Proceedings of the 4th International Conference on Nonlinear Dynamics
ND-KhPI2013
June 19-22, 2013, Sevastopol, Ukraine

Numerical Calculations of Creep Damage at Cyclic Loading by Use of Tensor Damage Parameter Model

D.Breslavsky[1*], Yu. Korytko[1], V.Mietielov[1], O.Morachkovsky[1], O.Tatarinova[1]

Abstract
Creep of cyclically loaded bodies and their fracture due to creep mechanisms are studied. Creep damage equations are built by use of tensor parameter for titanium alloy. Similar equations were derived for stress cyclic varying using the method of asymptotic expansions and averaging in a period. 2d plane stress problems were solved by FEM and fields of stress, strain, displacement, damage parameter components as well as time to rupture values were obtained

Keywords
Creep, damage, cyclic loading, tensor damage parameter, FEM, plane stress, numerical simulation

[1] National Technical University 'Kharkiv Polytechnic Institute', Kharkiv, Ukraine
* **Corresponding author:** brdm@kpi.kharkov.ua

Introduction

New structural materials like light weight alloys and metallic composites are widely used in industrial applications. Their creep-damage behavior is strongly anisotropic which demands using of tensor models for description of creep strain rate as well as damage accumulation. Such models had been suggested by Murakami [1], Cordebois-Sidoroff [2], O.Morachkovsky [3] and others. In recent years these approaches were used for the case of static loading.

Otherwise, very often structural elements are loaded by joint action of static and cyclic loading from their forced vibrations. As it is known [4], in this case so-called 'dynamic creep' processes are developed in the material. They are characterized by essential acceleration of creep strain rate as well as of damage accumulation which leads to decreasing of time to rupture values in comparison with pure static loading.

Constitutive equations for description of dynamic creep-damage processes as well mathematical procedures for their adequate numerical simulation were suggested by D.Breslavsky and O.Morachkovsky [5, 6]. In this paper these approaches were applied for tensor creep-damage model [3].

1. Anisotropic creep-damage laws

Let us regard creep-damage laws for anisotropic materials which correspond to general linear tensor dependences for principal axes of symmetry of creep and damage properties of anisotropic material. Damage accumulation in this case will be described by tensor of second rank ϖ_{ij}.

Let as accept that damage tensor ϖ_{ij}, as an internal parameter, corresponds to the associated with it tensor of thermodynamic stresses in damaged media R_{ij}. For materials with initial anisotropy equivalent stress σ_V is determined by mutual invariants of stress tensor σ_{ij} with material creep properties tensors a_{ij}, b_{ijkl}, as well as equivalent stress R_V is determined by mutual invariants of tensor R_{ij} with material damage properties g_{ij}, d_{ijkl}:

$$\sigma_V=\sigma_1+\sigma_2,\ \sigma_1=a_{ij}\sigma_{ij},\ \sigma_2=\sqrt{b_{ijkl}\sigma_{ij}\sigma_{kl}};\ R_V=R_1+R_2,\ R_1=g_{ij}R_{ij},\ R_2=\sqrt{d_{ijkl}R_{ij}R_{kl}}.$$

Using above mentioned suggestions let us present the dissipation potentials for creep strain and damage parameter's rates in the following forms:

$$\dot{D}=\dot{D}(\sigma_V;\varpi,T)=\sigma_{ij}\dot{c}_{ij}\,,\quad \dot{\Omega}=\dot{\Omega}(R_V;\varpi,T)=R_{ij}\dot{\varpi}_{ij}\,. \tag{1}$$

Thereat the flow rule and damage equations can be presented using the gradiental law as follows:

$$\dot{c}_{ij}=\dot{\lambda}\frac{\partial\dot{D}}{\partial\sigma_{ij}},\quad \dot{\varpi}_{ij}=\dot{\lambda}_*\frac{\partial\dot{\Omega}}{\partial R_{ij}},$$

where scalar multipliers $\dot{\lambda},\dot{\lambda}_*$ are determined from eqs (1) after specification of the form of each potential.

