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# METHOD OF INTEGRO-DIFFERENTIAL EQUATIONS FOR INTERPRETING THE RESULTS OF VERTICAL ELECTRICAL SOUNDING OF THE SOIL

*The paper is devoted to* ***the problem*** *of determining the geoelectric structure of the soil within the procedure of testing the grounding arrangements of existing power plants and substations to the required depth in conditions of dense development. To solve the problem, it was proposed to use the Schlumbergers method , which has a greater sounding depth compared to the Wenner electrode array.* ***The purpose*** *of the work is to develop a mathematical model for interpreting the results of soil sounding by the Schlumberger method in the form of a four-layer geoelectric structure.* ***Methodology****. To construct a mathematical model, it is proposed to use the solution of a particular problem about the field of a point current source, which, like the observation point, is located in the first layer of a four-layer soil. Based on this expressions, a system of linear algebraic equations of the 7-th order with respect to the unknown coefficients $a_i$ and $b_i$ was compiled. On the basis of its analytical solution, an expression for the potential of the electric field was obtained for conducting VES (the point current source and the observation point are located only on the soil surface).* ***Results****. Comparison of the results of soil sounding by the Schlumberger installation and the interpretation of its results for the same points shows a sufficient degree of approximation: the maximum relative error does not exceed 9.7 % (for the second point), and the average relative error is 3.6 %.* ***Originality****. Based on the obtained expression, a test version of the program was implemented in Visual Basic for Applications to interpret the results of VES by the Schlumberger method. To check the obtained expressions, the interpretation of the VES results was carried out on the territory of a 150 kV substation of one of the mining and processing plants in the city of Kriviy Rih.* ***Practical significance.*** *The developed mathematical model will make it possible to increase the sounding depth, and, consequently, the accuracy of determining the standardized parameters of the grounding arrangements of power stations and substations.* References 13, figures 3.

*Key words:* **electrical substation, grounding arrangements, vertical electrical sounding, Schlumberger method, method of integro-differential equations.**

*Робота присвячена проблематиці визначення геоелектричної структури ґрунту в межах випробування заземлювальних пристроїв діючих електричних станцій та підстанцій на необхідну глибину в умовах щільної забудови. Для вирішення проблеми запропоновано використати установку Шлюмберже, яка має більшу глибину зондування у порівнянні з установкою Веннера. За допомогою методів інтегро-диференційних рівнянь було отримано аналітичні вирази для інтерпретації результатів зондування ґрунту установкою Шлюмберже у випадку чотиришарового ґрунту. Для перевірки отриманих виразів була проведена інтерпретація результатів вертикального електричного зондування на території підстанції 150 кВ одного з гірничо-збагачувальних комбінатів: максимальна відносна похибка не перевищує 9,7 %, а середня – 3,6 %.* Бібл. 13, рис. 3.

*Ключові слова:* **електрична підстанція, заземлювальний пристрій, вертикальне електричне зондування, установка Шлюмберже, метод інтегро-диференційних рівнянь.**

**Formulation of the problem**. The procedure for determining the soil resistivity as a component of testing of the grounding arrangement (GA) for power stations and substations is regulated in the IEEE standards [1, 2]. In this case, it is recommended to use the Wenner installation for conducting vertical electrical sounding (VES) of the soil. Although, in the general case, the soil is a multilayer structure with many anisotropic inclusions, the expressions to interpret VES curves in the form of a two-layer geoelectric space with plane-parallel interfaces between layers are mainly used. The quality of VES and the interpretation of its results significantly affect the accuracy of calculating the parameters of GA, and, consequently, on the electrical safety of personnel and the reliability of the substation equipment.

