# MATHEMATICAL MODELING OF THE PROFILE OF A GEAR CUTTING ROLLING TOOL FOR MACHINING OF NON-INVOLUTE GEAR WHEELS

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**ABSTRACT:** *In this article, the relevance of the study of gears with a complex non-involute profile of the tooth flanks is justified. For the subsequent study of non-involute gearings, the task of mathematical modeling of the profile of a gear-cutting rolling tool for machining of non-involute gear wheels is solved. As a profile of the tooth flank of a rack, a certain section of one of the previously simulated flat kinematic curves is considered. Based on the algorithm for calculating the profile of envelope surfaces, a method for geometric analysis of the process of surfaces shaping by rolling gear cutting tools has been developed. Using computer graphics, the sequential shaping of the teeth space of non-involute gear wheels processed by rolling gear cutting tools with a given tooth profile was studied.*

**KEYWORDS:** shaping of gear wheels, mating surfaces, tooth profile, flat kinematic curve.

## 1. INTRODUCTION

The most common mechanical transmissions which are used in mechanical engineering are gears with an involute profile of the teeth flanks. Transmissions composed of such wheels differ in a number of advantages: they have a wide range of possible applications; maintain the constancy of angular velocities and gear ratio even in the presence of fluctuations in the axle distance; satisfy the condition of change of wheels with a different number of teeth, but with the same module; maintain correct engagement when regrinding; their manufacture is high-tech (this is due to the versatility and simplicity of the profile of the gear cutting tool).

However, along with certain advantages, involute gears have a number of significant disadvantages. These include: a large pressure coefficient between the teeth, i.e. high specific pressures on the teeth flank due to the small radii of their curvature and hence the insufficient ability to contact strength, poor conditions for the transfer of forces; a large coefficient of sliding between the teeth and hence significant friction losses in engagement, large wear with insufficient lubrication; low coefficient of overlap of the wheels and hence the insufficient smoothness of engagement, the inability to use a small number of teeth, the large dimensions of the transmission from the point of view of placing it in the overall design of the mechanism; the sensitivity of the wheels to manufacturing and installation errors, as well as elastic deformations of shafts, housings and other parts due to linear, rather than point contact of the teeth. Therefore, one of the current trends is the study of gears with a complex non-involute profile of the teeth flanks, which in some applications have advantages over involute gears and are deprived of some of their disadvantages.

There are two methods of cutting the teeth of cylindrical gear wheels: the copy method and the rolling method. The profile of the tool working by the rolling method does not depend on the number of teeth of the machined wheel, therefore, with the same tool, you can cut gear wheels with any number of teeth. The accuracy of the wheels made by the rolling method is significantly higher than the accuracy of the wheels made by the copy method.

In this paper, we solve the task of mathematical modeling of the profile of a gear cutting rolling tool for machining of non-involute gear wheels.

## 2. GEOMETRIC MODELING OF PLANE KINEMATIC CURVES

The authors of the article previously solved the task of geometric modeling of plane kinematic curves as potential profiles of the teeth flanks of gear cutting tools for shaping of non-involute gear wheels [1, 2].

To generalize the geometric modeling of curves, the mathematical apparatus of multi-parameter mappings of space was used, developed for solving the issues of surface shaping by cutting by Dr. Eng., Prof. of NTU "KhPI" B.A. Perepelitsa [3], as well as a generalized unified mapping structure for working and machine gearings, proposed by PhD, Sen. Staff Scientist of ISM NASU (Kiev) A.V. Krivosheya [4]. In this case, the kinematic curve was considered as a continuous trajectory of the complex motion of a point in a three-link gearing.

The use of a generalized unified structure and methods of multi-parameter mappings made it possible to model the field of various plane curves by the structural method without deriving their specific analytical equations [1, 2].

## 3. FORMATION OF THE INITIAL CONTOUR OF A RACK TOOL FOR THE MACHINING OF NON-EVOLVENT GEAR WHEELS

The task of mathematical modeling of the tooth profile of a gear-cutting tool for machining of non-involute gear wheels is solved.

