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MATHEMATICAL MODEL OF H-POLARIZED WAVE RADIATION FROM LONGITUDINAL SLOTS OF A CYLINDRICAL ANTENNA

In the paper mathematical model is developed for straight perfectly conducting circular cylindrical antennas with the finite number of longitudinal slots, in the case of H – polarized wave. Boundary hyper-singular integral equations of boundary value problem for the Helmholtz equations are introduced in the case the boundary is the cylindrical surface with longitudinal slots. Using the boundary integral equation we construct the discrete mathematical model of the problem of electromagnetic waves radiation from the structure slots, also the numerical experiment is conducted. The near and far fields are plotted for different parameters of the structure.

Key words: circular cylindrical antenna, discrete mathematical model, hypersingular integral equation, diffraction problems, method of discrete singularity, electromagnetic waves.

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МАТЕМАТИЧНА МОДЕЛЬ ВИПРОМІНЕННЯ Н-ПОЛЯРИЗОВАНОЇ ХВИЛІ З ПОЗДОВЖНІХ ЩІЛИН ЦИЛІНДРИЧНОЇ АНТЕНИ

Розглянуто циліндричну антенну структуру, яка має скінченну кількість подовжніх щілин. Розглянуто випадок випромінювання H – поляризованої хвилі. Отримано граничні гіперсингулярні та сингулярні інтегральні рівняння крайової задачі для рівняння Гельмгольця у випадку, коли границя – це циліндрична поверхня з поздовжніми щілинами. На основі граничного інтегрального рівняння побудовано дискретну математичну модель задачі випромінювання електромагнітних хвиль із щілин структури та проведено чисельний експеримент. Побудовані поля у ближній зоні та діаграми розсіяння для різних параметрів структури.

Ключові слова: кругова циліндрична антена, дискретна математична модель, гіперсингулярне інтегральне рівняння, щілинна циліндрична антена, метод дискретних особливостей, електромагнітні хвилі.

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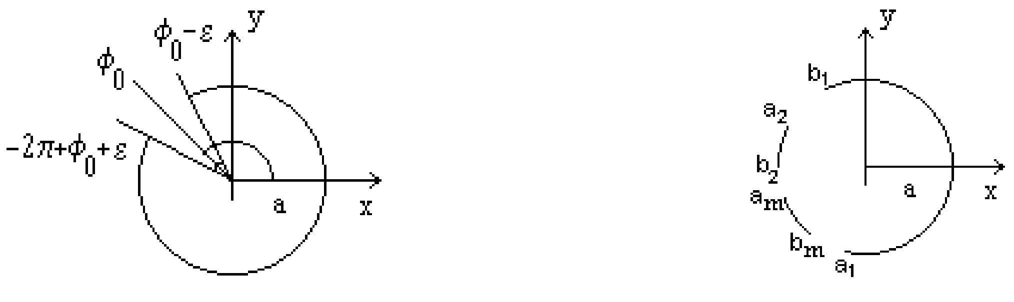
МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ИЗЛУЧЕНИЯ Н-ПОЛЯРИЗОВАННОЙ ВОЛНЫ ИЗ ПРОДОЛЬНЫХ ЩЕЛЕЙ ЦИЛИНДРИЧЕСКОЙ АНТЕННЫ

Рассмотрена цилиндрическая антенная структура кругового сечения, имеющая конечное число продольных щелей. Рассмотрен случай излучения H – поляризованной волны. Выведены граничные гиперсингулярные и сингулярные интегральные уравнения краевой задачи для уравнения Гельмгольца в случае, когда граница – цилиндрическая поверхность с продольными щелями. На базе граничного интегрального уравнения построена дискретная математическая модель задачи излучения электромагнитных волн из щелей структуры и проведен численный эксперимент. Построены поля в ближней зоне и диаграммы рассеяния для различных параметров электродинамической структуры.

Ключевые слова: дискретная математическая модель, круговая цилиндрическая антенна, гиперсингулярное интегральные уравнения, щелевая цилиндрическая антенна, метод дискретных особенностей, электромагнитные волны.

Introduction. Consider the boundary value problems for *the Helmholtz equations* and for *the Maxwell equations* in the case the border is a cylindrical surface with longitudinal slots. We reduced these problems to *the hyper-singular integral equation*. For an approximate (with controlled accuracy) solutions of such *boundary value problems* we constructed *the discrete mathematical models* by the boundary integral equation. These problems for the Helmholtz and Maxwell equations serve as mathematical models of cylindrical electrodynamic structures and have found wide application in designing and creating aperture and surface antennas, open resonators, slotted waveguides, and filters based on them. There are narrow-band filters, light detectors, and phase-sensitive elements in the light range. Based on the constructed discrete mathematical models, a numerical experiment was performed for these applied problems.

The geometry of a border. In the problems above the border is a surface of a circular cylinder with one or more longitudinal slots. We choose *a Cartesian coordinate system* such that the axis of the cylinder coincides with the z axis. In the section by the plane xOy we introduce the polar coordinates r and ϕ. The figures show the cross sections of the cylindrical surface with several slots by the plane perpendicular to the cylinder axis (fig. 1).



a *b*

Fig. 1 – The section of a cylindrical surface: *a* – with one slot of angular size 2ε ; *b* – with m slots.

