Nonlinear Vibration Problem of Launch Vehicle Carrying a Moving Time-Dependent Mass

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Abstract
In the present study, an analytical solution for nonlinear vibration problem of launch vehicle as a flexible beam model carrying moving with constant velocity mass which is time function is proposed. The nonlinear third and fifth order terms of partial differential equation with variable in time coefficients correspond to high amplitude and mid-plane stretching beam and the right term of initial equation of the problem represents the concentrated time-dependent moving mass effect. A hybrid asymptotic approach applied for an approximate analytical solution problem at given boundary and initial conditions are given. The resulting (approximate) solution has a form of a sum where each term consists of the product of two functions according to perturbation (on parameter at nonlinear terms) and phase-integral-Galerkin technique (on singular parameter at higher derivative) methods. The results of comparison of an approximate analytical solution and direct numerical integration of initial equation have shown a good enough accuracy as for “small” as well for “large” scalar parameters for asymptotic expansion of the desired function.

Keywords
Hybrid (P-WKB-G) asymptotic approach, nonlinear dynamic problem, Launch Vehicle, time dependent moving mass.

Introduction

Nonlinear vibration problem of Launch Vehicle with concentrated moving time dependent mass as a continuously distributed system has broad applications in engineering [1], including airspace structures, space tethers, flexible manipulators, highway bridges and others. For an overview of the publications in this sphere we refer [2, 3]. In [4] it was shown that the moving load solution is not an upper bound for the moving mass solution for the given boundary conditions. The existence of nonlinear terms in equations governing the vibration of flexible body motion with concentrated moving time dependent mass is important from the point of view modeling of flight dynamics effects. Most of studies were devoted to investigation of nonlinear vibration. Nonlinear cubic function term was considered in order to obtain analytical solution [2-4]. Effects of high degree terms as a rule are ignored. Here we refer the publications [1-5] which are connected with the subject of discussion and special attention we have paid to paper [2], in which employing of combination the homotopy method and traditional perturbation procedure for nonlinear analysis of the beam carrying a moving mass.

In the present study hybrid asymptotic solution of the vibrational motion of Launch Vehicle beam model, in which a concentrated moving time dependent mass with employing of the perturbation and phase integral (WKB) methods combined with Galerkin orthogonality principle (P-WKB-G method) has been obtained. An approximate analytical solution with direct numerical calculations of initial equation is compared. The results show that concentrated mass velocity and mass function from time are caused significant effects in nonlinear dynamic behavior of structure.

2. Formulation of the problem. Approximate analytical solution

The Launch Vehicle beam model with moving concentrated time dependent mass is shown in (Fig. 1).
The nonlinear differential equation with variable in time coefficients governing of Launch Vehicle vibration in high amplitude and mid-plain stretching at simply supported boundary conditions in which time dependent mass with constant velocity is moving on is as follows [1]:

\[
\frac{d^2 w(x, t)}{dt^2} + EI \frac{d^4 w(x, t)}{dx^4} - \left( \frac{EA}{2L} \left( \frac{\partial w(x, t)}{\partial x} \right) dx - \frac{EA}{8L} \left( \frac{\partial w(x, t)}{\partial x} \right)^2 dx \right) \frac{\partial^2 w(x, t)}{\partial x^2} =
\]

\[-M(t) \dot{w}(x, t) \delta[x-s] + Q_0 \sin \Omega t \]

where \( w(x, t) \) – transverse deformation of beam in time and coordinate.

If it is assumed [2] that deformation is large but performed slowly, the right part of initial equation (1) can be calculated as

\[
\dot{w}(x, t) = M(t) \left( \frac{\partial^2 w(x, t)}{\partial t^2} + \frac{x^2}{\partial x^2} \right) \delta[x-s]
\]

