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Project scope optimization model and method on criteria profit, time, cost, quality, risk

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Abstract

The paper presents a mathematical model of the structural optimization of the project scope, which includes five objective functions. One of the functions reflects the profit for the entire project life cycle. The others reflect the time of its realization, the cost of the project, the value of the generalized indicator of project product quality and the risk assessment associated with the project. The model takes into account the restrictions on the lack of financial debt after each phase completion, the duration of the project, the quality of the separate stages products.
It is assumed that the project scope is given in the form of a network model with the alternatives of the work execution.
The suggested model is a multi-objective, dynamic, containing Boolean variables, algorithmic and analytical objective functions and constraints.
A method of multi-objective structural optimization of the project scope with constraints was proposed to solve the problem.

Keywords: project; scope; multi-objective optimization; the network model; the dynamic problem.

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1. Project scope optimization in accordance with profit, time, cost, quality and risk criteria

Traditionally project management is represented in a shape of the so-called triangle formed by the following closely related factors: project scope, schedule and cost. So, all the provided factors are closely related. Moreover, in a Guide to Project Management Body of Knowledge (2008) risks related to these factors as well as quality of a project product should be taken into account. We can presume that project management is sure to be considered as a pyramid based on a project scope, while its sides comprise economic, socio-political, environmental, technological effects of the project, product quality, duration, budget and risks. The project scope is to be defined with consideration of all these factors.

The model of project scope optimization is proposed by Kononenko, Emelianova, Gricay (2007) concerning its execution time. Kononenko, Emelianova (2009) offered the model of project scope optimization according to cost criterion of its implementation within constraints on the schedule. In order to solve the above tasks the methods based on implicit enumeration are presented by Kononenko, Emelianova, Gricay (2007) and Kononenko, Emelianova (2009). The multicriteria model of project scope optimization has been invented by Kononenko, Mironenko (2010) regarding to time and cost criteria and within alternative work embodiment availability (or their complexes), provided in the form of network models. To find a proper solution the method based on a combination of minimax and implicit enumeration was developed. Kononenko, Protasov, Mironenko (2011) offered the model and method of project scope optimization concerning its schedule and budget performance within product quality constraints after a certain project phase has been completed. To solve this double criteria task a generalized criterion and implicit enumeration are supposed to be applied. In all above mentioned examples optimization-simulation approach of Tsvirkun, Akinfiev, Filippov (1985) has been implemented.

In this research we propose a mathematical model of the problem comprising five criterion functions subjected to minimization. The first function indicates business gross profit before taxation within product life cycle, the second one refers to project time calculated by means of critical path or any other method within a network model, the third concerns project costing, the fourth function denotes a value of the overall index of project product quality, and the fifth deals with risks estimation associated with the project. The model assumes that work execution alternatives (or their complexes) are specified in the form of network models. Moreover, it is supposed that after the completion of certain project phases financial debts are unlikely to exist. In addition, the maximum duration of project execution can serve as a model constraint. Constraints on project product quality are specified as well. It is assumed that only one alternative variant of work performance is possible to be conducted at each stage of the project.

The method of problem solving is provided below.

1.1. Mathematical model of the problem

The problem model is as follows:

$$\sum_{t=1}^{T}\sum_{l=1}^{L} C_t^{(l)} D_t^{(l)} - \sum_{h=1}^{H}\sum_{j=1}^{M_h} w_{hj} x_{hj} + \sum_{h=1}^{H}\sum_{j=1}^{M_h} E_{hj} x_{hj} - \sum_{t=1}^{T} U_t = P' \to \max_{x_{hj}}; \quad (1)$$

$$T_{pr} = \varphi_t(G, x_{hj}) \to \min_{x_{hj}}, \; j = \overline{1, M_h}, \; h = \overline{1, H}; \quad (2)$$

$$\sum_{h=1}^{H}\sum_{j=1}^{M_h} w_{hj} x_{hj} = F \to \min_{x_{hj}}; \quad (3)$$