We introduce the notation $L=\bigcup_{i=1}^{p}\left(\alpha_{i},\beta_{i}\right)$ and $CL=[-\pi,\pi]\setminus L$. Let S_a be a circle with the radius a and center at the origin. The arcs remaining after removing the slots are denoted by $\left(a_i, b_i\right)$, here $a_i=a\alpha_i$, $b_i=a\beta_i$ (fig. 1, *b*).

Neumann boundary value problem for the Helmholtz equation. The radiation problem of an $H-$polarized wave in the $2-D$ case reduces to the second boundary value problem for the Helmholtz equation (see, for example, [1]).

– *Formulation of the problem.*

In this problem we consider the dependence on time $e^{-i\omega t}$. The unknown function is represented in the form

$$u(r,\phi)=\begin{cases}u^{I}(r,\phi)+u_0(r,\phi), & r<a;\\ u^{II}(r,\phi), & r>a,\end{cases}$$

where $u^{I,II}(r,\phi)$ stands for the function inside (I) and outside (II) the structure respectively, the source of a cylindrical wave is a thread $u_0(r,\phi)=H_0^{(1)}(k^I r)$. The functions satisfy the following conditions.

– *The Helmholtz equation*:

$$\Delta u^{I,II}(r,\phi)+(k^{I,II})^2 u^{I,II}(r,\phi)=0, \tag{1}$$

in the region $R^2\setminus\left(\bigcup_{i=1}^{m}[a_i,b_i]\right)$ which is the exterior of the closed arcs of the circle S_a, here $k^{I,II}$ is the wave number and $(k^{I,II})^2=\varepsilon^{I,II}\mu\omega^2$.

– *The boundary condition*:

$$\left.\frac{\partial u(r,\phi)}{\partial r}\right|_{r=a}=0, \qquad \phi\in L. \tag{2}$$

– *The Sommerfeld radiation condition*:

$$\frac{\partial u(r,\phi)}{\partial r}-ik^{II}u(r,\phi)=o\left(\frac{1}{\sqrt{r}}\right),\ r\to\infty. \tag{3}$$

– The boundedness condition in each arbitrarily small region $\Omega\subset R^2$:

$$\int_{\Omega}\left[(k^{I,II})^2|u|^2+|\nabla u|^2\right]ds<\infty. \tag{4}$$

– «*Conjugate condition*»:

$$\left.u^{I}(r,\phi)\right|_{r=a}=\left.u^{II}(r,\phi)\right|_{r=a},\quad \phi\in C\overline{L}, \tag{5}$$

and

$$\left.\frac{1}{\varepsilon^{I}}\frac{\partial u^{I}(r,\phi)}{\partial r}\right|_{r=a}=\left.\frac{1}{\varepsilon^{II}}\frac{\partial u^{II}(r,\phi)}{\partial r}\right|_{r=a},\quad \phi\in C\overline{L}. \tag{6}$$

The derivation of the paired equation. The Fourier representations for functions satisfying the Helmholtz equation are obtained in the form of series (we mean convergence in the sense of distributions)

$$u^{I}(r,\phi)=\sum_{n=-\infty}^{\infty}C_n^{I}J_n\left(k^{I}r\right)e^{in\phi},\ r<a;\quad u^{II}(r,\phi)=\sum_{n=-\infty}^{\infty}C_n^{II}H_n^{(1)}\left(k^{II}r\right)e^{in\phi},\ r>a,$$

where $J_n(\zeta)$ is *the Bessel function* of order n, $H_n^{(1)}(\zeta)$ is *the Hankel function* of the first kind of order $|n|$.

With such choice of representation for the function, this function also satisfies radiation condition (3).

From conjugate condition (5 – 6) we get:

$$\begin{cases}\sum_{n=-\infty}^{\infty}C_n^{I}J_n\left(k^{I}a\right)e^{in\phi}+H_0^{(1)}\left(k^{I}a\right)=\sum_{n=-\infty}^{\infty}C_n^{II}H_n^{(1)}\left(k^{II}a\right)e^{in\phi}, & \phi\in C\overline{L};\\ \sum_{n=-\infty}^{\infty}\frac{k^{I}}{\varepsilon^{I}}C_n^{I}J_n'\left(k^{I}a\right)e^{in\phi}+\frac{k^{I}}{\varepsilon^{I}}H_0^{(1)\prime}\left(k^{I}a\right)=\sum_{n=-\infty}^{\infty}\frac{k^{II}}{\varepsilon^{II}}C_n^{II}H_n^{(1)\prime}\left(k^{II}a\right)e^{in\phi}, & \phi\in C\overline{L}.\end{cases} \tag{7}$$

