Peculiarities of Calculating Forced Electromagnets Shunt Windings Heating in Transient Modes

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Abstract—**An analytical expression is obtained that makes it possible to calculate the volumetric density of sources of transient heating of shunt windings, in which as they are heated up, the volumetric density of sources integrally decreases, but at the same time, at points with higher temperature, the source density is higher than at points with lower temperature. Using the example of temperature calculations of miniature gas distributors, it is shown that at forced heating of the electromagnets in shunt windings significant temperature drops, reaching 35% of the maximum temperature, as well as a slight effect on the heating of surrounding objects, in particular, steel of the magnetic core, are observed.**

***Index Terms*—Finite Element Method, forced heating, shunt windings.**

I. INTRODUCTION

In the International Electrotechnical Vocabulary (IEV) [1], as well as in the well-known explanatory dictionaries [2], the term “shunt winding” directly refers to electric machines, although the use of this term can naturally be extended to other objects, in which there are windings connected in parallel to the load or power source. In the same IEV, the term “shunt release” is defined as “a release energized by a source of voltage: IEV 441-16-41”. Structurally, shunt release is a device containing an electromagnet, the winding of which is connected through a switching element in parallel with a voltage source. It should be borne in mind that "release" is “a device, mechanically connected to a mechanical switching device, which releases the holding means and permits the opening or the closing of the switching device: IEV 441-15-17”.

Shunt windings are also used in undervoltage releases of circuit breakers, in various relays [3], contactors, electromagnetic clutches and brakes, blocking electromagnets and in the designs of a number of other electrical apparatus. One example of the use of shunt windings of DC electromagnets are high-speed electromagnetic valves for gas distribution devices. Such valves are used, in particular, in modern rocket and space technology as jet micromotors for the executive bodies of the rocket stabilization systems [4], as well as for controlling movement of spacecrafts in space (orientation, stabilization, orbit correction, maneuver, etc.) [5].

At the design stage of objects with DC electromagnets, it is necessary to perform calculations of the thermal state of shunt windings, the accuracy of which can affect the size and mass of the object as a whole. Despite the wide prevalence of shunt windings of DC electromagnets, there are no publications on calculation of transients of their forced heating, as far as the authors know.

The computational capabilities of modern computers make it possible to perform mathematical modelling of static and dynamic thermal processes in various objects by numerical solution of corresponding equations of mathematical physics [6]. Here, despite high performance of modern computers, solution of 3D problems causes significant difficulties both in terms of describing geometry of the object, and in terms of time spent on calculations. At the same time, absolute majority of shunt windings of a wide variety of apparatus with high degree of adequacy can be considered as axisymmetrical 2D objects in a cylindrical coordinate system, which significantly speeds up computational procedures when calculating static and dynamic heating processes of such windings.

The peculiarity of the process of forced heating of shunt windings is the presence of significant temperature differences at various points in the cross section of the winding, and hence significant differences in values of volumetric densities of heat sources. Here, this should take into account the fact that in the process of heating of the winding its resistance increases, therefore, the volumetric density of sources integrally decreases, but at the same time, at points with higher temperature at each moment in time, the density of sources is greater than at points with lower temperature.

The goal of this paper is to clarify the mathematical model and the algorithm for calculating forced heating of shunt windings.

II. DESCRIPTION OF THE TECHNIQUE PROPOSED

Carrying out thermal calculations using computer codes that use the Finite Element Method requires careful preparatory work related to description of geometry of the electromagnet on the r - z plane passing through its axis of symmetry, dividing the cross section of the electromagnet with shunt winding and objects adjacent to the electromagnet involved in heat exchange process (in aggregate, the calculation regions) into subdomains that can be considered homogeneous from the point of view of physical properties, determination of boundaries of these subdomains and differential equations describing processes in these subdomains, as well as specifying initial and boundary conditions.