For the establishing of failure criterion in invariant scalar form let us consider in a some material point the dissipation's part which is responsible for creep damage, as well as its limit value in the rupture time moment:

$$\Omega(t)=\int_0^t\dot{\Omega}dt,\qquad \Omega_*=\int_0^{t_*}\dot{\Omega}dt.$$

Let us suppose that limit value of creep damage dissipation of the material Ω_* is completely definite material characteristic. In this case, assuming as the scalar damage measure the value of $\omega(t)=\Omega(t)/\Omega_*$, the failure criterion can be written in the following form: $\omega(t_*)=1$, where t_* is the value of failure time.

By use of thermodynamic assumptions for materials with initial creep anisotropy we obtain the constitutive equations:

$$\dot{c}_{ij}=\frac{\dot{D}}{\sigma_V}(a_{ij}+\frac{b_{ijkl}\sigma_{kl}}{\sigma_2}),\ \dot{\varpi}_{ij}=\frac{\dot{\omega}}{R_V}(g_{ij}+\frac{d_{ijkl}R_{kl}}{R_2}), \tag{2}$$

where c_{ij},ϖ_{ij} are symmetric tensors of irreversible creep strains and damage; $\dot{D}=\sigma_{ij}\dot{c}_{ij}$ is dissipation due to creep: $\dot{\omega}=R_{ij}\dot{\varpi}_{ij}$ is the dissipation due to creep damage, which is referred for its limit value Ω_* at failure moment $(0\le\omega\le1)$.

By using of the principle of strain equivalence the influence of damage tensor on effective stress is considered from the following equation: $R_{ij}=\sigma_{ij}/(1-\omega)$, and besides $\dot{\omega}=R_{ij}\dot{\varpi}_{ij}$, $(0\le\omega\le1)$, $\omega(t_*)=1$. Keeping of the demands of basic thermodynamic inequality $\dot{D}+\dot{\Omega}\ge0$ dissipation potentials (1) can be rewritten in the following form:

$$\dot{D}(\sigma_V;\omega,T)=\dot{D}(R_V),\quad \dot{\Omega}(\sigma_{*V};\omega,T)=\dot{\Omega}(R_{*V}), \tag{3}$$

where $R_V=\sigma_V/(1-\omega)$, $\sigma_V=\sigma_1+\sigma_2$, $\sigma_1=a_{ij}\sigma_{ij}$, $\sigma_2=\sqrt{b_{ijkl}\sigma_{ij}\sigma_{kl}}$; $R_{*V}=\sigma_{*V}/(1-\omega)$, $\sigma_{*V}=\sigma_{*1}+\sigma_{*2}$, $\sigma_{*1}=g_{ij}\sigma_{ij}$, $\sigma_{*2}=\sqrt{d_{ijkl}\sigma_{ij}\sigma_{kl}}$ are effective stress invariants.

Within the supposed assumptions the principal directions of the material anisotropy are considered constant directly to failure, as well as surfaces of the dissipation potentials in the stress space are expanded in time proportionally to one parameter, which is connected with damage measure $0\le\omega\le1$.

Let us regard further constitutive equations (2) for materials with creep transversally isotropy.

Dissipation potentials (3) are chosen as power functions from stress invariants:

$$\dot{D} = R_V^N, \quad \dot{\omega} = R_{*V}^k / (1-\omega)^S, \tag{4}$$

where N, k, S are the constants.