From the second conjugate condition (6) and the boundary condition (2) it follows that:

$$\sum_{n=-\infty}^{\infty}\frac{k^{I}}{\varepsilon^{I}}C_n^{I}J_n'\left(k^{I}a\right)e^{in\phi}+\frac{k^{I}}{\varepsilon^{I}}H_0^{(1)\prime}\left(k^{I}a\right)=\sum_{n=-\infty}^{\infty}\frac{k^{II}}{\varepsilon^{II}}C_n^{II}H_n^{(1)\prime}\left(k^{II}a\right)e^{in\phi},$$

we get

$$\frac{k^{I}}{\varepsilon^{I}}C_0^{I}J_0'\left(k^{I}a\right)+\frac{k^{I}}{\varepsilon^{I}}H_0^{(1)\prime}\left(k^{I}a\right)=\frac{k^{II}}{\varepsilon^{II}}C_0^{II}H_0^{(1)\prime}\left(k^{II}a\right)\equiv C_0;$$

$$\frac{k^I}{\varepsilon^I} C_n^I J_n'\left(k^I a\right) = \frac{k^{II}}{\varepsilon^{II}} C_n^{II} H_n^{(1)\prime}\left(k^{II} a\right) \equiv C_n, \quad n \in \mathbf{Z} \setminus \{0\}. \tag{8}$$

Acting in the same way as in [2], using the first relation of system (7) and boundary condition (2), we get the paired adder equation:

$$\begin{cases} \sum_{n=-\infty}^{\infty} C_n e^{in\phi} = 0, \quad \phi \in L; \\ \sum_{n=-\infty}^{\infty} C_n \left[\frac{\varepsilon^I J_n\left(k^I a\right)}{k^I J_n'\left(k^I a\right)} - \frac{\varepsilon^{II} H_n^{(1)}\left(k^{II} a\right)}{k^{II} H_n^{(1)\prime}\left(k^{II} a\right)} \right] e^{in\phi} = \frac{\varepsilon^I J_0\left(k^I a\right)}{k^I J_0'\left(k^I a\right)} H_0^{(1)\prime}\left(k^I a\right) - H_0^{(1)}\left(k^I a\right), \quad \phi \in CL. \end{cases} \tag{9}$$

Let

$$A_0 = C_0 \left[\frac{\varepsilon^I J_0\left(k^I a\right)}{k^I J_0'\left(k^I a\right)} - \frac{\varepsilon^{II} H_0^{(1)}\left(k^{II} a\right)}{k^{II} H_0^{(1)\prime}\left(k^{II} a\right)} \right] - \frac{\varepsilon^I J_0\left(k^I a\right)}{k^I J_0'\left(k^I a\right)} H_0^{(1)\prime}\left(k^I a\right) + H_0^{(1)}\left(k^I a\right);$$

$$A_n = C_n \left[\frac{J_n\left(k^I a\right)}{k^I J_n'\left(k^I a\right)} - \frac{H_n^{(1)}\left(k^{II} a\right)}{k^{II} H_n^{(1)\prime}\left(k^{II} a\right)} \right], \ n \in \mathbf{Z} \setminus \{0\},$$

then get definitively:

$$\begin{cases} \sum_{n=-\infty}^{\infty} A_n \Gamma_m e^{in\phi} = \left(\frac{\varepsilon^I J_0\left(k^I a\right)}{k^I J_0'\left(k^I a\right)} H_0^{(1)\prime}\left(k^I a\right) - H_0^{(1)}\left(k^I a\right) \right) \Gamma_0, \quad \phi \in L; \\ \sum_{n=-\infty}^{\infty} A_n e^{in\phi} = 0, \quad \phi \in CL. \end{cases} \tag{10}$$

Introduce the notation: $\Gamma_n = \left[\frac{\varepsilon^I J_n\left(k^I a\right)}{k^I J_n'\left(k^I a\right)} - \frac{\varepsilon^{II} H_n^{(1)}\left(k^{II} a\right)}{k^{II} H_n^{(1)\prime}\left(k^{II} a\right)} \right]^{-1} \equiv B_1 |n| + B_2 \frac{1}{|n|} + K_n$, where $B_1 = 1/(a\varepsilon^I + a\varepsilon^{II})$, and $B_2 = \frac{-a}{\varepsilon^I + \varepsilon^{II}} \left(\frac{\left(k^I\right)^2 + \left(k^{II}\right)^2}{2} - \frac{\varepsilon^I \left(k^{II}\right)^2 + \varepsilon^{II}\left(k^{II}\right)^2}{2\left(\varepsilon^I + \varepsilon^{II}\right)} \right)$, $K_n = \Gamma_n - B_1 |n| - B_2 \frac{1}{|n|}$, and from the asymptotic expansion for Γ_n [4] for large n it follows that $K_n = O\left(\frac{1}{n^2}\right)$.