$$\sum_{h=1}^{H}\sum_{j=1}^{M_h}\sum_{r=1}^{R_h} b_r \cdot \Psi_{hjr}^{norm} \cdot x_{hj} = Q \to \min_{x_{hj}}; \quad (4)$$

$$\sum_{h=1}^{H}\sum_{j=1}^{M_h}\sum_{i=1}^{I} P_{hji} \cdot V_{hji} \cdot x_{hj} = R_{neg} \to \min_{x_{hj}}; \quad (5)$$

$$S_h = S_{h-1} + K_h - \sum_{j=1}^{M} w_{hj} x_{hj}; \quad S_h \geq 0, \quad h = \overline{1,H}; \quad (6)$$

$$T_{pr} \leq T^{def}, \quad T_{pr} = \varphi_t(G, x_{hj}), \; j = \overline{1,M_h}, \quad h = \overline{1,H} \quad (7)$$

$$\Psi_{hjr} \cdot x_{hj} \lesseqgtr Q_{hr}^{def}, \; j = \overline{1,M_h}, \; h = \overline{1,H}, \; r = \overline{1,R_h}; \quad (8)$$

$$\sum_{j=1}^{M} x_{hj} = 1, \quad h = \overline{1,H}; \quad (9)$$

$$x_{hj} \in \{0,1\}, \quad j = \overline{1,M_h}, \quad h = \overline{1,H}; \quad (10)$$

where T denotes duration of maintenance or product consumption phase;
T_{pr} – project activity duration at investment phase;

l – product type, in totality equal to L;
$C_t^{(l)}$ – l- type production cost within t- year, $t = \overline{1,T}$;
$D_t^{(l)}$ – l- type production sales volume within t- year, $l = \overline{1,L}$, $t = \overline{1,T}$;

$$D_t^{(l)} = \begin{cases} A_t^{(l)}, if \; A_t^{(l)} \leq B_t^{(l)}; \\ B_t^{(l)}, if \; A_t^{(l)} > B_t^{(l)}; \end{cases} \quad (11)$$

$$A_t^{(l)} = \varphi_A(G, x_{hj}), t = \overline{1,T}; \quad (12)$$

$$x_{hj} \in \{0,1\}, h = \overline{1,H}, j = \overline{1,M_h}; \quad (13)$$

$B_t^{(l)}$ – relevant forecast of product demand within t- year;
$A_t^{(l)}$ – l- type production capacity within t- year;
M_h – quantity of operation execution options at phase h, $h = \overline{1,H}$;
h – operation execution phase number;
H – project phase quantity;
x_{hj} – Boolean variable equal to one in case j-option of operating activity is performed at h- phase, and zero otherwise;

G – network model of project activity including their execution alternatives $G = \{A, Z, \tau, W\}$;
A – network knot set,

$$A = \{ a_{hij} \}, \; i = \overline{1, n_j}, \; h = \overline{1,H}, \; j = \overline{1,M_h}, \quad (14)$$

where a_{hij} – stands for i- operation performed at h-phase within j-option (alternative) of network;

n_j – operation set within j-option (alternative) of network model;

Z – quantity of directed arcs,

$$Z = \left\{ z_{hi^j, pm^f} \right\}, \; i = \overline{1, n_j}, \; m = \overline{1, n_f}, \; h, p = \overline{1, H}, \; j = \overline{1, M_h}, \; f = \overline{1, M_p}, \tag{15}$$

z_{hi^j, pm^f} – stands for the arc directing from i-knot at h-phase of j-option (alternative) and entering m-knot at p-phase of f-alternative; $i \neq m$ provided $p = h$; $p \geq h$;

τ – quantity of operation execution terms within the knot,

$$\tau = \left\{ \tau_{hi^j} \right\}, i = \overline{1, n_j}, h = \overline{1, H}, j = \overline{1, M_h}, \tag{16}$$

here τ_{hij} – stands for execution terms of i -operation at h-phase of j-option of operating activity;