Equations (2) are regarded as constitutive ones for materials with initial anisotropy and creep-damage properties asymmetry, in particular for tension and compression. If these effects are absent, then $a_{ij} = g_{ij} = 0$. In this case creep-damage equations for materials with creep transversally isotropy can be written in the following form:

$$\underline{\dot{c}} = b_{1111}^{(N+1)/2} \frac{\overline{\sigma}_2^{N-1}}{(1-\omega)^N} [B]\underline{\sigma}, \quad \underline{\dot{\varpi}} = d_{1111}^{k/2} \frac{\sigma_{*2}^{k-2}}{(1-\omega)^{k+S-1}} [D]\underline{\sigma}, \tag{5}$$

$$\dot{\omega} = d_{1111}^{k/2} \frac{\sigma_{*2}^{k}}{(1-\omega)^{k+S}}, \quad \omega(0) = 0, \quad \omega(t_*) = 1, \tag{6}$$

where $\underline{\dot{c}} = (\dot{c}_{11}, \dot{c}_{22}, 2\dot{c}_{12})^T$, $\underline{\sigma} = (\sigma_{11}, \sigma_{22}, \sigma_{12})^T$, $\underline{\dot{\varpi}} = (\dot{\varpi}_{11}, \dot{\varpi}_{22}, 2\dot{\varpi}_{12})^T$ are the vectors of creep strain rates, stresses and damage rates $\overline{\sigma}_2^2 = \underline{\sigma}^T [B]\underline{\sigma}$, $\sigma_{*2}^2 = \underline{\sigma}^T [D]\underline{\sigma}$ are the stress invariants;

$$[B] = \begin{vmatrix} 1 & \beta_{12} & 0 \\ \beta_{21} & \beta_{22} & 0 \\ 0 & 0 & 4\beta \end{vmatrix}, \quad \beta_{12} = -\frac{1}{2} b_{1111}, \beta_{22} = \frac{b_{2222}}{b_{1111}}, 4\beta = \frac{b_{1212}}{b_{1111}},$$

$$[D] = \begin{vmatrix} 1 & \delta_{12} & 0 \\ \delta_{21} & \delta_{22} & 0 \\ 0 & 0 & 4\delta \end{vmatrix}, \quad \delta_{12} = -\frac{1}{2} d_{1111}, \delta_{22} = \frac{d_{2222}}{d_{1111}}, 4\delta = \frac{d_{1212}}{d_{1111}}.$$

Further let us regard the cyclic loading, where stress tensor consists from pure static part σ_0 and fast varying part σ_1 : $\sigma = \sigma_0 + \sigma_1$. The frequency f is the frequency of forced oscillations: $\sigma_1 = \sigma^a \sin(2\pi f t)$, where σ^a is amplitude stress. Such process is referred as dynamic creep-damage process [4]. It had been mathematically described in [6] for scalar damage parameter by use of method of asymptotic expansions jointly with averaging in a period technique.

Let us use this approach for eqs (5)-(6). After introducing the small parameter $\mu = T/t_*$ we present processes in two time scales (slow t and fast $\xi = \tau/T$, where $\tau = t/\mu$,) in the following form of expansions:

$$c \cong c^0(t) + \mu c^1(\xi), \quad \omega \cong \omega^0(t) + \mu \omega^1(\xi), \tag{7}$$

where $c^0(t)$, $\omega^0(t)$, $c^1(\xi)$, $\omega^1(t,\xi)$ are the functions which coincide to basic creep-damage process in slow (0) and fast (1) time scales.

Considering the dependence of creep strain and damage parameters only from 'slow' time [5], after averaging we have:

$$\left\langle c^0(\xi) \right\rangle = \int_0^1 c^0(t) d\xi \cong c^0(t), \quad \left\langle c^1(\xi) \right\rangle = \int_0^1 c^1(\xi) d\xi \cong 0, \quad \left\langle \omega^0(\xi) \right\rangle = \int_0^1 \omega^0(t) d\xi \cong \omega^0(t),$$

$$\left\langle \omega^1(\xi) \right\rangle = \int_0^1 \omega^1(\xi) d\xi \cong 0, \quad \left\langle \varpi^0(\xi) \right\rangle = \int_0^1 \varpi^0(t) d\xi \cong \varpi^0(t), \quad \left\langle \varpi^1(\xi) \right\rangle = \int_0^1 \varpi^1(\xi) d\xi \cong 0.$$