Derivation of the hypersingular integral equation of the Neumann problem. Using the representation for Γ_n rewrite the first equation in system (10):

$$A_0 \Gamma_0 + B_1 \sum_{n=-\infty}^{\infty}{}' A_n |n| e^{in\phi} + B_2 \sum_{n=-\infty}^{\infty}{}' A_n \frac{1}{|n|} e^{in\phi} + \sum_{n=-\infty}^{\infty}{}' K_n A_n e^{in\phi} = \left(\frac{\varepsilon^I J_0\left(k^I a\right)}{k^I J_0'\left(k^I a\right)} H_0^{(1)\prime}\left(k^I a\right) - H_0^{(1)}\left(k^I a\right) \right) \Gamma_0, \ \phi \in L. \tag{11}$$

The prime over the summation sign means that $n \neq 0$.

Continuing as in [2], we introduce a new unknown function

$$v(\phi) = \sum_{n=-\infty}^{\infty} A_n e^{in\phi}; \tag{12}$$

$$v(\phi) = 0, \quad \phi \in CL,$$

with the unknown coefficients A_n given by the formula:

$$A_n = \frac{1}{2\pi} \int_L v(\phi) e^{-in\phi} d\phi, \quad n \in \mathbf{Z}. \tag{13}$$

Using parametric representations for a hypersingular integral operator and an integral with a logarithmic kernel [3] and substituting representation (12) for $v(\phi)$ in equation (11), we obtain the hypersingular integral equation:

$$-\frac{B_1}{2\pi} \left(h.f.p. \int_L \frac{v(\theta)}{2\sin^2 \frac{\theta - \phi}{2}} d\theta \right) - \frac{B_2}{\pi} \int_L v(\theta) \ln \left| \sin \frac{\theta - \phi}{2} \right| d\theta + \frac{\Gamma_0}{2\pi} \int_L v(\theta) d\theta -$$

$$-\frac{2B_2\ln 2}{2\pi}\int_L v(\theta)\,d\theta+\frac{1}{\pi}\int_L K(\theta,\phi)v(\theta)\,d\theta=f\;, \tag{14}$$

where $K(\theta,\phi)=\sum_{n=1}^{\infty}\left[\Gamma_n-B_1|n|-B_2\frac{1}{|n|}\right]\cos(n(\phi-\theta))$, $f=\left(\frac{\varepsilon^I J_0\left(k^I a\right)}{k^I J_0'\left(k^I a\right)}H_0^{(1)\prime}\left(k^I a\right)-H_0^{(1)}\left(k^I a\right)\right)\Gamma_0$, here the first integral is understood in the sense of *Hadamard finite* part ($h.f.p.$).

We write equation (14) in the form:

$$-\frac{B_1}{\pi}*h.f.p.\int_L\frac{v(\theta)}{(\theta-\phi)^2}d\theta-\frac{B_2}{\pi}\int_L v(\theta)\ln|\theta-\phi|\,d\theta+$$

$$+\left[-\frac{B_1}{2\pi}\int_L v(\theta)\left\{\frac{1}{2\sin^2\frac{\theta-\phi}{2}}-\frac{2}{(\theta-\phi)^2}\right\}d\theta-\frac{B_2}{\pi}\int_L v(\theta)\ln\left|\frac{\sin\frac{\theta-\phi}{2}}{\frac{\theta-\phi}{2}}\right|d\theta+\right.$$

$$\left.+\frac{\Gamma_0}{2\pi}\int_L v(\theta)\,d\theta+\frac{1}{\pi}\int_L K(\theta,\phi)v(\theta)\,d\theta\right]=f\;. \tag{15}$$

From (15) note that the integrals in square brackets, have smooth kernels.

The Meixner edge condition will be satisfied if the restriction of the function $v(\theta)$ to the interval (α_q,β_q) is represented in the form: $v(\theta)\big|_{(\alpha_q,\beta_q)}=w_q(\theta)\sqrt{(\beta_q-\theta)(\theta-\alpha_q)},\;\alpha_q<\theta<\beta_q$.

Hence we obtain the sum of integrals over each interval (α_q,β_q)

$$-\frac{B_1}{\pi}\sum_{q=1}^{m}h.f.p.\int_{\alpha_q}^{\beta_q}\frac{w_q(\theta)}{(\theta-\phi)^2}\sqrt{(\beta_q-\theta)(\theta-\alpha_q)}d\theta-\frac{B_2}{\pi}\sum_{q=1}^{m}\int_{\alpha_q}^{\beta_q}w_q(\theta)\ln|\theta-\phi|\sqrt{(\beta_q-\theta)(\theta-\alpha_q)}d\theta+$$

$$+\left[-\frac{B_1}{2\pi}\sum_{q=1}^{m}\int_{\alpha_q}^{\beta_q}w_q(\theta)\left\{\frac{1}{2\sin^2\frac{\theta-\phi}{2}}-\frac{2}{(\theta-\phi)^2}\right\}\sqrt{(\beta_q-\theta)(\theta-\alpha_q)}d\theta-\right.$$

$$-\frac{B_2}{\pi}\sum_{q=1}^{m}\int_{\alpha_q}^{\beta_q}w_q(\theta)\ln\left|\frac{\sin\frac{\theta-\phi}{2}}{\frac{\theta-\phi}{2}}\right|\sqrt{(\beta_q-\theta)(\theta-\alpha_q)}d\theta+\frac{\Gamma_0}{2\pi}\sum_{q=1}^{m}\int_{\alpha_q}^{\beta_q}w_q(\theta)\sqrt{(\beta_q-\theta)(\theta-\alpha_q)}d\theta+$$

$$\left.+\frac{1}{\pi}\sum_{q=1}^{m}\int_{\alpha_q}^{\beta_q}w_q(\theta)K(\theta,\phi)\sqrt{(\beta_q-\theta)(\theta-\alpha_q)}d\theta\right]=f\;,\quad\phi\in L\,. \tag{16}$$