In the process of cutting of gear wheels by a gear rack with a standard profile, the involute profile of the wheel teeth is reproduced [5]. In the general case, not only sections of straight lines, but also other arbitrary types of curves (for example, kinematic) can be considered as a profile of a rack tooth flanks [1, 2]. The use of such a tool allows to cut the wheels with a more complex non-involute teeth profile.

Let's consider some section of one of the previously modeled flat kinematic curves as a profile of the rack tooth flank. In the selected frame $x_1y_1z_1$ each curve point can be defined by a radius vector $m_{r_1}$ of its coordinates $x_1$, $y_1$, $z_1$ and the geometric properties of the curve at this point, including:

- projections of the tangents vectors $K_x$, $K_y$, $K_z$ and normals vectors $N_x$, $N_y$, $N_z$;

- unit vectors of tangents $\vec{\kappa}$ and normals $\vec{n}$, their directions are set using the direction cosines of the vectors:

$$\cos(\vec{x}_1 \wedge \vec{\kappa}),\ \cos(\vec{y}_1 \wedge \vec{\kappa}),\ \cos(\vec{z}_1 \wedge \vec{\kappa}),$$
$$\cos(\vec{x}_1 \wedge \vec{n}),\ \cos(\vec{y}_1 \wedge \vec{n}),\ \cos(\vec{z}_1 \wedge \vec{n});$$

- curvature radius of the curve $\rho$ (or curvature $\frac{1}{\rho}$).

The initial producing rack contour with the profile of the tooth flank in the form of a certain kinematic curve is shown in fig. 1. In a normal section $x_1y_1$, the profile of the rack tooth flank is defined by arrays of coordinates $x_{1j}$, $y_{1j}$ ($j = \overline{1, n}$), where $n$ – the number of points of the curve. The point located on the midline of the staff has a number $n_{\mathrm{mid}}$.

The angle of the tooth flank profile $\alpha_j$ is variable. The value $\alpha_j$ at a particular point in the profile is defined as the angle between the negative direction of the axis $x_1$ and the direction of the tangent vector to the profile at that point. Knowing the direction cosines of the tangent vectors at the profile points, we can determine $\alpha_j$ from the relation

$$\alpha_j = arctg\left( \frac{\cos(\vec{y}_1 \wedge \vec{\kappa})_j}{-\cos(\vec{x}_1 \wedge \vec{\kappa})_j} \right), \ j = \overline{1,n}. \tag{1}$$

The angles of the profile at the boundary points of tooth flank are called $\alpha_{01}$ and $\alpha_{02}$.