W – denotes set of network operating activity costs,

$$W = \left\{ w_{hi^j} \right\}, \; i = \overline{1, n_j}, h = \overline{1, H}, j = \overline{1, M_h}, \tag{17}$$

where w_{hi^j} – stands for i –operation cost at h-phase of j-option of operating activity;

E_{hj} – denotes depreciated cost of outgoing fixed assets at h-phase of j-option of operating activity;

U_t – current costs of production;

$U_t = \varphi_c(G, x_{hj}), t = \overline{1, T}$;

w_{hj} – stands for operation expenditures at h-phase of j-option within the network model (can be achieved by totaling a set of operational expenditures);

b_r – denotes weight of r- quality coefficient, $0 \leq b_r \leq 1, \sum_{r=1}^{R_h} b_r = 1$;

Ψ_{hjr}^{norm}– denotes the normalized value of r- product-quality index, resulted from j-option operating activity at h-phase, $r = \overline{1, R_h}$,

$$\Psi_{hjr}^{norm} = \frac{\Psi_{max,r} - \Psi_{hjr}}{\Psi_{max,r} - \Psi_{min,r}}, \forall r \in I_{h1};$$

$$\Psi_{hjr}^{norm} = \frac{\Psi_{hjr} - \Psi_{min,r}}{\Psi_{max,r} - \Psi_{min,r}}, \forall r \in I_{h2}; \tag{18}$$

Ψ_{hjr} – stands for values of r- product-quality index of j-option (alternative) project operating activity or their complexes at h-phase;

$\Psi_{min,r}$ – stands for minimum possible value of r- product-quality index;

$\Psi_{max,r}$ – stands for maximum possible value of r- product-quality index;

R_h – denotes quantity of product-quality indexes resulted from h-phase completion;

I_{h1}– set of numbers of product-quality indexes which are maximized at h-phase;

I_{h2}– set of numbers of product-quality indexes which are minimized at h-phase;

P_{hji} – stands for probability of i-risk occurrence within j-option operating activity of the network model at h-phase, $i = \overline{1, I}$;

V_{hji} – stands for negative impacts resulted from i-risk occurrence within j-option operating activity of the network model at h-phase of the project, $i = \overline{1, I}$;

S_h – implies cash balance after performance at h-phase completion;

K_h – denotes volume of money appropriated at *h*-phase.

The value of objective function (1) reflects company profit before taxation within product life cycle.

The value of objective function (2) $T_{pr} = \varphi_t(G, x_{hj})$ acts for duration of project investment phase calculated by means of critical path or any other method within a network model $G = \{A, Z, \tau, W\}$.

The value of objective function (3) equals to lump-sum costs within project implementation.

The value of objective function (4) denotes overall index value of project product quality.

The value of objective function (5) presents risks estimation, related to the project implementation.

Constraint (6) assumes that within project execution the financial indebtedness is not likely to occur after each stage completion.

Constraint (7) assumes that the terms of investment phase performance is likely to be not more than value T^{def} indicated by the customer beforehand.

Form (8) defines constraints, according to which product quality resulted from *h*- phase operating activity is likely to meet predetermined edge value of the *r* quality coefficient Q_{hr}^{def}. For each *h*- phase of project work completion or their complexes, $h = \overline{1, H}$, requirements to the value of *r*- product-quality index, are specified, where $r = \overline{1, R_h}$.

Example (9) attributes constraints, according to which just one option of work is likely to be performed at each *h* phase.

In models (1)-(10) some more constrains are likely to occur, as for resources expenditures, staff, equipment, primary goods, raw materials, spare parts as well as sequence of option work performance.

The proposed model is five-criteria, dynamic, comprising Boolean variables, algorithmic and analytical objective functions as well as algorithmic and analytical constraints.

To solve problems (1)-(10) the method of multi-criteria project scope optimization has been proposed. In this case the following criteria of its implementation should be taken into account: profit, time, cost, quality and risks. The method proposed is based on application of overall index and implicit enumeration, considering constraints and alternative options of work performance, presented in the form of network models.