Acting in the same way as in [5], we introduce the mappings:

$$g_q:\;(-1,1)\to(\alpha_q,\beta_q):t\mapsto\theta=\frac{\beta_q-\alpha_q}{2}t+\frac{\beta_q+\alpha_q}{2}\,, \tag{17}$$

$$w_q(\theta)\sqrt{(\beta_q-\theta)(\theta-\alpha_q)}=\frac{\beta_q-\alpha_q}{2}\gamma_q(t)\sqrt{1-t^2}\,, \tag{18}$$

$$d\theta=\frac{\beta_q-\alpha_q}{2}dt\,.$$

As a result we get:

$$-\frac{B_1}{\pi}\left(h.f.p.\int_{-1}^{1}\frac{\gamma_p(t)}{(t-t_0)^2}\sqrt{1-t^2}dt\right)-\frac{B_2}{\pi}\left(\frac{\beta_q-\alpha_q}{2}\right)^2\int_{-1}^{1}\gamma_p(t)\ln|t-t_0|\sqrt{1-t^2}dt+$$

$$+\frac{1}{\pi}\sum_{q=1}^{m}\left(\frac{\beta_q-\alpha_q}{2}\right)^2\int_{-1}^{1}\tilde{K}_{pq}(t_0,t)\gamma_q(t)\sqrt{1-t^2}dt=f\;, \tag{19}$$

where

$$\tilde{K}_{pq}(t_0,t)=\begin{cases} K^1\left(g_p(t_0),\,g_p(t)\right)-B_2\ln\dfrac{\beta_p-\alpha_p}{2}, & p=q;\\ K^1\left(g_p(t_0),\,g_q(t)\right)-\dfrac{B_1}{\left(g_p(t_0)-g_q(t)\right)^2}-B_2\ln\left|g_p(t_0)-g_q(t)\right|, & p\neq q,\end{cases}$$

and

$$K^1\left(g_p(t_0),g_q(t)\right)=K\left(g_p(t_0),g_q(t)\right)-\frac{B_1}{2}\left\{\frac{1}{2\sin^2\dfrac{g_p(t_0)-g_q(t)}{2}}-\frac{2}{\left(g_p(t_0)-g_q(t)\right)^2}\right\}-$$

$$-B_2\ln\left|\frac{\sin\dfrac{g_p(t_0)-g_q(t)}{2}}{\dfrac{g_p(t_0)-g_q(t)}{2}}\right|+\frac{\Gamma_0}{2}.$$

Discrete mathematical model. To discretize the system of integral equations (19), we introduce into consideration an unknown function $\gamma_p^{n_p-2}(t)$, where $\gamma_p^{n_p-2}(t)$ is a polynomial of degree n_p-2.

We also replace smooth kernels of equations by their interpolation polynomials, for each of the variables with nodes $t_{oj}^{n_p}$, $j=1,\dots,n_p-2$, here $t_{oj}^{n_p}$ are zeros of *the Chebyshev polynomial* of the second kind $U_{n_p-2}(t_0)$. We obtain the system of hyper-singular integral equations with respect to the unknown functions $\gamma_p^{n_p-2}(t)$, $p=1,\dots,m$:

$$-\frac{B_1}{\pi}\left(a.f.p.\int_{-1}^{1}\frac{\gamma_p^{n_p-2}(t)}{\left(t-t_{0j}^{n_p}\right)^2}\sqrt{1-t^2}\,dt\right)-\frac{B_2}{\pi}\left(\frac{\beta_q-\alpha_q}{2}\right)^2\int_{-1}^{1}\gamma_p^{n_p-2}(t)\ln\left|t-t_{0j}^{n_p}\right|\sqrt{1-t^2}\,dt+$$

$$+\frac{1}{\pi}\sum_{q=1}^{m}\left(\frac{\beta_q-\alpha_q}{2}\right)^2\int_{-1}^{1}P_{n-2}\left[\tilde{K}_{pq}\right]\left(t_{0j}^{n_p},t\right)\gamma_q^{n_q-2}(t)\sqrt{1-t^2}\,dt=f\,,\quad j=1,\dots,n_p-2,\quad p=1,\dots,m. \tag{20}$$