Thermal calculation or determination of temperature fields ϑ in an object containing a shunt winding is performed by numerically solving a system of transient heat transfer equations within computational region. This system consists of n (corresponding to the number of subdomains) Fourier differential heat transfer equations (for subdomains with internal heat sources) and Laplace (for subdomains without internal sources). This system is the following [7], [8]:

$$c_{\mathrm{w}} \cdot \gamma_{\mathrm{w}} \cdot \frac{\partial \vartheta}{\partial t} + \mathrm{div}(-\lambda_{\mathrm{w}} \cdot \mathrm{grad}\vartheta) = q\,, \tag{1}$$

$$c_k \cdot \gamma_k \cdot \frac{\partial \vartheta}{\partial t} + \mathrm{div}(-\lambda_k \cdot \mathrm{grad}\vartheta) = 0\,,\ k = 1, 2, \ldots, n-1, \tag{2}$$

where (1) refers to the subdomain that corresponds to the cross section of the winding, and totality of equations (2) refers to other subdomains. In (1), (2): c_k is the specific heat of the material at arbitrary point of the k-th subdomain, γ_k is the material density of the k-th subdomain, λ_k is the specific thermal conductivity of the material at arbitrary point of the k-th subdomain, c_{w} is the specific equivalent heat capacity of the winding considered as a uniform body, γ_{w} is the equivalent density of the winding, λ_{w} is the equivalent thermal conductivity of the winding, q is the volumetric density of heat sources at arbitrary point of the subdomain that corresponds to the cross section of the winding.

To perform thermal calculation of the electromagnet in transient mode, it is necessary to set initial and boundary conditions [7], [8]. For the beginning of transient (t = 0), initial conditions should be formulated. We assume that before start of the heating process, the electromagnet was continuously in ambient air environment with initial temperature ϑ_{i}, therefore all parts of the electromagnet had the same temperature, that is, for all points we have the following initial conditions [7], [8]:

$$\vartheta(0) = \vartheta_{\mathrm{i}}. \tag{3}$$

Boundary conditions should be specified on the surface of calculation area. It is considered that the most correct are conditions of the third kind (the Robin conditions) [7], [8]:

$$-\lambda \cdot \mathrm{grad}_{\mathrm{n}}\vartheta = K \cdot (\vartheta - \vartheta_{\mathrm{a}})\,, \tag{4}$$

where λ is the thermal conductivity of the material of the body located near the boundary with environment, $\mathrm{grad}_{\mathrm{n}}\vartheta$ is the normal component of the temperature gradient near surface of the body, ϑ is the temperature on surface of the body, ϑ_{a} is the ambient air temperature, K is the heat transfer coefficient from surface of the body.

Some multiphysics codes provide the option to calculate the heat transfer coefficient at natural convection by solving the Navier-Stokes and thermal radiation equations, based on Stefan-Boltzmann law [8].

One of these subdomains, namely the subdomain that corresponds to cross section of the winding, differs from others in at least two ways: firstly, this subdomain is not homogeneous, because the winding consists of not one material like other subdomains (steel, insulation material, air, etc.), but of several materials with significantly different properties, at least of copper and insulation, and secondly, this subdomain is active – it contains internal heat sources with volumetric density q, which is non-uniformly distributed over cross section of the subdomain, and depends on temperature ϑ at particular point of the cross section. When performing thermal calculations, winding is usually considered as a kind of homogeneous body with some equivalent characteristics: the equivalent specific heat capacity c_{w}, the equivalent density γ_{w}, and the equivalent thermal conductivity λ_{w}.

In this paper, the authors proceed from assumption that the winding consists of only two components — copper and insulation, and volume fraction of copper is equal to the fill factor k_{w}, therefore, to calculate the equivalent density γ_{w} and the equivalent specific heat capacity c_{w}, we can recommend the following formulas:

$$\gamma_{\mathrm{w}} = \gamma_{\mathrm{cu}} \cdot k_{\mathrm{w}} + \gamma_{\mathrm{i}} \cdot (1 - k_{\mathrm{w}})\,, \tag{5}$$

$$c_{\mathrm{w}} = (c_{\mathrm{cu}} \cdot k_{\mathrm{w}} \cdot \gamma_{\mathrm{cu}} + c_{\mathrm{i}} \cdot (1 - k_{\mathrm{w}}) \cdot \gamma_{\mathrm{i}}) / \gamma_{\mathrm{w}}\,, \tag{6}$$

where γ_{cu}, γ_{i} are, respectively, the density of copper and internal insulation, c_{cu}, c_{i} are, respectively, the specific heat capacity of copper and internal insulation.