VES is carried out by injecting a test current by a generator between current electrodes A and B and measuring the voltage drop at a certain area of the soil surface at potential electrodes M and N. The value of the apparent resistivity is equal to the product of the ratio of the measured voltage and current by the geometric factor of the installation [3]:

$$\rho_k = \frac{U}{I} k, \tag{1}$$

where $U$ is the voltage drop across the potential electrodes M and N (see Fig. 1); $I$ is the current flowing through the current electrodes A and B; $k$ is the geometrical coefficient of installation. For the Wenner installation, $k = 2\pi L$, where $L$ – distance between the electrodes.

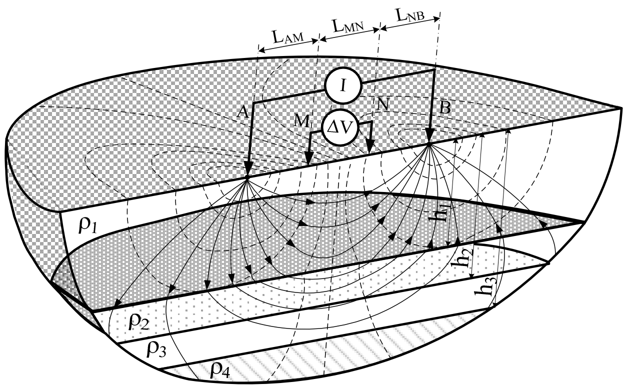


Fig. 1. The principle of conducting VES

The ratio of the spacing lengths of the current and potential electrodes depends on the choice of the VES installation, and the maximum distance between the current electrodes is determined by the required sounding depth. For the Wenner installation, the sounding depth is equal to 1/3 of the spacing length of the current electrodes [3]. Its advantages include:

- poor sensitivity to profile inclusions;
- direct relationship between electrode spacing and sounding depth;

• relatively simple expressions for calculating the apparent resistivity due to the equality of the interelectrode spacing between the current and potential electrodes.

In [4], based on the analysis of the experimental VES curves in the locations of more than 600 energy objects in Ukraine, it was shown that the vast majority of soils at the locations of power stations and substations have a three-layer structure (72,7 %), and another part (19 %) has more than three layers (usually four). Therefore, it is relevant to use interpretation tools with at least four layers, this will cover more than 90 % of energy objects in Ukraine. The authors in [5] based on the solution of the basic problem of the field of a point current source in a four-layer conducting half-space, expressions were obtained for interpreting the results of soil sounding by the Wenner installation. However, carrying out of VES for operating substations, as a rule, has to be performed in conditions of dense industrial or urban development, which does not allow providing the required sounding depth, which is several times greater than the largest diagonal of the GA [6]. Analysis of the literature shows that in the world, as a rule, models for the interpretation of VES curves built on numerical methods [7, 8] or using the method of images model [9] have found application. It was shown in [10] that the calculation error using such models can reach 20 %.

One of the ways to increase the sounding depth while maintaining the spacing of the current electrodes can be the use of a symmetric Schlumberger installation, which is a common case of the Wenner installation. However, a significant drawback is the lack of analytical expressions for interpreting the sounding results.

**The purpose of the work** is to develop a mathematical model for interpreting the results of soil sounding by the Schlumberger method in the form of a four-layer geoelectric structure.

**Research materials.** Interpretation of VES results is an inverse problem of electrical prospecting and, in the general case, is an ill-posed problem with many existing solutions that differ from the true one [5]. In this case, the relationship between the measured values of the apparent resistivity and the parameters of this model is expressed by integral equations.

To construct a mathematical model, it is proposed to use the solution of a particular problem about the field of a point current source, which, like the observation point, is located in the first layer of a four-layer soil [5]. When solving the problem, the following assumptions were made: the current does not pass through the boundary of the earth and the atmosphere, the interfaces between the layers are plane-parallel, and within each of them the electrical resistivity $\rho_i$ is uniform. It was assumed that a point current source $j$ is located in the first layer of a four-layer conducting half-space with plane-parallel interfaces (see Fig. 2). The electrical resistances of the first, second, third and fourth layers are denoted by $\rho_1$, $\rho_2$, $\rho_3$ and $\rho_4$, respectively. The depths of the interfaces of the first and second layers – $h_1$, the second and third – $h_2$, the third and fourth – $h_3$.