Using *interpolation-type quadrature formulas* [11, 12] we get the next system of linear algebraic equations with respect to the «approximations» of unknown functions $\gamma_p^{n_p-2}(t)$, $p=1,\dots,m$:

$$\frac{B_1}{n_p}\sum_{\substack{k=1\\k\neq j}}^{n_p-1}\gamma_p^{n_p-2}\left(t_{0k}^{n_p}\right)\left(1-\left(t_{0k}^{n_p}\right)^2\right)\frac{1-(-1)^{j+k}}{\left(t_{0j}^{n_p}-t_{0k}^{n_p}\right)^2}-B_1\frac{n_p}{2}\gamma_p^{n_p-2}\left(t_{0j}^{n_p}\right)-$$

$$-\frac{B_2}{n_p}\left(\frac{\beta_p-\alpha_p}{2}\right)^2\sum_{k=1}^{n_p-1}\gamma_p^{n_p-2}\left(t_{0k}^{n_p}\right)\left(1-\left(t_{0k}^{n_p}\right)^2\right)\left[\ln 2+2\sum_{r=1}^{n_p-1}\frac{1}{r}T_r\left(t_{0k}^{n_p}\right)T_r\left(t_{0j}^{n_p}\right)+\frac{(-1)^{k+j}}{2n_p}\right]-$$

$$-\sum_{q=1}^{m}\left(\frac{\beta_q-\alpha_q}{2}\right)^2\frac{1}{n_q}\sum_{k=1}^{n_q-1}\gamma_q\left(t_{0k}^{n_q}\right)\left(1-\left(t_{0k}^{n_q}\right)^2\right)K_{pq}\left(g_p\left(t_{0j}^{n_p}\right),g_q\left(t_{0k}^{n_q}\right)\right)=-f\,,\quad j=1,\dots,n_p-1,\ p=1,\dots,m. \tag{21}$$

Solving system (21) we find the values of the unknown functions $\gamma_p^{n_p}(t)$ at the nodes $t_{0j}^{n_p}$, then by the obtained values we reconstruct the function $\gamma_p^{n_p-2}\left(t_{0k}^{n_p}\right)$ as an interpolation polynomial.

Far-field directivity pattern and total scattering cross-section. Far-field directivity pattern is a function $D_H(\phi)$, by the definition $D_H(\phi)$ is the factor depending on ϕ for the leading term of the asymptotics [6] which depends only on r and the field $u(r,\phi)$ for large r:

$$u(r,\phi)=D_H(\phi)\sqrt{\frac{2}{\pi r}}e^{i\left(kr-\frac{\pi}{4}\right)}+O\left(\frac{1}{r}\right).$$

Using the representation of the field outside of the cylinder, we derive the far-field directivity pattern. Thus,

$$u^{-}(r,\phi)=\sum_{n=-\infty}^{\infty}A_n\frac{\Gamma_n}{H_n^{(1)\prime}(ka)}H_n^{(1)}(kr)e^{in\phi},$$

where $A_n=\frac{1}{2}\sum_{q=1}^{m}\frac{1}{n_q}\left(\frac{\beta_q-\alpha_q}{2}\right)^2\sum_{j=1}^{n_q-1}\left(1-\left(t_{0j}^{n_q}\right)^2\right)\gamma_q^{n_q-2}\left(t_{0j}^{n_q}\right)e^{-ing_q\left(t_{0j}^{n_q}\right)}$, $n\in\mathbf{Z}$, and the values of the interpolation polynomial $\gamma_q^{n_q-2}\left(t_{0j}^{n_q}\right)$ at the points $t_{0j}^{n_q}$ are determined from system of linear algebraic equations (21).

Using the principal term of the asymptotic expansion of the Hankel function [4]:

$$H_n^{(1)}(x)\sim\sqrt{\frac{2}{\pi x}}e^{i\left(x-\frac{\pi n}{2}-\frac{\pi}{4}\right)},\quad \text{as } x\to+\infty,$$

we obtain:

$$u^{-}(r,\phi)\sim\sqrt{\frac{2}{\pi kr}}e^{i\left(kr-\frac{\pi}{4}\right)}\sum_{n=-\infty}^{\infty}\frac{A_n\Gamma_n e^{-i\frac{\pi}{2}n}}{H_n^{(1)\prime}(ka)}e^{in\phi},\quad \text{as } r\to\infty.$$

Hence, the directivity pattern admits representation in the form:

$$D_H(\phi)=\sum_{n=-\infty}^{\infty}\Gamma_n A_n\frac{e^{-i\frac{\pi n}{2}}}{H_n^{(1)\prime}(ka)}e^{in\phi}. \quad (22)$$

The result of numerical calculations. For some values of angular-width of the gap and for the dimensionless parameter kR, we plotted the field diagrams in the far zone and the field near the cylindrical structure. In fig. 2, *a – d* it is seen that the increase of angular-width affects the location of the inner maximum.