In the new nondimensional variables initial equation (1) can be written as

\[
\frac{\partial^2 \eta}{\partial \tau^2} + \frac{\partial^2 \eta}{\partial \xi^2} - \left( \frac{1}{2} \beta \psi \frac{\partial \eta}{\partial \xi} \right) \frac{\partial \eta}{\partial \xi} + \frac{1}{8} \psi \beta \frac{\partial \eta}{\partial \xi} \frac{\partial^2 \eta}{\partial \xi^2} \frac{\partial^2 \eta}{\partial \xi^2} = -\alpha \left( \frac{\partial^2 \eta}{\partial \xi^2} + \frac{\partial^2 \eta}{\partial \tau^2} \right) \delta(\xi - \xi_0) + Q_0 \sin \Omega \tau
\]

where

\[
\eta = \frac{w}{L}, \quad \xi = \frac{x}{L}, \quad \xi_0 = \frac{s}{L}, \quad \alpha = \frac{M(t)}{m}, \quad \alpha = \frac{M(t)}{mL}
\]

\[
\beta = \frac{Al^2}{m}, \quad u^2 = \frac{m}{EI}, \quad \tau = \left( \frac{El}{m} \right)^{0.5} \sqrt{\frac{t}{l^2}}, \quad \Omega = \Omega \left( \frac{m}{EI} \right)^{0.5}
\]

Suppose that solution of the equation (2) satisfies to simply supported boundary conditions

\[
\eta(\xi, \tau) = q(\tau) \sin(\pi \xi)
\]

initial equation transforms to the equation in form

\[
\ddot{q}(t) + \dot{\gamma}(u, \xi_0) q(t) + f(u, \xi_0) q^3(t) - \gamma(u, \xi_0) q^3(t) = 0
\]

\[
\ddot{q}(t) + \alpha \gamma(t) q(t) = \mu \left[ \dot{f}(u, \xi_0) q^3(t) + \gamma(u, \xi_0) q^3(t) \right] + \dot{Q}(\xi) \sin \Omega \tau = \mu N[t] + Q(\xi, \tau)
\]

where coefficients are
In order to obtain an approximate analytical solution equation (6) it will be employed the hybrid asymptotic approach \[5\] based on perturbation (with respect to parameter at nonlinear terms)

\[ q(t) = q_0(t) + \mu q_1(t) \]  

and for the second step we employ phase integral (WKB) approximation for solutions of system singular non homogeneous linear equations with variable coefficients (with respect to the parameter near the highest derivative) \[6-7\]:

\[ \mu^0 : \varepsilon^2 q_0(t) + \omega_0^2(t)q_0(t) = \bar{Q}(\xi, \tau) \]

\[ \mu^1 : \varepsilon^2 q_1(t) + \omega_0^2(t)q_1(t) = \mu \left[ \int f(u, \xi_0)q_1(t) + \bar{f}(u, \xi_0)q_0(t) \right] = \mu N[t] \]

where

\[ \omega_0^2(t) = \frac{\pi^2 - 2u^2 \bar{a}(t) \sin^2(\pi \xi_0)}{1 + 2\bar{a}(t) \sin^2(\pi \xi_0)} \]

An asymptotic solution of initial equation (6) will be consider at initial conditions

\[ q_0(0) = 1, \quad \dot{q}_0(0) = 0 \]

The system of ordinary singular differential equations with variable in time coefficient \(\omega_0^2(t)\) is solved by two terms WKB-approximation. Finally we have obtained the solution of nonlinear forced vibration problem on the basis of hybrid perturbation – (two-term) WKB method in the form:

\[ q(\tau) = \frac{1}{[\omega_0^2(\tau)]^{\frac{1}{25}}} \left[ \sin K(\tau) \left[ c_1 + \frac{1}{\varepsilon} \int \bar{Q}(\xi, \tau) \cos K(\tau) d\tau \right] + \mu \int N(q_0, \tau) \cos K(\tau) d\tau \right] + \]

\[ + \cos K(\tau) \left[ c_2 - \frac{1}{\varepsilon} \int \bar{Q}(\xi, \tau) \sin K(\tau) d\tau - \mu \int N(q_0, \tau) \sin K(\tau) d\tau \right] \]

where

\[ K(\tau) = \int \varepsilon^{-1} \omega_0(\tau) d\tau \]

For the numerical calculations parameters of the system are:

\[ \bar{a}(t) = 10^{-3} (1 - \tau), \quad \beta = 1.2 \cdot 10^5, \quad u = 17.3, \quad v = 10 \text{ m / sec} \]

Some results of numerical calculations for free vibration are presented at Fig. 2-4.
Conclusions

In the present study an approximate analytical solution for nonlinear forced vibration problem of Launch Vehicle beam model carrying a moving time-dependent mass has been obtained. A hybrid perturbation and phase integral methods was employing. Some results for influence of amplitude mass function in time and velocity parameter of concentrated mass for free vibration of model are presented.

References