Methods for calculating the equivalent thermal conductivity λ_{w} of windings were considered in the works of a number of authors in the 30 s - 60 s of the last century [7] , [8], [9]. In those papers, empirical formulas obtained by processing experimental data for impregnated windings were presented. Among them there is the formula that we used in our calculations:

$$\lambda_{\mathrm{w}} = k \cdot \lambda_{\mathrm{i}} \cdot (d / \Delta)^p\,, \tag{7}$$

where k, p are, respectively, the empirical coefficient and the exponent (recommended values: k = 1.225; p = 0.5), λ_{i} is the thermal conductivity of internal insulation, d is the diameter of the winding wire, Δ is the thickness of the wire insulation.

In well-known classical studies on heat transfer in bodies with internal sources [7], [8], [9], two options for determining the volumetric density q of the heat sources are usually considered: either as some average constant values:

$$q = \mathrm{const} \tag{8}$$

or as linear dependencies:

$$q = q_0 \cdot (1 + \alpha_0 \cdot \vartheta), \tag{9}$$

where q_0 is the density of heat sources at temperature 0 °C, α_0 is the temperature coefficient of resistance of material of the winding wire, referred to temperature 0 °C, ϑ is the temperature at arbitrary point of the body.

At calculating such windings, the following factors should be taken into account: 1 – when the winding is heated, its resistance increases, and therefore, at constant supply voltage U, the power P and the average value of the volumetric density of heat sources decrease, but 2 – at specific time at point of the cross section of the winding with higher temperature, the resistance of the turn will be greater than at point with lower temperature, therefore, the volumetric density of heat sources at point with higher temperature will be greater than at point with lower temperature, too.

We take into account these two factors and the assumption of a uniform distribution of winding copper, and hence the current i over the surface S of the cross section of the winding or winding space of height a and width b, the area of which is equal to the product $a \times b$ (Fig. 1), that in each elementary area of size drdz copper occupies a part equal to the fill factor k_{w}, and the rest of the region is insulation.

Volumetric density of heating power source at arbitrary point with coordinates (r, z) on the winding section plane is equal to ratio of the elementary power dP allocated in the volume element d$V = l \cdot$ d$r \cdot$ dz, where l is the length of the current tube at this point, to value of this volume:

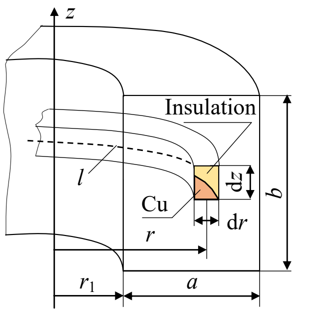

Figure 1: To determination of volumetric density of the heat sources in the winding.

$$q = \frac{\mathrm{d}P}{\mathrm{d}V}. \quad (10)$$

Volume dV is determined by the expression

$$\mathrm{d}V = l \cdot \mathrm{d}r \cdot \mathrm{d}z, \quad (11)$$

and the power dP can be determined using the current element di and the conductivity dG of elementary current tube:

$$\mathrm{d}P = \frac{(\mathrm{d}i)^2}{\mathrm{d}G}. \quad (12)$$

Expression for definition of dG can be represented as:

$$\mathrm{d}G = \frac{k_\mathrm{w} \cdot \mathrm{d}r \cdot \mathrm{d}z}{\rho_0 \cdot (1 + \alpha_0 \cdot \vartheta) \cdot l}, \quad (13)$$

where ρ_0 is the specific resistance of winding material at temperature 0 °C.

Calculated equivalent current density J in the cross section of the winding:

$$J = \frac{i \cdot N}{S} = \frac{U \cdot N}{R \cdot S}, \quad (14)$$

where i is the current in the winding at arbitrary time, R is the resistance of the winding at corresponding time, N is the number of turns of the winding, U is the voltage at the terminals of the winding.

Using (14), we can determine the current element

$$\mathrm{d}i = J \cdot \mathrm{d}r \cdot \mathrm{d}z = \frac{U \cdot N}{R \cdot S} \cdot \mathrm{d}r \cdot \mathrm{d}z, \quad (15)$$

and the elementary power:

$$\mathrm{d}P = \frac{(\mathrm{d}i)^2}{\mathrm{d}G} = \frac{U^2 \cdot N^2 \cdot \rho_0 \cdot (1 + \alpha_0 \cdot \vartheta) \cdot l \cdot \mathrm{d}r \cdot \mathrm{d}z}{R^2 \cdot S^2 \cdot k_\mathrm{w}}, \quad (16)$$