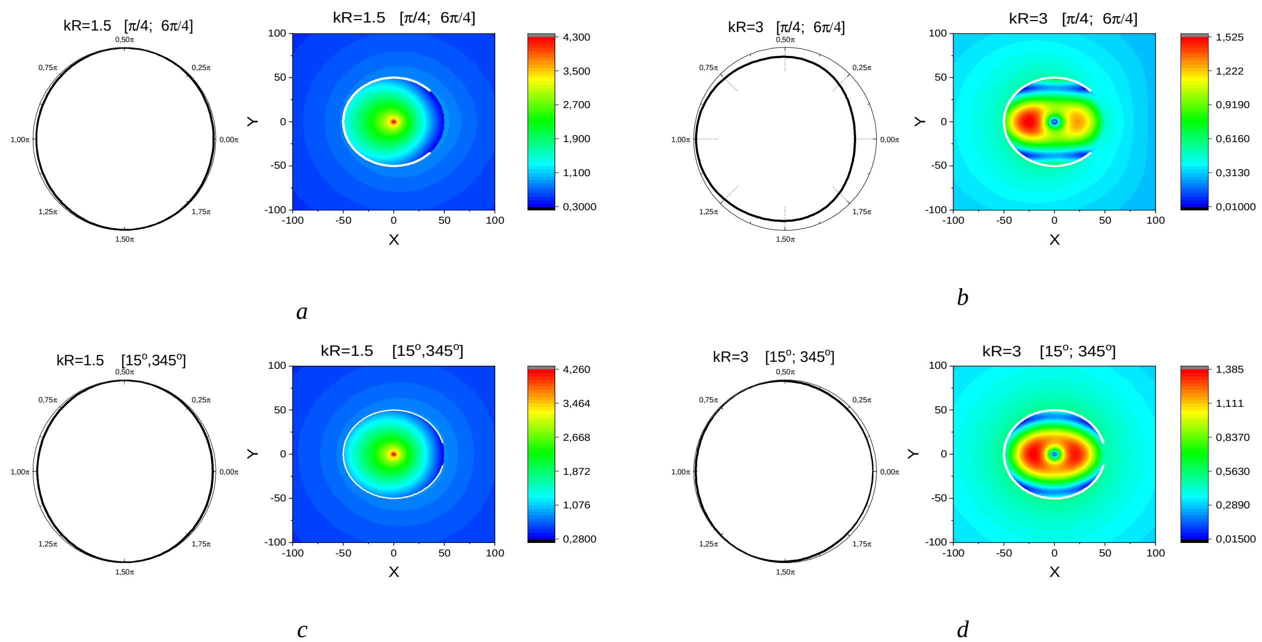


Fig. 2 – Far fields and near fields for the following parameters:
a – $\alpha=45^{\circ}$, $\beta=315^{\circ}$, $kR=1.5$; *b* – $\alpha=45^{\circ}$, $\beta=315^{\circ}$, $kR=3$;
c – $\alpha=15^{\circ}$, $\beta=345^{\circ}$, $kR=1.5$; *d* – $\alpha=15^{\circ}$, $\beta=345^{\circ}$, $kR=3$.

Summary. In this work, a boundary hypersingular integral equation was constructed for the problem of diffraction of a cylindrical wave from a round dielectric cylinder wrapped by a finite number of perfectly conducting infinite ribbons. Using the method of discrete singularities, we obtained a system of linear algebraic equations. The software was implemented and numerical calculations of some structures were carried out based on the numerical model.

Bibliography

1. *Гандель Ю. В., Ерёменко С. В., Полянская Т. С.* Математические вопросы метода дискретных токов. Обоснование численного метода дискретных особенностей решения двумерных задач дифракции электромагнитных волн : учебное пособие. Ч. 2. – Х. : ХГУ, 1992. – 145 с.
2. *Гандель Ю. В.* Введение в методы вычисления сингулярных и гиперсингулярных интегралов. – Харьков : ХНУ им. В. Н. Каразина, 2001. – 92 с.
3. *Абрамовиц М., Стиган И.* Справочник по специальным функциям с формулами, графиками и математическими таблицами // Пер. с англ. Под ред. *Диткина В. А.* и *Кармазинной Л. Н.* – Москва : Наука, 1979. – 832 с.
4. *Назарчук З. Т.* Численное исследование дифракции волн на цилиндрических структурах. – Київ : Наукова думка, 1989. – 256 с. – ISBN 5-12-000912-3.
5. *Носич А. И., Шестопалов В. П.* Электромагнитный аналог резонатора Гельмгольца // Докл. АН СССР. – 1977. – Т. 234. – №1. – С. 53 – 56.
6. *Носич А. И.* О влиянии резонансных режимов на характеристики рассеяния незамкнутого цилиндра // Радиотехника и электрон. – 1978. – Т. 23. – № 8. – С. 1733 – 1737.
7. *Духопельников С. В.* Математические модели для расчёта излучения из продольных щелей в волноводе круглого сечения // Вестник Харьковского национального университета. Сер. : Математическое моделирование. Информационные технологии. Автоматизированные системы управления. – 2005. – № 661. – Вып. 4. – С. 104 – 113.
8. *Духопельников С. В.* Математическая модель излучения цилиндрической волны из продольных щелей в прямом круговом цилиндре // Труды Международных школ-семенаров «Методы дискретных особенностей в задачах математической физики». – Орёл : ОГУ, 2005. – Вып. 4. – С. 45 – 50.
9. *Goldstone L. O., Oliner A. A.* Leaky wave antennas II : Circular waveguides // IRE Trans. Antennas Propagat. – 1961. – Vol. 9. – pp. 280 – 290.
10. *Ziolkowski R. V., Grant Brian J.* Scattering from Cavity-Backed Apertures : The Generalized Dual Series Solution of the Concentrically Loaded E-Pol Slit Cylinder Problem // IEEE Transactions on Antennas and Propagation. – 1987. – Vol. AP-35. – № 5. – pp. 504 – 528.
11. *Gandel Y. V., Kononenko A. S.* Justification of the numerical solution of a hypersingular integral equation // Differ. Equations. – 2006. – vol. 42. – no. 9. – pp. 1256 – 1262.
12. *Kostenko A. V.* Numerical method for the solution of a hypersingular integral equation of second kind // Ukrainian Mathem. Journal. – 2014. – vol. 65. – no. 9. – pp. 1373 – 1383.