The method is assigned to solve the problems of project scope optimization related to the criteria of profit, time, cost, quality and risks under conditions that any other following phase of project performance is unlikely to be initiated before the previous phase completion.

Then alternative options can be applied at either one phase of work scope or its set.

While applying the project scope optimization, a profit value of company before taxation up to the scheduled date F_1 is implemented. This value results from optimization of the project scope with consideration of a profit criterion, i.e. the result of problems solving (1, 6, 9, 10). Likewise the corresponding values of time F_2, cost F_3, quality F_4 and risks F_5 result from one-criteria tasks solving of project scope optimization in accordance with criteria of profit, time, cost, quality and risks consequently.

Let’s consider problem solving of project scope optimization according to profit criteria.

1.2. Problem of project scope optimization according to profit criteria

The model of project scope optimization according to profit criteria is as follows:

$$\sum_{t=1}^{T}\sum_{l=1}^{L} C_t^{(l)} D_t^{(l)} - \sum_{h=1}^{H}\sum_{j=1}^{M_h} w_{hj}x_{hj} + \sum_{h=1}^{H}\sum_{j=1}^{M_h} E_{hj}x_{hj} - \sum_{t=1}^{T} U_t = P' \to \max_{x_{hj}}; \quad (19)$$

$$S_h = S_{h-1} + K_h - \sum_{j=1}^{M} w_{hj}x_{hj}; \quad S_h \geq 0, \quad h = \overline{1,H}; \quad (20)$$

$$\sum_{j=1}^{M} x_{hj} = 1, \quad h = \overline{1,H}; \quad (21)$$

$$x_{hj} \in \{0,1\}, \quad j = \overline{1,M_h}, \quad h = \overline{1,H}; \quad (22)$$

To solve this problem a method of optimization, related to the implicit enumeration methods has been suggested. It is advisable to conduct information preparations in advance to reduce the amount of computation within the main cycle of this method and, respectively, to reduce time of decision taking. Information preparation for the project scope optimization method concerning profit is the calculation of the lower edge for expenditures related to the implementation of operating activities at each h phase, $h = \overline{1,H}$. To perform this, firstly we define expenditures related to the j option implementation at each h phase

$$З_{hj} = w_{hj} - E_{hjt} + \sum_{t=1}^{T} U_{hjt}, \; h = \overline{1,H}, \; \forall j = \overline{1,M_h}, \quad (23)$$

where U_{hit} – operating costs within t – year, related to the j option implementation at each h phase.

Then, at each h phase we take the option related to the least costs and identify them

$$З_{h\,min} = \min \{З_{hi}\}_{i=1}^{M_h}. \quad (24)$$

Values $З_{h\,min}$ will be used in project scope optimization method related to profit criterion.

1.3. The project scope optimization method related to profit

Within the project scope optimization method related to profit criterion we are considering options of operating activity at each h phase. If h increases, range of choice for future decisions decreases. Therefore, estimating the upper capacity edge to be obtained as a result of j option of operating activity at h phase, let's start with the upper edge which can be estimated at the earliest of all possible phases. For example, in a plant workshops construction, the first estimation of prospective plant production capacity can be completed after defining the size and number of floors. Further, moving gradually, this estimation is due to be specified, but not increased. The above logic should be taken into account while determining $A_t^{(l)}$ as a function of (G, x_{hj}).

The project scope optimization method related to profit criterion comprises a series of steps

1. $\Theta_H := 0$, P_H – set of j options, selected at all H phases of the project;
h:=1; f:=0; f^*:=+∞.
2. Start considering from the 1 option, i.e. $j_h := 1$.
3. Verify, if the limitation (20) is implemented

$S_h = S_{h-1} + K_h - w_{hj}$;
$S_h \geq 0$.

If the limitation is not implemented, go to step 8.

