} (1/\text{MPa})^m \cdot \text{h}$, $m = 6$. For a cylinder of length and external radius 0.2 m, and thickness $2.1 \cdot 10^{-2}$ m, Fig. 1 shows the change with time due to creep in the radial displacement of the point "A" at the internal surface of the wall for static loading $P = P^0$ and additional pressure oscillations $P = P^0 + p^1 \sin \Omega t$ ($P^0 = 22$ MPa, $p^1 = 11$ MPa, $\omega = 200$ Hz) — curve 2. One notices a substantial difference in the irreversible cylinder distortion due to the effect of creep of the imposed fast-oscillation pressure on it. The calculations were carried out by using a software package [4] designed to solve a large class of similar creep problems of axially symmetric elements of machine structures often encountered in technological applications.

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DESIGN OF WAVEGUIDES WITH AN AXISYMMETRIC LONGITUDINAL-BENDING VIBRATION MODE

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An approach to the design of a class of waveguides used in production processes of the ultrasonic welding of plastic components around a circular perimeter is proposed in this discussion [1]. Their configurations are shown in Fig. 1. It consists of two cones, mated along surface S_2 . In the working regime, the vibration mode of the upper cone is close to the first (half-wave) longitudinal mode of a conical rod, and the mode of the lower cone approaches that of the first bending mode of a linearly tapering disk.

The design problem consists in determining the remaining geometric parameters from assigned values of the outside radius R_3 , which defines the perimeter of the weld, the resonant frequency f_r , and the amplification factor of longitudinal displacements from the upper end S_1 to the effective section S_3 .

In addition to these functional constraints, there are constraints of a strength character. In our study, we examine the condition, as defined by the limiting value of the intensity of the stress amplitude for a given normal displacement of the effective edge.

In the proposed approach, the desired configuration is determined by synthesizing the upper and lower cones whose dimensions are concretized by the method of superposing spectral (resonance) curves corresponding to the required vibration modes.

1. Statement of Problem and Method of Its Solution. Let isotropic linearly elastic body V , which is bounded by surface S , be given. The system of relationships describing the harmonic vibrations of the body include the equation

$$\nabla^2 \vec{u} + \frac{1}{1-2\nu} \text{grad div } \vec{u} + \rho \omega^2 G^{-1} \vec{u} = 0 \quad (1.1)$$

and a boundary condition, which can be assigned in the form

$$\vec{\sigma} \cdot \vec{n} = \vec{\sigma}_0 \text{ on } S_1; \quad \vec{\sigma} \cdot \vec{n} = 0 \text{ on } S - S_1. \quad (1.2)$$

Here, $\vec{u}$ is the displacement amplitude, $\vec{\sigma}$ is the stress-amplitude tensor, $\vec{\sigma}_0$ is the amplitude of the external-load vector, $\vec{n}$ is the external normal to S , G is the shear modulus, ν is Poisson's ratio, ρ is density, and ω is the angular frequency ($\omega = 2\pi f$).

Problem (1.1)-(1.2) is solved by the finite-element method (FEM); in this case, an isoparametric tetragonal eight-node element is used with quadratic approximation of displacement [2]. The resonance characteristics are calculated within the framework of the problem of forced oscillations [4]. The $F(\omega)$ function, which is equal to the characteristic displacement component u_α in the region of its antinode and which is readily determined in