The formulation of the problem under consideration consists of the Laplace equation and additional conditions. The electric field of a point current source in a four-layer medium has axial symmetry, and the potential does not depend on the $\psi$-coordinate; therefore, the Laplace equation in a curvilinear orthogonal cylindrical coordinate system takes the form:

$$\frac{\partial^2 \varphi}{\partial z^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{\partial^2 \varphi}{\partial r^2} = 0 . \quad (2)$$

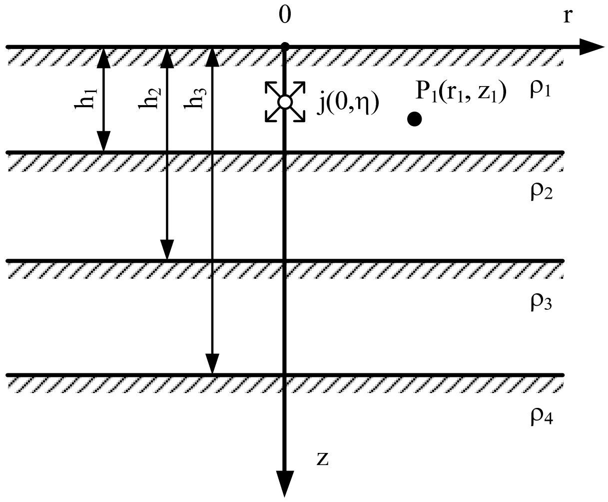


Fig. 2. A point current source $j$ located in the first layer of the four-layer structure; $P_1$ ($r_1$, $z_1$) – observation point

Solution (2) is found by the Fourier method for separation of variables [11]:

$$\varphi(r,z) = \int_0^\infty J_0(\lambda r)\cdot\left(a_i\, e^{\lambda z} + b_i\, e^{-\lambda z}\right) d\lambda , \quad (3)$$

where $a_i$ and $b_i$ are constants determined by soil parameters, coordinates of a point current source and observation point; $\lambda$ is a separation parameter of variables; $J_0$ is a zero-order Bessel function of the first kind.

The form of function (3) is common for all layers of the conducting half-space. However, in each layer, depending on the relative position of the point current source and the observation point, the constants take on their particular values. To find the constants $a_i$ and $b_i$ in the first layer, we use the additional conditions:

• with an unlimited increase in the $z$-coordinate the potential $\varphi$ tends to zero, therefore

$$a_4 = 0 ; \quad (4)$$

• in accordance with the principle of electric current continuity at the interface between the $i$-th and ($i$+1)-th layers the normal components of the vectors of the electric current density are equal to each other:

$$\frac{1}{\rho_i}\frac{\partial \varphi_i}{\partial z} = \frac{1}{\rho_{i+1}}\frac{\partial \varphi_{i+1}}{\partial z} ; \quad (5)$$

• from the condition of equality of the tangential components of the electric field strength vector at the boundaries of adjacent layers, at the interface between the $i$-th and ($i$+1)-th the potentials are equal:

$$\varphi_i = \varphi_{i+1} ; \quad (6)$$

• the condition on the boundary of the conducting half-space has the form

$$\left.\frac{\partial \varphi_1}{\partial z}\right|_{z=0} = 0 . \quad (7)$$

Based on expressions (4) – (7), a system of linear algebraic equations of the 7-th order with respect to the unknown coefficients $a_i$ and $b_i$ was compiled. On the basis of its analytical solution, an expression for the potential of the electric field was obtained in [5]. Considering that when conducting VES, the point current source and the observation point are located only on the soil surface (i.e. $z = 0$ and $\eta = 0$), this expression will take the form:

$$\varphi_{1,1}(r,0) = \frac{I\rho_1}{2\pi}\left[\frac{1}{r} + 2K_{2,1}\sum_{n=0}^{m}\frac{K_n}{\sqrt{r^2+(2h_1+H_n)^2}} + 2K_{3,2}\sum_{n=0}^{m}\frac{K_n}{\sqrt{r^2+(2h_2+H_n)^2}} + 2K_{4,3}\sum_{n=0}^{m}\frac{K_n}{\sqrt{r^2+(2h_3+H_n)^2}}\right] \quad (8)$$

where $K_{i+1,i}$ is the coefficient of soil heterogeneity equal to $K_{i+1,i} = \frac{\rho_{i+1}-\rho_i}{\rho_{i+1}+\rho_i}$; $K_n$ and $H_n$ are the coefficients obtained as a result of the expansion of the function characterizing the multilayer medium; $n$ is the number of the term of the series; $m$ is the number of terms of the series.

The values of $K_n$ and $H_n$ are found by the least squares method [12] when approximating the function $F_4(\lambda)$, which characterizes a four-layer soil at $\lambda \to \infty$:

$$F_4(\lambda) = \frac{1}{F_Z(\lambda)}, \quad (9)$$

where $F_Z(\lambda)$ is

$$F_Z(\lambda) = 1 - K_{2,1}e^{-2\lambda h_1} - K_{3,2}e^{-2\lambda h_2} - K_{4,3}e^{-2\lambda h_3} + K_{2,1}K_{3,2}e^{-2\lambda(h_2-h_1)} + K_{2,1}K_{4,3}e^{-2\lambda(h_3-h_1)} + K_{3,2}K_{4,3}e^{-2\lambda(h_3-h_2)} - K_{2,1}K_{3,2}K_{4,3}e^{-2\lambda(h_3-h_2-h_1)}.$$

Thus, a basic expression was obtained for the development of a mathematical model for interpreting the VES results in the form of a four-layer geoelectric structure.

To develop a model that allows us to interpret the results obtained using the Schlumberger method, we will use the expression for determining the apparent resistivity (1), the geometric configuration of the installation itself (see Fig. 1) and the expression for determining the potential on the soil surface (8).

Based on the principle of superposition, the voltage at the potential electrodes $M$ and $N$ will be determined as (10), where $\varphi_{AM}$, $\varphi_{BM}$, $\varphi_{AN}$ and $\varphi_{BN}$ are the values of potential at electrodes $M$ and $N$, induced from current electrodes $A$ and $B$, respectively.

Substituting the expression for potential (8) into (10) and taking into account the symmetry of the Schlumberger installation, the voltage drop will have the form (11).

Taking into account (11), the geometric coefficient of installation and transformations, we obtain expression (1) in the following form (12).

$$U = \varphi_M - \varphi_N = \left(\varphi_{AM}\Big|_{r=L_{AM}} - \varphi_{BM}\Big|_{r=L_{BM}}\right) - \left(\varphi_{AN}\Big|_{r=L_{AN}} - \varphi_{BN}\Big|_{r=L_{BN}}\right), \quad (10)$$

$$U = \frac{I\rho_1}{2\pi}\left[\frac{1}{L_{AM}} - \frac{1}{L_{AN}} + 2\sum_{i=1}^{4}K_{i+1,i}\left(\sum_{n=0}^{m}\frac{K_n}{\sqrt{L_{AM}^2+(2h_i+h_n)^2}} - \sum_{n=0}^{m}\frac{K_n}{\sqrt{L_{AN}^2+(2h_i+h_n)^2}}\right)\right], \quad (11)$$

$$\rho_k = \frac{L_{AM}L_{AN}}{(L_{AM}-L_{AN})}\left[\frac{1}{L_{AM}} - \frac{1}{L_{AN}} + 2\sum_{i=1}^{4}K_{i+1,i}\left(\sum_{n=0}^{m}\frac{K_n}{\sqrt{L_{AM}^2+(2h_i+h_n)^2}} - \sum_{n=0}^{m}\frac{K_n}{\sqrt{L_{AN}^2+(2h_i+h_n)^2}}\right)\right]. \quad (12)$$

Based on the obtained expression, a test version of the program was implemented in Visual Basic for Applications to interpret the results of VES by the Schlumberger method. To check the obtained expressions, the interpretation of the VES results (see Table 1) was carried out on the territory of a 150 kV substation of one of the mining and processing plants in the city of Kriviy Rih (see Fig. 3).