So, by use of asymptotic expansions methods with subsequent averaging in a period of cyclic load, after transition to the case of multi-axial stress state, we obtain constitutive equations for dynamic creep-damage processes in alloys with orthotropic creep-damage properties:

$$\dot{\underline{c}} = b_{1111}^{(N+1)/2} K(A_n) \frac{\bar{\sigma}_2^{N-1}}{(1-\omega)^N} [B]\underline{\sigma}, \quad \dot{\underline{\varpi}} = d_{1111}^{k/2} H_1(A_k) \frac{\sigma_{*2}^{k-2}}{(1-\omega)^{k+S-1}} [D]\underline{\sigma}, \tag{8}$$

$$\dot{\omega} = d_{1111}^{k/2} H_2(A_k) \frac{\sigma_{*2}^{k}}{(1-\omega)^{k+S}}, \quad \omega(0)=0, \quad \omega(t_*)=1, \tag{9}$$

where $K(A_n) = \int_0^1 (1 + A_n \sin(2\pi\xi))^n \, d\xi$, $A_n = \frac{\bar{\sigma}_2^a}{\bar{\sigma}_2}$, $H_1(A_k) = \int_0^1 (1 + A_k \sin(2\pi\xi))^{k-1} d\xi$, $H_2(A_k) = \int_0^1 (1 + A_k \sin(2\pi\xi))^k \, d\xi$, $A_r = \frac{\sigma_{*2}^a}{\sigma_{*2}}$. $(\bar{\sigma}_2^a)^2 = (\underline{\sigma}^a)^T [B]\underline{\sigma}^a$, $(\sigma_{*2}^a)^2 = (\underline{\sigma}^a)^T [D]\underline{\sigma}^a$ are the invariants of cyclic stresses.

2. FEM solution for plane stress problem

Let us regard the problem statement for structural element, made from material with anisotropic creep-damage properties, which is in plane stress state. The volume V is fixed in a surface part S_1 and is loaded by traction p $\{p_1, p_2\}$ on another surface part S_2. In co-ordinate system OX_1X_2 the motion of material points under the creep conditions is described by use of Lagrange approach. The vectors of displacements $u=\{u_1, u_2\}^T$ and their rates $v=\{v_1, v_2\}^T$ are introduced. In these assumptions creep problem for the case of small displacements and strains is described by following boundary – initial value problem:

$$\sigma_{ij,j} + f_i = \rho \dot{v}_j, \quad (i,j=1,2) \quad x_1, x_2 \in V; \quad \sigma_{ij} n_j = p_i + p_i^a \sin 2\pi f t, \quad x_1, x_2 \in S_2. \tag{10}$$

$$v_1 = du_1/dt = \dot{u}_1; \quad v_2 = du_2/dt = \dot{u}_2 \tag{11}$$

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}, \quad \varepsilon_{22} = \frac{\partial u_2}{\partial x_2}, \quad \varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right), \tag{12}$$

The level of stresses which don't exceed yield limit will be considered. Let us accept, that elastic e_{ij} and creep c_{ij} strain components as well as their rates are additive:

$$\dot{\varepsilon}_{ij} = \dot{e}_{ij} + \dot{c}_{ij}, \tag{13}$$

Obtained system (10-13) has to be solved jointly with constitutive equations (8-9). Boundary conditions on surface parts $S_1 : \dot{u}_i = \dot{u}_i^*$ and S_2: $\dot{\sigma}_{ij} n_j = \dot{p}_i$ as well initial ones at t=0 considering the elastic strain-stress distribution have to be added. Constitutive equations (8-9) are added to above system.