Substituting (16) into (11), we obtain:

$$q = \frac{\mathrm{d}P}{\mathrm{d}V} = \frac{U^2 \cdot N^2 \cdot \rho_0 \cdot (1 + \alpha_0 \cdot \vartheta) \cdot l \cdot \mathrm{d}r \cdot \mathrm{d}z}{R^2 \cdot S^2 \cdot k_\mathrm{w} \cdot l \cdot \mathrm{d}r \cdot \mathrm{d}z} =$$

$$= \frac{U^2 \cdot N^2 \cdot \rho_0 \cdot (1 + \alpha_0 \cdot \vartheta)}{R^2 \cdot S^2 \cdot k_\mathrm{w}}. \quad (17)$$

As a result of simple transformations, we obtain the final expression for determining volumetric density of heat sources at arbitrary point of the winding section at arbitrary point in time:

$$q = \frac{U^2}{R_\mathrm{i} \cdot V} \cdot \frac{(1 + \alpha_0 \cdot \vartheta) \cdot (1 + \alpha_0 \cdot \vartheta_\mathrm{i})}{(1 + \alpha_0 \cdot \vartheta_\mathrm{av})^2}, \quad (18)$$

where R_i is the winding resistance at the initial temperature; ϑ_av is the average temperature of the winding; V is the winding volume:

$$V = a \cdot b \cdot \pi \cdot (2 \cdot r_1 + a), \quad (19)$$

where r_1 is the inner diameter of winding space (Fig. 1).

For windings with known dimensions and winding data, (18) can be written as:

$$q = \frac{U^2}{R_0 \cdot V} \cdot \frac{(1 + \alpha_0 \cdot \vartheta)}{(1 + \alpha_0 \cdot \vartheta_\mathrm{av})^2}, \quad (20)$$

where R_i is the winding resistance at temperature 0 °C.

Formula (20) can be used in the calculations of both non-stationary and stationary temperature fields.

III. Numerical Results Obtained

As an example, we present results of thermal calculations of one of designed gas distribution devices. Calculations are carried out by numerical solution of the heat transfer equations (1) and (2) with initial condition (3) and boundary conditions (4). Heat transfer coefficients took into account natural convection (solving the Navier-Stokes equations) and thermal radiation (Stefan-Boltzmann law). The calculations took into account features of heating shunt windings discussed above.

Initial data:

- area of the winding space of the winding – $S = 72.6 \cdot 10^{-6}$ m^2 (72.6 mm^2)
- winding volume – $V = 4650 \cdot 10^{-9}$ m^3 (4650 mm^3)
- number of turns of the winding – $N = 1155$
- winding wire diameter – $d = 0.2 \cdot 10^{-3}$ m
- winding fill factor – $k_\mathrm{w} = 0.5$
- ambient air temperature, initial winding temperature – $\vartheta_\mathrm{a} = 35$ °C
- winding resistance at temperature 35°C – $R_{35} = 42.87$ Ω
- ambient air pressure – $p_\mathrm{a} = 0.1$ MPa
- winding supply voltage – $U = 33.5$ V
- temperature coefficient of resistance wire (copper) at 0 °C – $\alpha_0 = 0.00423$ K^{-1}

Sketch of the gas distribution device and its geometric model are shown in Fig. 2.

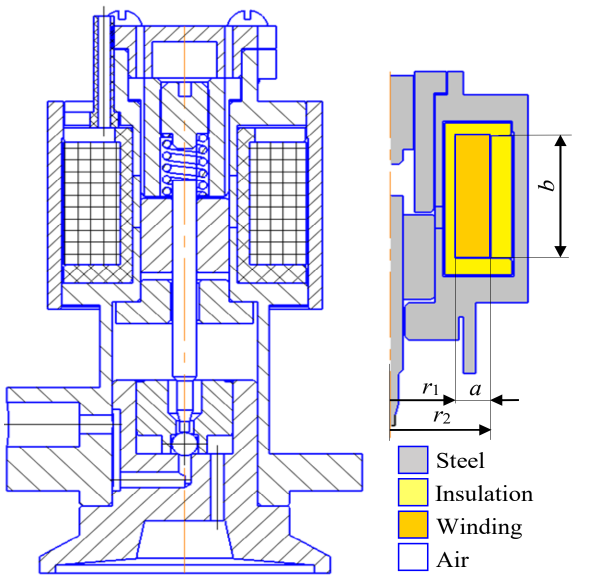

Figure 2: The sketch of the gas distribution device and its geometric model.