4. Define the profit, that may be obtained resulting from operating activity at phases from the *1* to *h* phases

$$P_h' = \sum_{t=1}^{T}\sum_{l=1}^{L} C_t^{(l)} D_t^{(l)} - \sum_{k=1}^{h}\sum_{j=1}^{M_k} w_{kj} x_{kj} + \sum_{k=1}^{h}\sum_{j=1}^{M_k} E_{kj} x_{kj} - \sum_{k=1}^{h}\sum_{j=1}^{M_k}\sum_{t=1}^{T} U_{kjt}\, x_{kj},$$

where $D_t^{(l)} = \begin{cases} A_t^{(l)}, if\ A_t^{(l)} \le B_t^{(l)}; \\ B_t^{(l)}, if\ A_t^{(l)} > B_t^{(l)}; \end{cases}$

$$A_t^{(l)} = \varphi_A(G, x_{hj}), t = \overline{1,T};$$

$$x_{hj} \in \{0,1\}, h = \overline{1,H}, j = \overline{1,M_h};$$

where U_{kjt} – current costs per *t* year, associated with the *j* option of operating activity at *h* phase.

5. Estimate lower edge for possible expenditures resulted from the rest phases performance, i.e. starting with *h+1* to *H*.

$$З_h' = З_{h+1,min} + З_{h+2,min} + \cdots + З_{H,min}.$$

Values $З_{h+1,min}, З_{h+2,min}, \ldots, З_{H,min}$ were defined at the information preparation phase.

Value $\pi = P_h' - З_h'$ is profit upper edge estimate that may be obtained resulting from operating activity at the phases from 1 to *H*. If $-\pi \ge f^*$, so the use of *j* the option will not provide better solution than the record one, go to step 8.

6. If $h < H$, examine the next phase of the project, $h \coloneqq h + 1$ and we pass back to step 2.
7. Put a new record value $f^* \coloneqq f$ and keep in mind set $\Theta_H \coloneqq \{j_h\}_{h=1}^{H}$.
8. If $j_h < M_h$, analyze the following option, i.e. $j_h \coloneqq j_h + 1$ and pass back to step 3.
9. If $h > 1$, pass back to the previous phase, i.e. $h \coloneqq h\text{-}1$. Take j_h value memorized and pass back to step 8. If $h = 1$ and $\Theta_H = \varnothing$, the task is infeasible, otherwise the optimal solution is obtained. The value of objective function $P' = f^*, F_1 \coloneqq P'$.

1.4 The method of multicriteria project scope optimization

1. Assume that

$\Theta_H \coloneqq 0$;

$h \coloneqq 1$;

$f^* \coloneqq +\infty$.

2. Take $j_h \coloneqq 1$.
3. Verify the implementation of limitation (6)

$S_h = S_{h-1} + K_h - w_{hj}$;

$S_h \ge 0$.

Introduce imaginary knot «finish», denoting the end of all operations at *h* phase.

Define the total time of operation performance t_h from *1* to *h* phase by critical path length calculation.

Bind value $f' \coloneqq t_h$. Define

$$T'_{pr_h} = t_{min_{h+1}} + \ldots + t_{min_H}.$$

Name $T_{pr} = f^t + T'_{pr_h}$ and verify limitation performance (7)
$T_{pr} \leq T^{def}$.
Verify limitation performance (8):

$$\Psi_{hjr} x_{hj} \lesseqgtr Q^{def}_{hr}, \; r = \overline{1, R_h}.$$

Verify other limitations, if any in this model. If at least one limitation fails, pass to step 12.