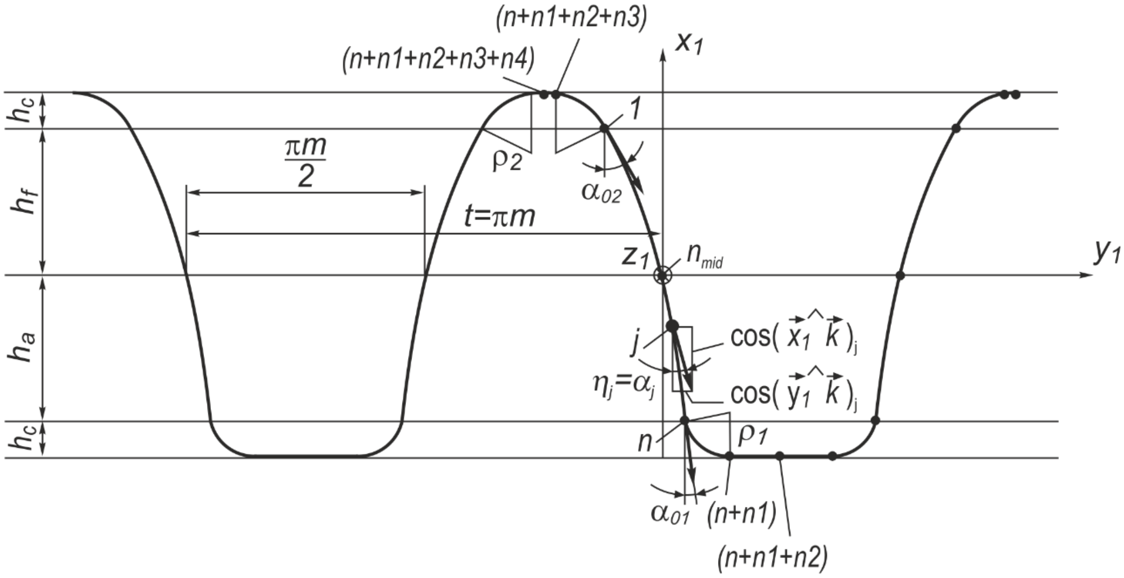


***Fig. 1.*** *The initial producing contour of a rack tool for the machining of non-evolvent gear wheels*

As can be seen from fig. 1, on the tooth flank the angle of the profile $\alpha$ coincides with the angle of inclination of the tangent to the profile $\eta$ :

$$\eta_j = \alpha_j, \ j = \overline{1,n}. \tag{2}$$

The full profile of the rack tooth, in addition to the sections corresponding to the tooth flanks, also contains rounding sections on the additional protrusion of the tooth and fillet and straight sections on the head and leg of the tooth. Let's determine the coordinates of the points in these sections and the values of the angle $\eta$ in them.

Angles $\alpha_{01}$ and $\alpha_{02}$ determine the values of the rounding radii at the boundary points of the tooth flank profile (we will call them $\rho_1$ and $\rho_2$). They are generally different in value and can be determined from the relations:

$$\rho_1 = \frac{h_c}{1 - \sin \alpha_1}; \quad \rho_2 = \frac{h_c}{1 - \sin \alpha_2}. \tag{3}$$

Let's consider the rounding on the profile of the additional protrusion of the tooth. By analogy with the tooth flank, we will consider it as a discrete set of points. Let the number of points in this section be equal $n1$. Then the coordinates of these points and the values of the angle $\eta$ in them for geometric reasons can be calculated using the following formulas (fig. 1):

$$x_{1(j+n)} = x_{1n} + \rho_1 \sin\alpha_1 - \rho_1 \sin\left(\alpha_1 + \frac{\pi/2-\alpha_1}{n1-1}(j-1)\right);$$
$$y_{1(j+n)} = y_{1n} + \rho_1 \cos\alpha_1 - \rho_1 \cos\left(\alpha_1 + \frac{\pi/2-\alpha_1}{n1-1}(j-1)\right); \quad (4)$$
$$\eta_{(j+n)} = \alpha_1 + \frac{\pi/2-\alpha_1}{n1-1}(j-1);$$
$$j = \overline{1, n1}.$$

Let the rectilinear section of the profile on the head of the rack tooth contains $n2$ points. Their coordinates and values of the angle $\eta$ at these points can be determined from the following equalities:

$$x_{1(j+n+n1)} = x_{1(n+n1)};$$
$$y_{1(j+n+n1)} = y_{1(n+n1)} + \frac{(j-1)\left|t/4 - y_{1(n+n+1)}\right|}{n2-1}; \quad (5)$$
$$\eta_{(j+n+n1)} = \eta_{(n+n1)} = \pi/2;$$
$$j = \overline{1, n2}.$$

Similarly, the coordinates of the points and values of the fillet profile angle $\eta$ containing $n3$ points are determined:

$$x_{1(j+n+n1+n2)} = x_{11} - \rho_2 \sin\alpha_2 + \rho_2 \sin\left(\alpha_2 + \frac{\pi/2-\alpha_2}{n3-1}(j-1)\right);$$
$$y_{1(j+n+n1+n2)} = y_{11} - \rho_2 \cos\alpha_2 + \rho_2 \cos\left(\alpha_2 + \frac{\pi/2-\alpha_2}{n3-1}(j-1)\right); \quad (6)$$
$$\eta_{(j+n+n1+n2)} = \alpha_2 + \frac{\pi/2-\alpha_2}{n3-1}(j-1);$$
$$j = \overline{1, n3}.$$