Table 1

The results of experimental measurements by the Schlumberger method

| $L_{MN}/2$, m | 0,1 | 0,13 | 0,17 | 0,22 | 0,27 | 0,33 | 0,4 | 0,5 | 0,6 | 0,8 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $L_{AB}/2$, m | 0,3 | 0,39 | 0,51 | 0,66 | 0,81 | 0,99 | 1,2 | 1,5 | 1,8 | 2,4 | 3 |
| $U/I$, Ω | 56,01 | 49,03 | 34,69 | 27,91 | 23,07 | 19,72 | 16,39 | 13,53 | 11,87 | 9,491 | 8,231 |
| $\rho_k$, Ω·m | 70,38 | 80,1 | 74,11 | 77,16 | 78,27 | 81,78 | 82,39 | 85,01 | 89,5 | 95,41 | 103,43 |

| $L_{MN}/2$, m | 1,3 | 1,7 | 2,2 | 2,7 | 3,3 | 4 | 5 | 6 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $L_{AB}/2$, m | 3,9 | 5,1 | 6,6 | 8,1 | 9,9 | 12 | 15 | 18 | 24 | 27 |
| $U/I$, Ω | 6,834 | 5,199 | 4,046 | 3,342 | 2,861 | 2,162 | 1,539 | 1,176 | 0,658 | 0,537 |
| $\rho_k$, Ω·m | 111,64 | 111,07 | 111,86 | 113,39 | 118,64 | 108,67 | 96,7 | 88,67 | 66,15 | 60,73 |

Comparison of the results of soil sounding by the Schlumberger installation and the interpretation of its results for the same points shows a sufficient degree of approximation (see Fig. 3): the maximum relative error does not exceed 9.7 % (for the second point), and the average relative error is 3.6 %. The results obtained can

be used to determine the electrical properties of the soil, including the propagation of an electromagnetic wave with a short front, created by a special generator [13] that simulates the lightning current.

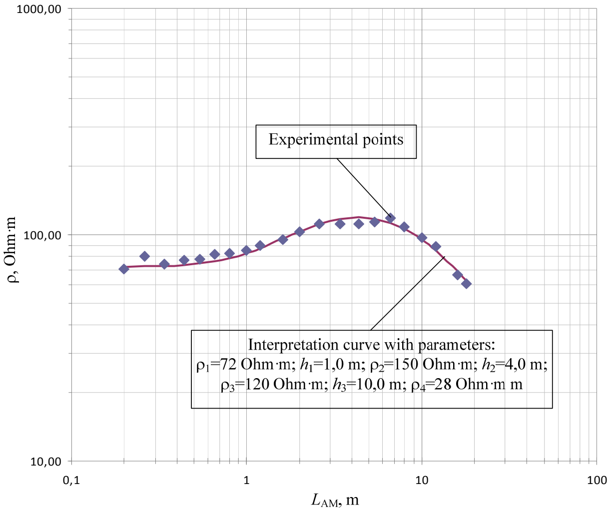


Fig. 3. Interpretation of VES results obtained by the Schlumberger method

**Conclusions.**

1. Based on the analytical solution of the problem of the field of a point current source located in the first layer of a four-layer geoelectric structure, a mathematical model has been developed for interpreting the results of soil sounding by the Schlumberger installation in the form of a four-layer geoelectric structure.

2. Based on experimental studies carried out at the existing 150 kV substation, the correctness of the developed mathematical model was confirmed. A test computer program has been developed for the interpretation of soil sounding results in an interactive mode.

