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# Interval frequency estimation by cross-correlation analysis

**I A Kostiukov**

Department of Electrical Insulation and Cable Engineering, National Technical University “Kharkiv Polytechnic Institute”, 2 Kyrpychova street, Kharkiv, Ukraine

E-mail: [iakostiukow@gmail.com](mailto:iakostiukow@gmail.com),

**Abstract.** This paper presents a substantiation of interval frequency estimation which can be carried out based on frequency dependence of calculated values of cross-correlation coefficient between analyzed signal and reference sine function with zero initial phase. In considered approach the interval of frequencies which contains the exact frequency of analyzed signal is established by finding the range of frequencies limited by the maximum value of the main lobe and the closest zero crossing of this lobe. The results of carried out simulations for interval frequency estimation of a multifrequency signal are also given.

## 1. Introduction

The problem of accurate frequency estimation remains actual for many engineering applications. In practice mentioned problem can arise both for signals which contain oscillation with a single frequency and for multifrequency signals with various values of signal-to-noise ratio and, in fact, can be regarded as a part of a more common problem of estimation the spectral density of various signals and time series. In technical issues that concern various aspects of electrical engineering the problem of frequency estimation is of a significant interest primarily from the point of view of assessment the quality of electrical energy [1,2], measurements of active power in case of processing the results of measurements in frequency domain [3] and the analysis of time series of consumed active and reactive power [4].

In case of processing digital signals frequency estimation usually can be carried out by applying properties of autocorrelation functions [5], by means of using various techniques based on the results of linear prediction theory [6], including the Pisarenko frequency estimator, which employs eigenvalues and eigenvectors of correlation matrix [7]. Other methods rely on the applying of the discrete Fourier transform. Methods based on classical results of the linear prediction theory, which imply the estimation of autoregressive coefficients for analyzed signal which consists of sine oscillation with noise, can exhibit poor noise stability, depending on the value of the initial phase of sine oscillation and also splitting of spectral lines, which can lead to the false conclusion about the presence of some oscillation in analyzed signal. The accuracy of methods which are based on the applying of the discrete Fourier transform depends on spectral leakage and finite frequency resolution. The problem of undesirable spectral leakage partially can be mitigated by applying spectral windowing [8]. The increasing of accuracy is also attained by applying special interpolation algorithms of the results of Fourier transform. The interpolation of the discrete Fourier transform coefficients can be carried out either without the applying of preliminary spectral windowing [9], or after such applying [10]. The results of comparative analysis of noise stability of various approaches for frequency estimation carried out in [11] have shown higher level of noise stability of methods which rely on the applying of the discrete Fourier transform with further interpolation in comparison with the methods which rely on the applying of autocorrelation function.



Though the applying of spectral windowing combined with interpolation algorithms usually allows us to increase the accuracy of frequency estimation, the problem of accuracy still remains actual, especially for the case of analysis time series which contain several sine oscillations with undesirable noise component. Since the problem of accuracy remains actual, as the developed algorithms for frequency estimation still lead to some more or less significant errors, in some practical applications it can be relevant to carry out interval estimation of frequency by establishing the lower and the upper boundaries of some range of frequencies which will contain the exact value of frequency. The substantiation of such interval estimation is carried out in this paper.

## 2. Materials and methods

For digital signals the applying of Fourier transform allows to compute a sequence of elements which are obtained by calculating the values of cross-correlation between the samples of analyzed signal and orthogonal cosine and sine functions. In idealized case, when analyzed signal contains oscillations with certain frequency, which is equal to the frequency of cosine and sine functions applied in the Fourier transform, and also in case of absence of spectral leakage and sufficient frequency resolution, the module of computed element of Fourier transform with some frequency component will exhibit maximum, due to the high cross-correlation with corresponding frequency component of analyzed signal. In this case efficient methods for frequency estimation take into consideration both estimation accuracy and computational efficiency. According to the latter requirement the number of frequencies of applied in Fourier transform sine and cosine functions is selected to be equal to the number of samples of analyzed signal. The values of these frequencies are selected depending on the value of sampling frequency  $f_s$ . Usually the values of these frequencies are computed according to (1).