The system (8-13) is solved by use of two time scales method jointly with averaging in a period of forced vibration *1/f*. Full description of the procedure can be found in [5,6]. The problem derives to the simplified one which is similar to problem of static loading, but with constitutive equations (8-9).

Anisotropic creep-damage behavior is simulated by FEM software had been developed in NTU 'KhPI'. Triangular linear element is used. The problem which is formulated in rates is solved:

$$[K]\dot{\underline{\delta}} = \dot{\underline{F}} + \dot{\underline{F}}_c, \tag{14}$$

where $[K] = \sum_e \int_{V^e} [B]^T [D][B] dV$ is global stiffness matrix; $\dot{\underline{F}} = \sum_e \int_{\Sigma_2^e} [N]^T \dot{\underline{p}} d\Sigma$, $\dot{\underline{F}}_c = \sum_e \int_{V^e} [B]^T \underline{\sigma}^* dV$ are the loading vectors from traction and forces which determined by creep strains.

3. Anisotropic creep-damage in titanium plates

Developed method and software were used for creep-damage modeling in titanium plates made from BT1-0 alloy which is similar to IMI125 or T40 grades. The properties of creep and long term strength of the specimens made from the plane billet in three directions were experimentally obtained for temperature T=773K by O.Morachkovsky and V.Konkin [7]. The constants values which had been obtained after data processing are the following: $b_{1111}=2{,}303\cdot 10^{-4}$, $b_{1122}=-1{,}151\cdot 10^{-4}$, $b_{2222}=1{,}924\cdot 10^{-4}$, $b_{1212}=2{,}058\cdot 10^{-4}$, $(\mathrm{MPa})^{-2N/N+1}/\mathrm{h}^{2/N+1}$; $d_{1122}=-1{,}771\cdot 10^{-5}$, $d_{2222}=3{,}324\cdot 10^{-5}$, $d_{1212}=3{,}127\cdot 10^{-5}$, $(\mathrm{MPa})^{-2}/(\mathrm{h})^{2/k}$; $k=N=5$, $s=1$. Comparison between experimental data at static loading for specimens had been cut in different directions with numerical simulations of rectangular plates in tension simulating the above test data were done at first in order to verify numerical method. A satisfactory agreement which doesn't exceed 25-32% had been obtained for time variation of strains and time to rupture values. Table 1 contains some examples of these comparisons for time to rupture values for net stress 60 MPa.

Table 1. Comparison of time to rupture values for static loading

Orientation of anisotropy axes, ϑ°	Stress, MPa	Experimental value of time to rupture t_*, h	Numerical value of time to rupture t_*, h
0	60	24	25.9
45	60	19	20.5
90	60	28.7	19.7

Further the static and dynamic creep of titanium plate (0.8mx0.8m) with a central hole (radius is equal to 0.1m) were numerically simulated. One quarter of a plate was considered and stress-strain state and damage redistribution on time were analyzed. Fig.1 contains the distribution of damage measure ω for the case of pure static loading by net stress 10MPa at the moment t=220h which is nearest to the failure moment t_*=221h. Essential stress concentration near the hole leads to failure in this area. The creep –damage behavior of the plate is strongly non-linear: increasing of the traction from 10 to 13MPa leads to decreasing of time to failure from 221h to 68.5h.