Mode of operation of the gas distribution device is short-term, the operating time is 70 s. In accordance with technical requirements, the average winding temperature at maximum allowable voltage of power supply (33.5 V) should not exceed 150 °C.

Figure 3 shows the calculated curve of the transient of average winding temperature (solid line). As it can be seen, average temperature of the winding during 70 s of heating increases from 35 °C to 112 °C and does not reach maximum permissible value. On the same graph, solid lines show curves demonstrating how the resistance *R* of the winding and the power *P* dissipated in it change in time.

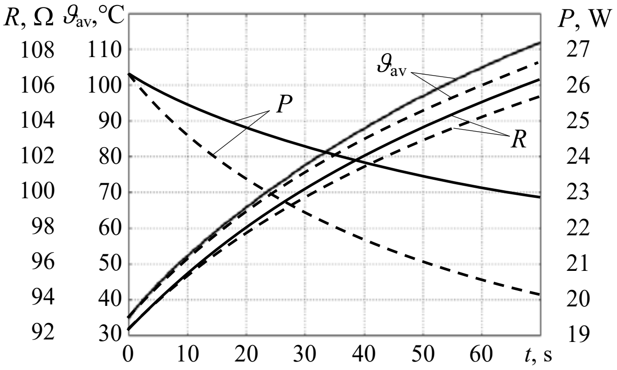

Figure 3: Transients of the average temperature of the winding, the resistance of the winding and the power dissipated in it.

A characteristic feature of this calculation is that heating is calculated not for a separate winding, but for the entire electromagnet and adjacent objects, taking into account heat exchange between the winding, magnetic core and adjacent objects.

At forced heating of shunt windings, significant temperature drops are observed both inside and outside the winding, as evidenced by the picture of the temperature field of the electromagnet 70 s after the beginning of the transient, shown in Fig. 4.

It must be emphasized that the picture of temperature field presented in Fig. 4, refers to the entire electromagnet, and from this picture it is difficult to separate the temperature distribution directly in the winding and estimate the temperature rise in it. Such estimation can be done by graphs presented in Fig. 5 (solid lines) showing temperature distribution along the winding perimeter, as well as radial temperature distribution in the cross section of the winding with maximum temperature.

From these graphs it can be seen that in this winding, despite its miniature (winding thickness is only 4.4 mm, and width is 16.5 mm), the temperature rise in it 70 s after the start of forced heating is 29.6 °C with average temperature of 112 °C, the maximum temperature of 120.6 °C and the lowest temperature of 91 °C at the corner point.

Such a significant temperature rise is characteristic specifically for forced heating process of the winding, in which the objects surrounding the winding, first of all the magnetic core, due to the significant heat capacity, do not have time to heat up.

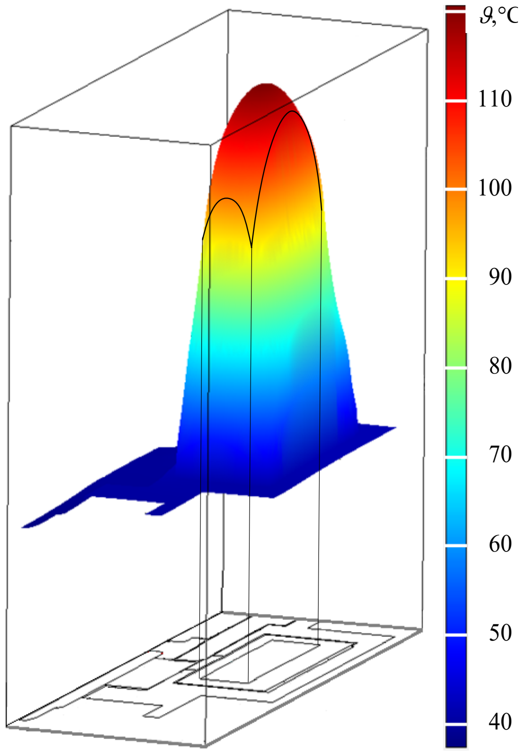

Figure 4: The spatial image of the picture of temperature field of the electromagnet 70 s after the beginning of the transient.