3. The developed mathematical model will make it possible to increase the sounding depth, and, consequently, the accuracy of determining the standardized parameters of the grounding arrangements of power stations and substations.

**Conflict of interest.** The authors declare that they have no conflicts of interest.

REFERENCES

***1.*** *IEEE Std 80-2013. Guide for Safety in AC Substation Grounding*. New York, IEEE, 2013. 206 p. doi: https://doi.org/10.1109/IEEESTD.2015.7109078.

***2.*** *IEEE Std 81-2012. Guide for Measuring Earth Resistivity, Ground Impedance, and Earth Surface Potentials of a Grounding System*. New York, IEEE, 2012. 86 p. doi: https://doi.org/10.1109/IEEESTD.2012.6392181.

***3.*** Kolesnikov V.P. *Osnovy interpretatsii elektricheskikh zondirovanii* [The base of interpretation of electrical soundings]. Moscow, Scientific World Publ., 2007. 248 p. (Rus).

***4.*** Koliushko D.G., Rudenko S.S., Koliushko G.M. Analysis of electrophysical characteristics of grounds in the vicinity electrical substation of Ukraine. *Electrical Engineering & Electromechanics*, 2015, no. 3, pp. 67-72. (Rus). doi: https://doi.org/10.20998/2074-272x.2015.3.10.

***5.*** Koliushko D.G., Rudenko S.S. Interpretation the results of the vertical electrical sounding as the geoelectrical half space with four layer. *Bulletin of NTU «KhPI»*, 2015, no. 12 (1121), pp. 324-329. (Rus).

***6.*** Koliushko D.G., Rudenko S.S., Asmolova L.V., Tkachova T.I. Determination of the soil sounding depth for the earthing resistance calculation of substations 35 kV. *Electrical Engineering & Electromechanics*, 2020, no. 1, pp. 52-55. doi: https://doi.org/10.20998/2074-272x.2020.1.08.

***7.*** Lagace P.J., Fortin J., Crainic E.D. Interpretation of resistivity sounding measurements in N-layer soil using electrostatic images. *IEEE Transactions on Power Delivery*, 1996, vol. 11, no. 3, pp. 1349-1354. doi: https://doi.org/10.1109/61.517490.

***8.*** Colominas I., Navarrina F., Casteleiro M. Analysis of transferred Earth potentials in grounding systems: a BEM numerical approach. *IEEE Transactions on Power Delivery*, 2005, vol. 20, no. 1, pp. 339-345. doi: https://doi.org/10.1109/TPWRD.2004.835035.

***9.*** Arnautovski-Toseva V., Grcev L. Image and exact models of a vertical wire penetrating a two-layered earth. *IEEE Transactions on Electromagnetic Compatibility*, 2011, vol. 53, no. 4, pp. 968-976. doi: https://doi.org/10.1109/TEMC.2011.2149533.

***10.*** Arnautovski-Toseva V., Grcev L. On the image model of a buried horizontal wire. *IEEE Transactions on Electromagnetic Compatibility*, 2016, vol. 58, no. 1, pp. 278-286. doi: https://doi.org/10.1109/TEMC.2015.2506608.

***11.*** Burgsdorf V.V., Yakobs A.I. *Zazemlyayushchie ustroystva elektroustanovok* [Grounding arrangement of electrical installations]. Moscow, Energoatomizdat Publ., 1987. 400 p. (Rus).

***12.*** Rencher A.C., Christensen W.F. *Methods of multivariate analysis, Third Edition.* John Wiley & Sons, Inc., 2012. 768 p. doi: https://doi.org/10.1002/9781118391686.

***13.*** O. Rezinkin, M. Rezinkina, A. Danyluk, R. Tomashevskyi, Formation of high-voltage pulses with nanosecond fronts in low-impedance loads. *IEEE 2nd Ukraine Conference on Electrical and Computer Engineering (UKRCON)*, 2019, pp. 464-467 doi: https://doi.org/10.1109/UKRCON.2019.8880015.

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