For a straight section of the profile on the tooth leg containing $n4$ points

$$x_{1(j+n+n1+n2+n3)} = x_{1(n+n1+n2+n3)};$$
$$y_{1(j+n+n1+n2+n3)} = y_{1(j+n+n1+n2+n3)} - \frac{(j-1)\left|y_{1(j+n+n1+n2+n3)} + t/4\right|}{n4-1}; \quad (7)$$
$$\eta_{(j+n+n1+n2+n3)} = \eta_{(j+n+n1+n2+n3)} = \pi/2;$$
$$j = \overline{1, n4}.$$

For a symmetrical section of the rack tooth profile, also containing $(n+n1+n2+n3+n4)$ points, we obtain:

$$x_{1(j+n+n1+n2+n3+n4)} = x_{1j};$$
$$y_{1(j+n+n1+n2+n3+n4)} = -y_{1j} + t/2; \quad (8)$$
$$\eta_{(j+n+n1+n2+n3+n4)} = \pi - \eta_j;$$
$$j = \overline{1, (n+n1+n2+n3+n4)}.$$

Thus, the full tooth profile of a rack tool in a normal section $x_1 y_1$ is defined by a set of $2(n+n1+n2+n3+n4)$ coordinates of points $x_{1j}$ and $y_{1j}$ and a set of values of the angle of inclination of the tangent to the profile $\eta_j$ at these points.

## 4. GEOMETRIC ANALYSIS OF THE SURFACES SHAPING PROCESS BY ROLLING GEAR CUTTING TOOLS

Based on the previously developed algorithm for calculating the profile of envelope surfaces [6], a method for geometric analysis of the process of surfaces shaping by rolling gear cutting tools has been developed. In this case, a new structural approach to find the formable and tool surfaces as envelopes, which does not require the derivation of specific analytical equations, is used [7, 8].

Using computer graphics, the sequential shaping of the space between the teeth of the gear wheel is studied. A plane geometric task is being solved. The workpiece and the shaping tooth of the tool (for example, tool rack) can be considered as geometric figures, i.e. bounded subsets of plane points. As noted earlier, the boundaries of the tooth of the tool can be not only straight line segments, but also various other types of curves. In motion relative to the workpiece, the tool tooth as a geometric figure sweeps on the plane a region representing a set of trajectories of points. The boundaries of this area. which is swept, are either the trajectories of single points of the tooth, or the envelopes of certain curves that bound the tooth.

In fig. 2, a, b the initial profile of the tool rack with a concave profile of the tooth flanks and the successive shaping of the space between the teeth of the gear wheel with the module *m*=5 and the number of teeth *z*=20 is shown. In the figures, the workpiece points, in which, at the current time, the condition of touching the surfaces is fulfilled, are highlighted. The combination of these points represents the envelopes of certain curves that limit the tool tooth. They are marked in fig. 2, c. Fig. 2, d images the boundaries of the desired space between the wheel teeth. Fig. 3 and 4 represent the space shaping between the teeth of gears with *m*=5 and *z*=20 by tool rack with a convex and concave-convex profile of the tooth flanks, respectively.

The proposed method allows to analyze the process of shaping the surface of the gear wheel, detect the appearance of undercuts, and also determine the range of gear wheels which can be obtained using a specific rolling tool.

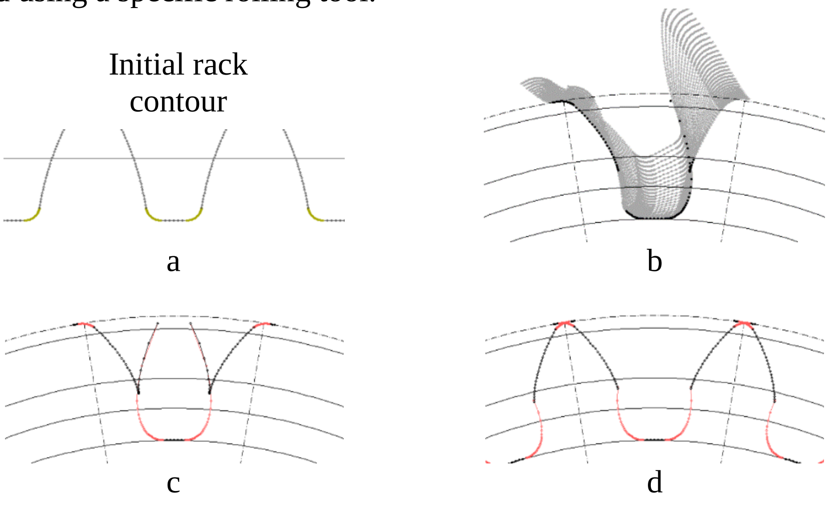


***Fig. 2.*** *The shaping of the space between the teeth of the gear wheel with m=5 and z=20 by tool rack with a concave profile of the tooth flanks*