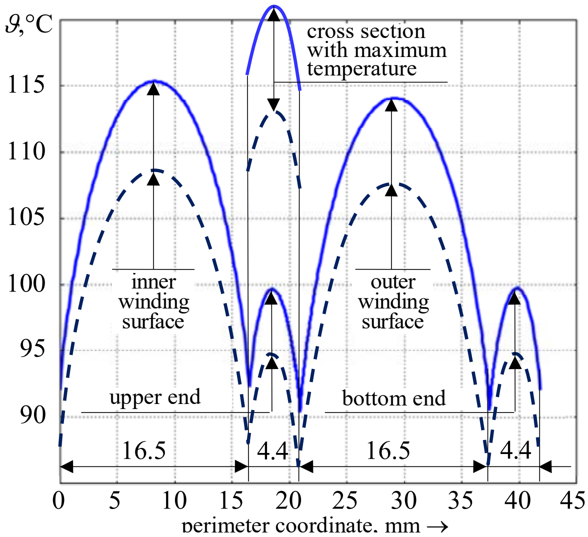

Figure 5: Temperature distribution around the perimeter of the electromagnet winding and in the cross section with the maximum temperature 70 s after the beginning of the transient.

As can be seen from the picture of temperature field of the electromagnet shown in Fig. 4, 70 s after the beginning of the transient, the temperature of the magnetic core increased by no more than 5 °C and, thus, does not perform the function of a radiator, which helps to cool the winding and equalize the temperature at various points of its cross section.

The initial value of volumetric density of heat sources at all points in the cross section of the winding is constant: $q_i = U^2 / (R_i \cdot V) = 5.63 \cdot 10^6$ W/m³. 70 s after the beginning of transient, the volumetric density of heat sources is distributed irregularly over the cross section of the winding. The graphic picture of this distribution for the considered example is shown in Fig. 6.

Attention is drawn to the fact that degree of this irregularity is not very significant. For example, if the maximum temperature value (122 °C) in the cross section of the winding (Fig. 5) exceeds the lowest temperature value (91 °C) by 34%, then the maximum value of volumetric density of heat sources ($5.00 \cdot 10^6$ W/m^3) exceeds the smallest value ($4.75 \cdot 10^6$ W/m^3) by only 5.2%.

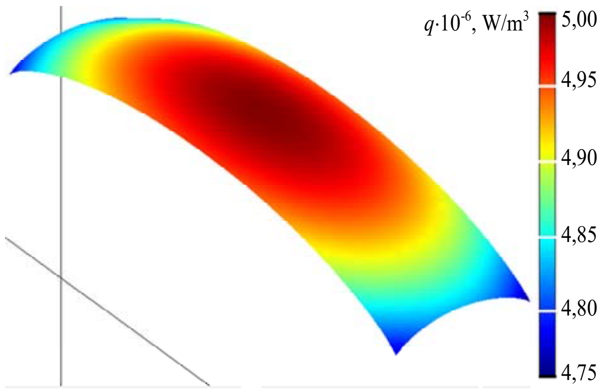

Figure 6: Volumetric density of heat sources distribution 70 s after the beginning of the transition process.

Above calculations took into account specifics of heating of shunt windings — integral decrease in the volumetric density of sources as the winding is heated up and irregular distribution of the source density over the cross section of the winding at any given time, namely, at points with higher temperature the source density is greater than at points with lower temperature.

It is of interest to compare obtained results with results of calculations, provided that at each particular moment in time the density of heat sources is assumed to be the same at all points of the cross section. In this case, the density of heat sources should be calculated by the formula:

$$q = \frac{U^2}{R \cdot V} = \frac{U^2 \cdot (1+\alpha_0 \cdot \vartheta_1)}{R_1 \cdot (1+\alpha_0 \cdot \vartheta_{av}) \cdot V} = \frac{U^2}{R_0 \cdot (1+\alpha_0 \cdot \vartheta_{av}) \cdot V} \ . \quad (21)$$

In this case, at calculating heating of the electromagnet with shunt winding, the volumetric density of heat sources, decreasing in time from initial value ($5.00 \cdot 10^6$ W/m^3), at each moment in time is distributed irregularly over the winding cross section and 70 s after the beginning of the transient drops to $4.33 \cdot 10^6$ W/m^3 which is 11% less than the average q value ($4.88 \cdot 10^6$ W/m^3) in the first version.

