Resonance behavior of the non-ideal system which contains a snap-trough truss as absorber

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Abstract. A resonance behavior of a system containing the linear oscillator, the Mises girder as absorber of elastic vibrations and the source of energy with a limited power-supply is analyzed. Stationary resonance regimes of vibrations near stable equilibrium position are considered, namely, vibrations near the resonance 1:1 between the linear oscillator and the motor, vibrations near the resonance 1:1 between the absorber and the motor. The stationary regime of snap-through motion is also considered.

Introduction

The systems with a limited power-supply are characterized by interaction of source of energy and elastic sub-system which performs motions is under action of the source. Such systems are named also as the non-ideal systems (NIS). One of the most important effects observed in such systems is the Sommerfeld effect [1], when the stable resonance regime with large amplitudes is appeared in the elastic sub-system, and the large part of the vibration energy passes from the energy source to the resonance vibrations. Resonance dynamics of such systems was first analytically described by V.O.Kononenko [2]. Investigations on the subject were continued in different publications. In particular, overviews on numerous studies of the non-ideal systems dynamics can be found in [3-5]. The Mises girder as absorber of free and forced vibrations is analyzed in [6,7]. Here a resonance behavior of a system containing the linear oscillator, the Mises girder as absorber and the source of energy with a limited power-supply is analyzed. Two resonance regimes of vibrations near stable equilibrium position are considered, namely, vibrations near the resonance 1:1 between the linear oscillator and the motor, and near the resonance 1:1 between the absorber and the motor. The stationary regime of snap-through motion is considered too. The multiple scale method is used. Outcome from the resonance behavior near the system equilibrium positions to the snap-through motion is presented.

Analysis of the near resonance vibrations

A motion of the system is determined by variables x, y and φ, corresponding to motions of the linear oscillator, the snap-through truss (or the Mises girder), and to the motor rotation respectively. There is here an interaction of the linear elastic sub-system and the energy source. The motor acts to the elastic sub-system having a mass M by the crank shaft of radius r. Besides, the system contains the Mises girder of the mass m as absorber which is attached to the linear sub-system and to the motionless ground by springs of the length L. The angle γ corresponds to two girder's stable equilibrium states. The mass of the absorber is essentially smaller than one of the elastic sub-system. Potential and kinetic energies of the systems can be written as

$$\Pi = \sum_{i=0}^{3} c_i \,\Delta L_i^2\,,\;\; T = \tfrac{1}{2}\left(M\,\dot{x}^2 + m\,\dot{y}^2 + I\,\dot{\varphi}^2\right), \tag{1}$$

where $\Delta L_0 = x,\;\; \Delta L_1 = x - r\sin\varphi,\qquad \Delta L_2 = L - \sqrt{(L\cos\gamma - x)^2 + y^2},\;\; \Delta L_3 = L - \sqrt{(L\cos\gamma)^2 + y^2}$.
One introduces $K(\dot{\varphi})$ as the driving moment of the energy source, and $H(\dot{\varphi})$ as the moment of resistance to rotation. For a simplification the characteristic and the moment of resistance are taken as linear ones.

The small parameter ε is introduced to equations of motion. This parameter characterizes a smallness of the angle γ, corresponding to the girder's stable equilibrium, the ratio μ of the absorber mass with respect to the elastic sub-system mass, the normalized values of springs stiffness c_2, and c_3, the dissipation coefficients and the normalized radius of the shaft r. Besides, it is assumed that the vibration components in function of the angular velocity are small with respect to the constant component. Then the multiple scales method is used to describe the system behavior near resonances between the rotation frequency and two fundamental frequencies, namely, fundamental frequency of the elastic sub-system ω_u (the first resonance), and fundamental frequency of the Mises girder small vibrations ω_w (the second resonance). To analyze stability of the obtained stationary regimes corresponding equations in variations are considered, and multiplicators are calculated. The same system is also considered without the assumption that frame stiffness is small.

Amplitude of the elastic sub-system versus the engine characteristic q near the first resonance is shown in Fig.1. Effect of absorption near the first resonance is presented in Fig. 2. Vibrations of the elastic sub-system and rotations of the engine both with the absorber, and without one are shown Fig. 2. The system parameters are chosen, in particular, as the following: the ratio $\mu = 0.025$; $\gamma = \pi/6$; L=1; r= 0.03; q= 1.697; normalized value of the spring stiffness is essentially smaller than one of the elastic sub-system. We can see from the Fig. 2 that in the presence of the Mises girder amplitudes of vibrations of the elastic sub-system are almost three times less than ones of the sub-system without absorber. Vibrations of the Mises girder are significant. An influence of the absorber to the motor is not essential here, but it is possible to obtain both essential reduction of the elastic vibrations, and fast passage of the motor to the maximal frequency of rotation.