$$f_m = \frac{mf_s}{N}, \quad (1)$$

where  $N$  denotes the number of samples of analyzed signal,  $m$  denotes the number of Fourier transform component. Due to the symmetry of obtained results of calculations with respect to the Nyquist frequency, only one half of obtained by applying Fourier transform sequence of complex numbers is applied. Instead of calculating the values of cross-correlation of the analyzed signal with orthogonal sine and cosine functions it is possible to calculate the values of cross-correlation only with respect to sine function but for the doubled number of applied in Fourier transform frequencies. On one hand such increasing of numbers will increase the frequency resolution and, consequently, will allow to get more detailed information about the size and the location of sidelobes, however, due to the refusal from the applying of the complete system of orthogonal functions, the results of such calculations will exhibit a significant dependence on the value of the initial phase of analyzed harmonic oscillation.

### 2.1 Frequency properties of cross-correlation between the analyzed harmonic signal and sine function

In general case the relation for cross-correlation between some analyzed signal with certain angular frequency  $\omega_2$  and some sine function with zero initial phase can be determined according to (2).

$$B = \int_0^b \sin(\omega_1 t) A_2 \sin(\omega_2 t + \varphi_2) dt \quad (2)$$

where  $A_2$  denotes the amplitude of analysed signal,  $\omega_1$  denotes the angular frequency of applied sine function,  $\varphi_2$  is the initial phase of analyzed signal. Further analysis leads to consideration of the influence of relations between the values of  $\omega_1$  and  $\omega_2$  on the value of  $B$ . In case when the frequency of applied sine function is not equal to the frequency of analyzed signal the solution of (1) can be written in the following form:

$$B = \frac{A_2 (2 \sin(\varphi_2) \omega_1 + \sin(\zeta) \omega_1 + \sin(\zeta) \omega_2 - \sin(\xi) \omega_1 + \sin(\xi) \omega_2)}{2\theta} \quad (3)$$

where  $\zeta$ ,  $\xi$  and  $\theta$  can be determined by using (4-8):

$$\zeta = b\omega_1 - b\omega_2 - \varphi_2 \quad (4)$$

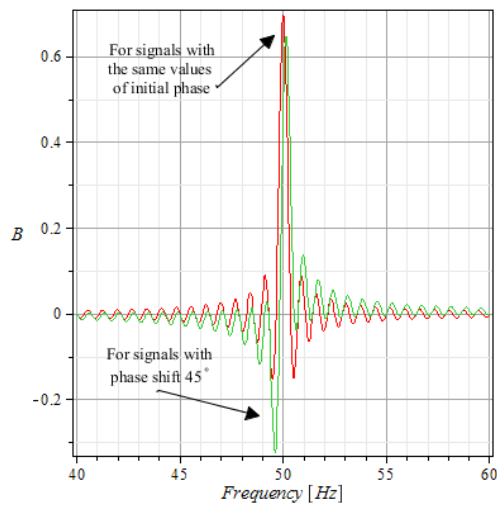
$$\xi = b\omega_1 + b\omega_2 + \varphi_2 \quad (5)$$

$$\theta = \theta_1\theta_2 \quad (6)$$

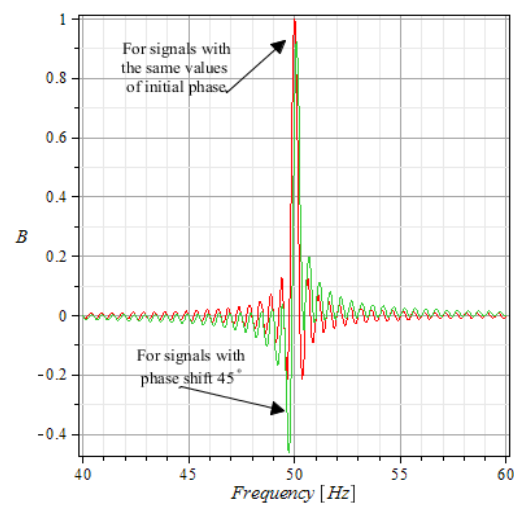
$$\theta_1 = \omega_1 - \omega_2 \quad (7)$$

$$\theta_2 = \omega_1 + \omega_2 \quad (8)$$

The results of calculations which illustrate frequency properties of  $B$  are presented on Figures 1-2.