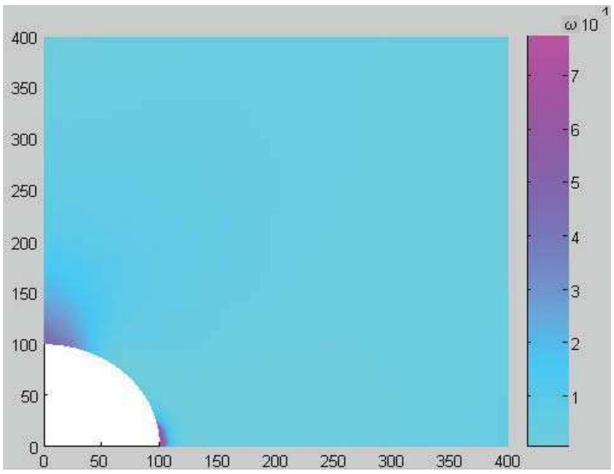

Figure 1. Damage parameter at t=220h

Let us present the results of dynamic creep simulation of the similar plate. The axial loading in this case consists from static part p_0=16.9MPa and the cyclic one which is varying due to sine law with frequency 10Hz. Let us introduce the loading parameter $L=p_0/p_a$, where p_a is amplitude value of the loading. The varying of L from 0 to 0.3 was analyzed.

The time to failure values in this loading program decreases with increasing of *L* from 0.166 h for *L*=0 to 0.122h for *L*=0.3. It happens for material with anisotropic creep-damage properties just the same in dynamic creep processes of elements with isotropic ones [6]. The character of stress and damage redistribution is similar for the static case, but both processes are run faster.

However, for smaller values of net stress, and, respectively, for smaller stress values in a plate, character of a process differs from above analyzed one. Dynamic loading accelerates the stress relaxation in area near the hole as well as in conventional dynamic creep [6], wherein the obtained stress level became so small, that rate of damage accumulation decrease significantly. For example, for p_0=13MPa the value of time to failure for *L*=0 is equal to t_*=68.5 h, but adding of small cyclic part (*L*=0.05) leads to it's increasing and t_*=72.6 h. Subsequent increasing of *L* leads to similar growth of time to failure values due to fast dynamic relaxation at first time of a process: for *L*=0.1 t_*=120.1 h.

Conclusions

An approach for the effective mathematical modeling of dynamic creep-damage processes in structural elements made from the titanium alloy with anisotropic properties is presented. The use of two time scale method with asymptotical expansions on the small parameter and subsequent averaging in a period allows us to avoid the direct integration through a cycle and simulate only static problems with flow rule and damage equation of special form. Effects of essential acceleration of strain and damage rate growth as well as stress relaxation due to dynamic creep had been established for plate with a hole made from material with anisotropic creep-damage properties.

References

[1] Murakami S. Notion of Continuum Damage Mechanics and Its Application to Anisotropic Creep Damage Theory *J. Engng. Mater. Techn,* Vol. 105, pp. 99-105, 1983.

[2] Cordebois J.P., Sidoroff F. Damage Induced Elastic Anisotropy *Mechanical Behaviour of Anisotropic Solids (*J. P. Boehler, editor), Colloque Euromech 115, Villard-de-Lans, June 19-22, Martinus Nijhoff Publishers, pp. 761- 774, 1979.

[3] Morachkovsky O.K., Pasynok M.A. Investigation of the influence of the obtained anisotropy due to precurcive creep on the creep of materials *Proc. KhSPU* , KhSPU, Kharkiv, Vol. 27, pp. 197-203, 1998 (in Russian).

[4] Taira S., Ohtani R. *Theory of High Temperature Strength of Materials,* Metallurgia, Moscow, 1986. (in Russian, translated from Japan).

[5] Breslavsky D., Morachkovsky O. A new model of nonlinear dynamic creep. *IUTAM Symposium on Anisotropy, Inhomogenity and Nonlinearity in Solid Mechanics*, Kluwer Academic Publishers, Dordrecht, pp. 161-166, 1995.

[6] Breslavsky D., Morachkovsky O. Dynamic creep continuum damage mechanics: FEM-based design analysis. *Computational Plasticity: Fundamentals and Applications. Proc. of the Fifth International Conference on Computational Plasticity held in Barselona, Spain, 17-20 March 1997*, IMNE, Barselona IMNE, Part 1, pp.1071-1076, 1997.

[7] Konkin V.N., Morachkovsky O.K. Creep and long term strength of light weight alloys with anisotropic properties. *Problems of Strength*, Vol. 5, pp. 38-42, 1987. (in Russian)