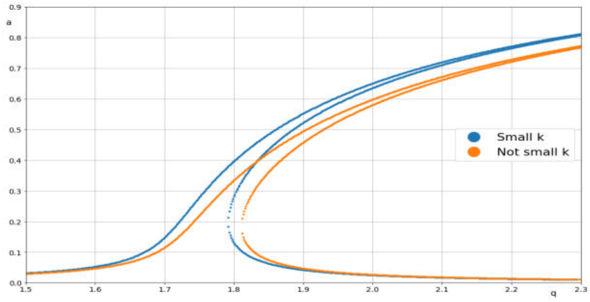

Figure 1: Amplitude of the elastic sub-system vibrations versus the motor characteristic for different valuations of the parameter *k*. The first resonance region.

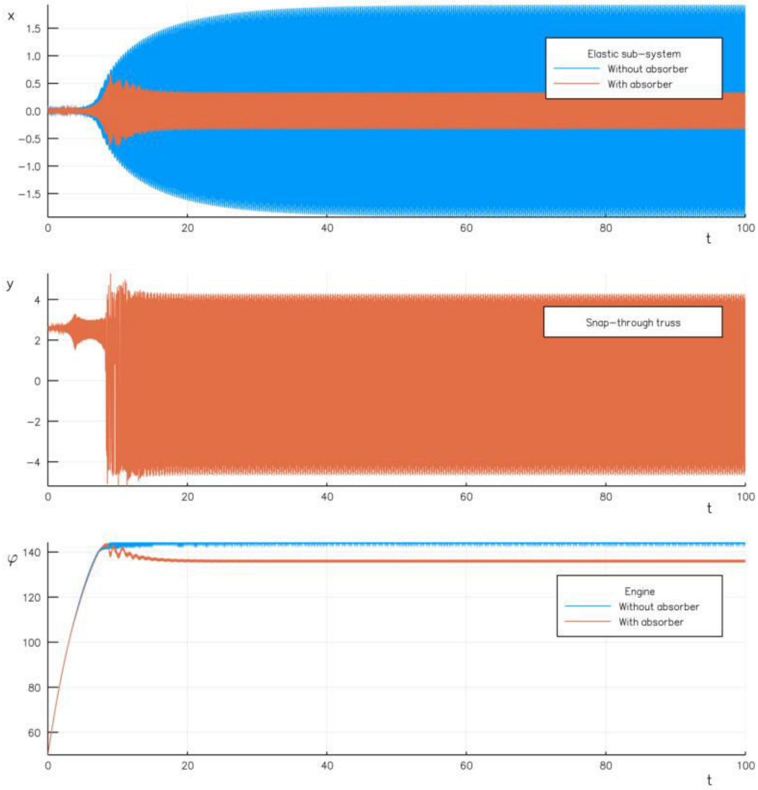

Figure 2: Absorption of vibrations of the elastic sub-system by the snap-through truss.

Conclusions

Resonance behaviour of the non-ideal system which contains the snap-through truss (Mises girder) as absorber is considered. Resonances between the motor rotation frequency and two fundamental frequencies, namely, fundamental frequency of the elastic sub-system and fundamental frequency of the Mises girder are analyzed. Corresponding frequency responses are obtained. Effect of the absorber to reduction of elastic vibrations is shown.

References

[1] Sommerfeld, A. (1902) Beiträge zum dynamischen ausbau der festigkeitslehe. *Physikal Zeitschr* **3**: 266-286.
[2] Kononenko V.O. (1969) Vibrating Systems with Limited Power Supply. Illife Books, London.
[3] Nayfeh, A.H., Mook, D.T. (1979) Nonlinear Oscillations. Wiley, New York.
[4] Eckert M. (1996) The Sommerfeld effect: theory and history of a remarkable resonance phenomenon. European J. Phys. **17** (5): 285–289
[5] Balthazar et al. (2003) An overview on non-ideal vibrations. *Meccanica* **38** (6): 613–621.
[6] Avramov K.V., Mikhlin Y.V. (2004) Snap-through truss as a vibration absorber. *J. of Vibration and Control* **10**: 291–308.
[7] Avramov K.V., Mikhlin Y.V. (2006) Snap-through truss as an absorber of forced oscillations. *J. of Sound and Vibrations* **290**: 705–722.