Results of calculation of transients of the change in the average temperature, resistance of the winding and the power released by it are shown in Fig. 3 by dotted lines. In this case, the average temperature 70 s after the beginning of the transient is equal to 105.6 °C at the maximum temperature of 113.3 °C and the lowest temperature of 82 °C, which is 7% - 8% less than values obtained in calculations using (20) in determination q values.

The results of calculation of temperature on the perimeter of the winding (Fig. 5, dashed lines) turn out to be about as small, although the value of the temperature difference between the most and least heated points remains almost unchanged.

Figure 7 shows the calculated temperature distribution curves in sections with maximum temperatures 70 s after the beginning of the transition process, obtained using (20) to determine q (dotted line), and using (21) to determine q (dashed line).

In the considered examples, 70 s after the beginning of the transient, a significant part of the heat dissipated in the winding is given to the insulation frame and the compound filling the space between the outer part of the winding and the yoke of the electromagnet, where the temperature rises from 35 °C to 70 °C - 100 °C. Therefore, forced heating process in these examples cannot be considered as adiabatic. Calculations show that during adiabatic heating, winding temperature 70 s after the beginning of transient reaches 162 °C, which is much higher than calculated average temperature (112 °C).

Figure 7 shows one more curve – the result of calculating temperature distribution in the cross section with the maximum temperature in stationary mode (thin line). This calculation was carried out at reduced voltage of the power source of 10.15 V. This voltage was chosen so that the maximum temperature value in stationary mode was equal to the maximum temperature under conditions of forced heating 70 s after the beginning of the transient (120.6 °C) when the power supply voltage is 33.5 V. As you can see, under conditions of forced heating, the temperature of the magnetic core increased by only 5 °C - 6° C, while under conditions of stationary heating, the temperature of the magnetic core increased significantly – to 106 °C - 107 °C , and the temperature rise in the winding did not exceed 5 °C - 6 °C.

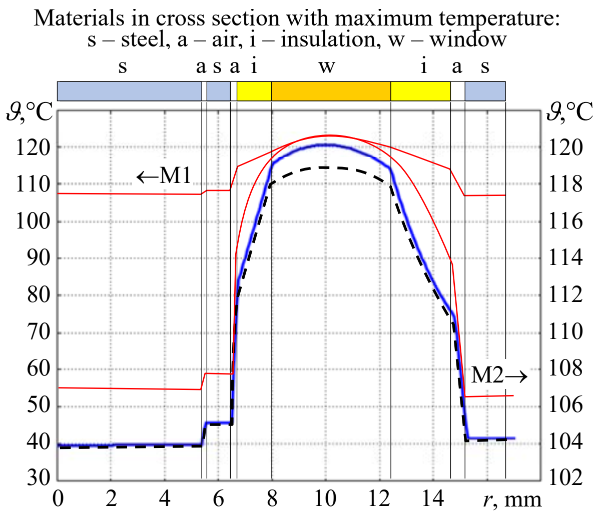

Figure 7: Comparison of temperature distribution curves in cross section with maximum temperature in conditions of forced and stationary heating.

It should be borne in mind that thermal calculations are not an end in themselves – they allow to form initial data for electromagnetic calculations and for determining traction forces and response times, which in many cases is the main result of the calculation of electromagnet. As a result, the errors made during thermal calculations lead to errors in subsequent electromagnetic, force and dynamic calculations.

For example, lowering of the result of calculating the average temperature leads to overestimation of the result of calculation of the traction force of electromagnet, and as a result, the designed electromagnet may turn out to be inoperable. Calculating the temperature fields of the windings, including at their forced heating, by numerical solution of differential heat transfer equations using the Finite Element Method taking into account the approaches considered in this paper, improves accuracy of calculating thermal and, as a result, electromagnetic, force and dynamic characteristics of electromagnets, and increases quality of their design.

IV. Conclusions

1. An analytical expression is obtained that allows to calculate the volumetric density of sources of transient heating of shunt windings, in which as they are heated, the volumetric density decreases integrally, but at the same time, at points with higher temperatures, the source density is greater than at points with less temperature.

2. Calculations carried out show that at the forced heating of electromagnets, in shunt windings significant temperature rises, reaching 35% of the maximum temperature, as well as a weak effect on the heating of objects surrounding the winding, in particular, steel of the magnetic core, are observed.

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