**Figure 1.** The dependence of cross-correlation (2) on the value of frequency of applied sine function for analyzed signal with the duration equal to 1.4 second and for two values of phase shift  $\varphi_2$ .



**Figure 2.** The dependence of cross-correlation (2) on the value of frequency of applied sine function for analyzed signal with the duration equal to 2 seconds and for two values of phase shift  $\varphi_2$ .

The results of carried out calculations illustrate the displacement of maximum value of  $B$  from the value of fundamental frequency of analyzed signal. This displacement is caused by the initial phase of analyzed signal and the refusal from the applying of the complete system of orthogonal functions, i.e. both sine and cosine functions. However, from the results presented on figure 2 it can be concluded that deleterious impact of the initial phase  $\varphi_2$  on the accuracy of frequency estimation can be mitigated by the increasing of the duration of analyzed signal. As it can be concluded from the analysis of (4) and (5), in case of sufficiently large value of the upper limit of integration  $b$  the values of sine functions from  $\zeta$  and  $\xi$  practically have a very slight dependence on the value of the initial phase of analyzed signal. Consequently, due to the influence of the initial phase of analyzed signal, for comparatively short signals the direct applying of (2) for exact frequency estimation can be inaccurate. However, (2) can be used for interval frequency estimation, as it can be shown that the exact frequency of analyzed signal stays within the range of frequencies limited by maximum and minimum values in frequency dependence of  $B$ . Further narrowing of frequency interval can be attained by the analysis of antiderivative function determined for  $B = f(\omega_1)$

## 2.2 Frequency properties of antiderivative function for $B = f(\omega_1)$

The relation for antiderivative function derived for  $B = f(\omega_1)$  can be written according to (9):

$$B_{0s} = B_{00} + B_{01} + B_{02} + B_{03} + B_{04} + B_{05} + B_{06} + B_{07} + B_{08}, \quad (9)$$

where all parameters  $B_{00}..B_{09}$  can be calculated according to (10-18).

$$B_{00} = \frac{1}{2} A_2 \sin(\varphi_2) \ln((\omega_1 - \omega_2)(\omega_1 + \omega_2)); \quad (10)$$

$$B_{01} = \frac{1}{2} A_2 \sin(b\omega_2)^2 \cos(\varphi_2) \text{Si}(b\omega_1 - b\omega_2); \quad (11)$$

$$B_{02} = -\frac{1}{2} A_2 \sin(b\omega_2)^2 \cos(\varphi_2) \text{Si}(b\omega_1 + b\omega_2); \quad (12)$$

$$B_{03} = -\frac{1}{2} A_2 \cos(b\omega_2)^2 \sin(\varphi_2) \text{Ci}(b\omega_1 - b\omega_2); \quad (13)$$

$$B_{04} = -\frac{1}{2} A_2 \cos(b\omega_2)^2 \sin(\varphi_2) \text{Ci}(b\omega_1 + b\omega_2); \quad (14)$$