4. The income (excluding taxation) is likely to be gained resulting from the execution of work scope according to the option accepted at phases from 1 to *h*

$$P'_h = \sum_{t=1}^{T}\sum_{l=1}^{L} C_t^{(l)} D_t^{(l)} - \sum_{k=1}^{h}\sum_{j=1}^{M_k} w_{kj} x_{kj} + \sum_{k=1}^{h}\sum_{j=1}^{M_k} E_{kj} x_{kj} - \sum_{k=1}^{h}\sum_{j=1}^{M_k}\sum_{t=1}^{T} U_{kjt} x_{kj}.$$

Estimate the lower edge for expenditures, that may be incurred as a result of all phase's performance starting with $h+1$ to *H*.

$$З'_h = З_{h+1,min} + З_{h+2,min} + \cdots + З_{H,min}.$$

Values $З_{h+1\,min}, \; З_{h+2\,min}, \ldots, З_{H\,min}$ were defined at the stage of information preparation.
Value $\pi = P'_h - З'_h$ stands for upper profit edge estimation that is likely to be obtained resulting from operating activity at 1to *H* phases.

Norm f^1 so: $f^1_{norm} = \frac{F_1}{\pi}$.

5. Bind $f^2 := T_{pr}$. Norm f^2 so: $f^2_{norm} = \frac{f^2}{F_2}$.

6. Define w'_h. Bind $f^w := w'_h$.
Calculate $W'_{pr_h} = w_{min_{h+1}} + \cdots + w_{min_H}$.
Name $f^3 = f^w + W'_{pr_h}$.
Norm f^3 so: $f^3_{norm} = \frac{f^3}{F_3}$.

7. Define the quality of product q'_h that is defined with work scope performed at phases from the 1 to *h*

$$q'_h = \sum_{k=1}^{h}\sum_{j=1}^{M_h}\sum_{r=1}^{R_h} b_r \cdot \Psi^{norm}_{kjr} \cdot x_{kj}.$$

Bind $f^q := q'_h$.
Define $Q'_{pr_h} = Q_{h+1,min} + Q_{h+2,min} + \cdots + Q_{H,min}$.
Name $f^4 = f^q + Q'_{pr_h}$.
Norm f^4 as: $f^4_{norm} = \frac{f^4}{F_4}$.

8. Estimate project risks, that are likely to occur after work completion at phases from 1 to *h*

$$r'_h = \sum_{k=1}^{h}\sum_{j=1}^{M_h}\sum_{i=1}^{I} P_{kji} \cdot V_{kji} \cdot x_{kj}.$$

Bind $f^r := r_h'$.
Define $R_h' = R_{h+1,min}^{neg} + \cdots + R_{H,min}^{neg}$.
Name $f^5 = f^r + R_h'$.
Norm f^5 as: $f_{norm}^5 = \frac{f^5}{F_5}$.

9. Define $f = \lambda_1 f_{norm}^1 + \lambda_2 f_{norm}^2 + ... + \lambda_5 f_{norm}^5$. If $f \geq f^*$, pass to step 12.

10. Provided $h < H$ analyze the following project phase, i.e. $h := h + 1$ and pass back to step 2.

11. Bind a new value $f^* := f$ to variable f^* and fix set $\Theta_H := \{j_h\}_{h=1}^{H}$.

12. Provided $j_h < M_h$ consider the following option, i.e. $j_h := j_h + 1$ and pass to step 3.

13. Provided $h > 1$ pass to the previous phase, i.e. $h := h - 1$. Take $\dot{j}_h$ memorized and pass to step 12. Provided $h = 1$ и $\Theta_H = \varnothing$ the task is unfeasible, otherwise the optimal solution is obtained. In this case values of variables fp, ft, f^w, fq and fr for Θ_H define project performance profit, time, cost, product quality and risks related to the project consequently.

As a result the mathematical model and method of multi-criteria project scope optimization related to profit, time, cost, quality and risks have been proposed, providing the availability of constraints and alternatives of work scope performance as network models form.

The proposed model is multi-criteria and dynamic. It comprises Boolean variables, algorithmic and analytical objective functions as well as algorithmic and analytical constraints.

The method is based on the generalized criterion application along with the implicit enumeration method.

The method is assigned to solve the problems of project scope optimization providing any following project phase is unlikely to be initiated before the previous phase completion.

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