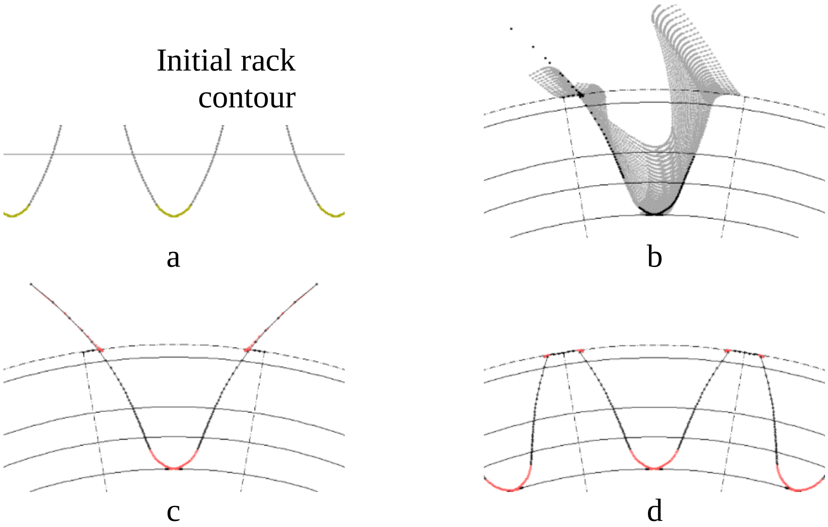


***Fig. 3.*** *The shaping of the space between the teeth of the gear wheel with m=5 and z=20 by tool rack with a convex profile of the tooth flanks*

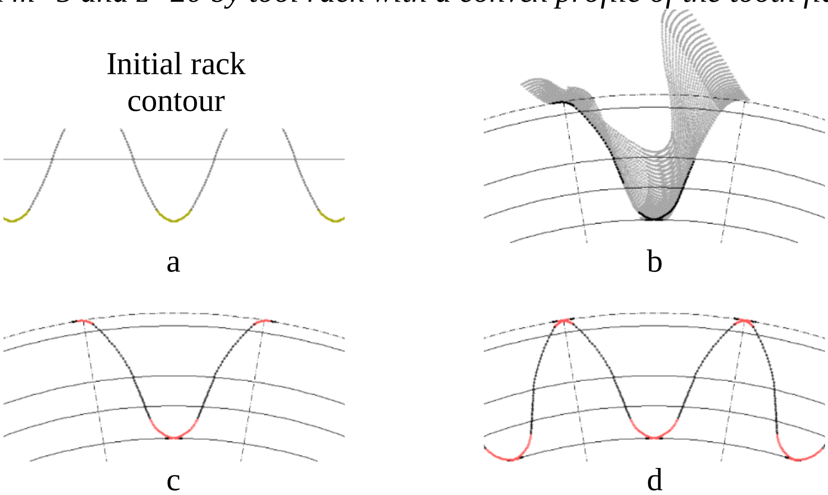


***Fig. 4.*** *The shaping of the space between the teeth of the gear wheel with m=5 and z=20 by tool rack with a concave-convex profile of the tooth flanks*

**5. CONCLUSION**

A mathematical model for calculating the tooth profile of a gear-cutting tool for machining of non-involute gear wheels has been developed. As a profile of the rack tooth flank, a certain section of one of the modeled kinematic curves is considered. Based on the algorithm for calculating the profile of envelope surfaces, a method for geometric analysis of the process of shaping surfaces by rolling gear cutting tools has been developed. Using computer graphics, the sequential shaping of the space between the teeth of non-involute gear wheels processed by rolling-in gear-cutting tools with a given tooth profile has been investigated.

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