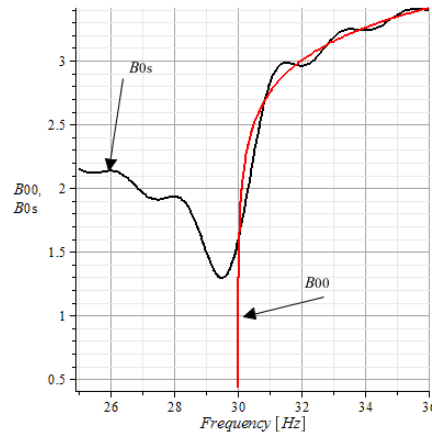
$$B_{05} = \frac{1}{2} A_2 \cos(b\omega_2)^2 \cos(\varphi_2) \text{Si}(b\omega_1 - b\omega_2); \quad (15)$$

$$B_{06} = -\frac{1}{2} A_2 \cos(b\omega_2)^2 \cos(\varphi_2) \text{Si}(b\omega_1 + b\omega_2); \quad (16)$$

$$B_{07} = -\frac{1}{2} A_2 \sin(b\omega_2)^2 \sin(\varphi_2) \text{Ci}(b\omega_1 - b\omega_2); \quad (17)$$

$$B_{08} = -\frac{1}{2} A_2 \sin(b\omega_2)^2 \sin(\varphi_2) \text{Ci}(b\omega_1 + b\omega_2). \quad (18)$$

The results of calculation of frequency dependence of  $B_{0s}$  and  $B_{00}$  are presented on figure 3.



**Figure 3.** Frequency dependence of  $B_{0s}$  and  $B_{00}$  for signal with the initial phase equal to  $\pi/4$ , frequency equal to 30 Hz, and the duration equal to 0.5 s.

From the analysis of presented on figure 3 frequency dependence it can be also concluded that due to the presence of term  $B_{00}$  the exact frequency of analyzed signal stays within the range of frequencies limited by the maximum value of the main lobe and the closest zero-crossing of this lobe, which corresponds to the minimum value in frequency dependence of antiderivative determined for  $B = f(\omega_1)$ . This conclusion has been made due to the approaching  $B_{00}$  to the exact value of frequency of analyzed signal, whereas the closest zero crossing of the main lobe corresponds to the lower value of frequency.

### 3. The results of carried out simulations for interval frequency estimation

All simulations concerning interval frequency estimation have been carried out with respect to the arbitrarily generated function which contains components with different values of frequency:

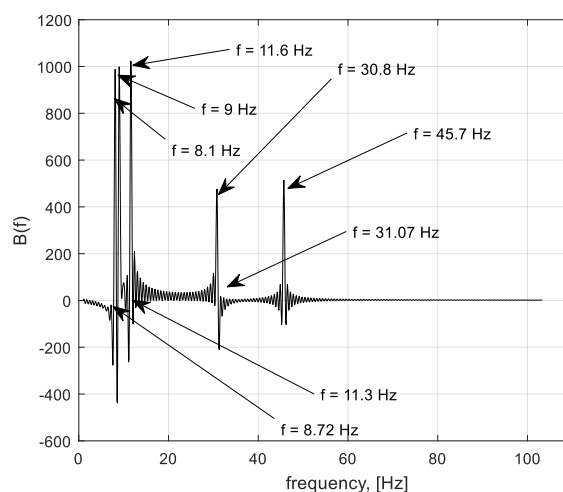
$$Y(t) = 2\sin\left(\omega_3 t + \frac{\pi}{8}\right) + 2\sin\left(\omega_4 t + \frac{\pi}{8}\right) + 2\sin\left(\omega_5 t + \frac{\pi}{8}\right) + \sin\left(\omega_6 - \frac{\pi}{4}\right) + \sin(\omega_7 t). \quad (19)$$

The exact values of frequency components  $f_3..f_7$  that correspond to angular frequencies  $\omega_3.. \omega_7$  in (19) are listed in table 1, which also contains the results of interval frequency estimation and estimated by means of applying the discrete Fourier transform values of frequency. All calculations have been carried out for 1024 samples of  $Y(t)$ .