References (transliterated)

1. Gandel' Yu. V., Eremenko S. V., Polyanskaya T. S. *Matematicheskie voprosy metoda diskretnykh tokov. Obosnovanie chislennogo metoda diskretnykh osobennostey resheniya dvumernykh zadach difraktsii elektromagnitnykh voln : uchebnoe posobie. Ch. II* [Mathematical problems of the method of discrete currents. Justification of the numerical method of descrete singularities for solving two dimensional problems of diffraction of electromagnetic waves: textbook. Part 2]. Kharcov, KhGU Publ., 1992. 145 p.
2. Gandel' Yu. V. *Vvedenie v metody vychisleniya singulyarnykh i gipersingulyarnykh integralov* [Introduction to the methods of evaluating singular and hypersingular integrals]. Kharkov, KhNU im. V.N. Karazina Publ., 2001. 92 p.
3. Abramowitz M., Stegun I. *Spravochnik po spetsial'nym funktsiyam s formulami, grafikami i matematicheskimi tablitsami. Per. s angl.* Ed. Ditkin V. A., Karmazina L. N. [Handbook of mathematical functions with formulas, graphs, and mathematical tables. Translated from English]. Moscow, Nauka Publ., 1979. 832 p.
4. Nazarchuk Z. T. *Chislennoe issledovanie difraktsii voln na tsilindricheskikh strukturakh* [Numerical study of wave diffraction on cylindrical structures]. Kyiv, Naukova dumka Publ., 1989. 256 p. ISBN 5-12-000912-3.
5. Nosich A. I., Shestopalov V. P. Elektromagnitnyy analog rezonatora Gel'mgol'tsa [Electromagnetic analogue of Helmholtz resonator]. *Dokl. AN SSSR*. 1977, vol. 234, no. 1, pp. 53–56.
6. Nosich A. I. O vliyanii rezonansnykh rezhimov na kharakteristiki rasseyaniya nezamknutogo tsilindra [On the impact of resonant modes on the diffraction characteristics of open cylinder]. 1978, vol. 23, no. 8, pp. 1733–1737.
7. Dukhopel'nikov S. V. Matematicheskie modeli dlya rascheta izlucheniya iz prodol'nykh scheley v volnovode krugovogo secheniya [Mathematical model for computing propagation from longitudinal slots in circular cross section waveguide]. *Vestnik Khar'kovskogo natsional'nogo universiteta. Ser. : Matematicheskoe modelirovanie. Informatsionnye tekhnologii. Avtomatizirovannye sistemy upravleniya* [Bulletin of the Kharkov National University. Series: Mathematical modeling. Information technologies. Automated control systems]. 2005, vol. 661., no. 4, pp. 104–113.
8. Dukhopel'nikov S. V. Matematicheskaya model' izlucheniya tsilindricheskoy volny iz prodol'nykh scheley v pryamom krugovom tsilindre [Mathematical model of propagation of cylindrical wave from longitudinal slots in straight circular cylinder]. *Trudy Mezhdunarodnykh shkol-seminarov "Metody diskretnykh osobennostey v zadachakh matematicheskoy fiziki"* [Proceedings of the International schools-workshops “Methods of discrete singularities for problems of mathematical physics”]. Orel, OGU Publ., 2005, no. 4, pp. 45–50.
9. Goldstone L. O., Oliner A. A. Leaky wave antennas II : Circular waveguides. *IRE Trans. Antennas Propagat*. 1961, vol. 9, pp. 280–290.
10. Ziolkowski R. V., Grant Brian J. Scattering from Cavity-Backed Apertures : The Generalized Dual Series Solution of the Concentrically Loaded E-Pol Slit Cylinder Problem. *IEEE Transactions on Antennas and Propagation*. 1987, vol. AP-35, no. 5, pp. 504–528.
11. Gandel Y. V., Kononenko A. S. Justification of the numerical solution of a hypersingular integral equation. *Differ. Equations*. 2006, vol. 42, no. 9, pp. 1256–1262.
12. Kostenko A. V. Numerical method for the solution of a hypersingular integral equation of second kind. *Ukrainian Mathem. Journal*. 2014, vol. 65, no. 9, pp. 1373–1383.

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