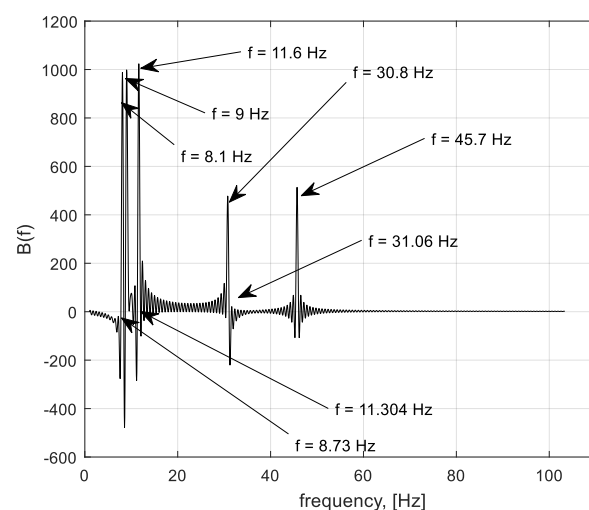
**Table 1.** Frequency components of  $Y(t)$  and the results of frequency estimation

Frequency component of $Y(t)$	The exact value of frequency, [Hz]	Interval estimation for 1024 values of frequency of reference sine function, [Hz]	Interval estimation for 2048 values of frequency of reference sine function, [Hz]	Values of frequency estimated by applying Fourier transform, [Hz]
$f_3$	8	7.77..8.1	7.77..8.1	8.182
$f_4$	9	8.72..9	8.73..9	9.126
$f_5$	11.56	11.3..11.6	11.304..11.6	11.737
$f_6$	30.9	30.8..31.07	30.8..31.06	30.63
$f_7$	45.7	45.7	45.7	45.618

The calculated frequency dependencies of  $B$  for 1024 and 2048 values of frequency of reference sine function are presented on figure 4 and figure 5.



**Figure 4.** The dependence of cross-correlation (2) on the value of frequency of applied reference sine function with 1024 different values of frequency.



**Figure 5.** The dependence of cross-correlation (2) on the value of frequency of applied reference sine function with 2048 different values of frequency.

#### 4. Discussion

The results of carried out simulations have shown that for all cases the established upper and lower boundaries of possible range of frequencies allowed to distinguish some frequency interval which contains the exact values of  $f_3..f_7$ . For all cases the values of frequencies estimated by means of using

approach based on the applying of the discrete Fourier transform did not match the established interval of frequencies which contains the exact values of  $f_3..f_7$ . Consequently, for all cases of estimation either the upper boundary or the lower boundary of estimated frequency interval has been more accurate in comparison with the value of frequency which has been estimated by means of applying the Fourier transform. Besides, the exact bias of estimated by applying Fourier transform values of frequency remains unknown. However, it should be noted that for the case of simulations the lower level of accuracy of estimated by applying the Fourier transform values of frequency in comparison with the values of either the lower or the upper boundaries of distinguished frequency interval could have been caused by the relevant selection of frequencies of reference sine function. The increasing of frequency resolution by the increasing of number of frequencies did not lead to any significant narrowing of estimated frequency interval, as it can be concluded from the comparison of data on figure 4 and figure 5. The high accuracy of  $f_7$  estimation has been caused by zero initial phase of corresponding sine component of  $Y(t)$ .

Considered approach for interval frequency estimation is based on the calculation of cross-correlation between the analyzed signal and sine function with variable frequency and zero initial phase is intended for the estimation of range of frequencies which contains the exact value of frequency of analyzed signal. This range of frequencies is limited by the maximum value of the main lobe and the closest zero crossing of this lobe. Though the applying of such approach allows us to establish a pretty wide boundaries which contain the necessary range of frequencies, especially for pretty short signals, it can be relevant in some technical applications, as it does not imply the necessity of taking into the consideration possible bias of estimated values of frequency which occur in case of applying point estimators.

## 5. Conclusions

Considered approach for interval frequency estimation implies the refusal from the applying of the complete system of orthogonal functions, which is usually used in point frequency estimators, for the sake of establishing some range of frequencies which contains the exact value of frequency of analyzed signal. This frequency is limited by the maximum value of the main lobe (with the highest amplitude) and the closest zero crossing of this lobe. Though such approach for interval estimation allows us to establish a pretty wide frequency interval, depending on the duration of analyzed signal, it can be narrowed by the increasing of the duration of analyzed signal.

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