

МЕХАНІКА / МЕХАНИКА / MECHANICS

THERMAL SHOCK ANALYSIS OF A FUNCTIONALLY GRADED PLATE USING FINITE ELEMENT METHOD

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Functionally gradient materials (FGMs) originally designed in the 80's for an improvement of temperature resistance of constructive elements of nuclear reactors and chemical plants have received now a status of high potential structural materials for variety of applications [1]. FGMs consist of two or more constituencies that make these composites microscopically inhomogeneous with smoothly varying mechanical properties along one or more defined directions. This is achieved by a gradual changing the volume fractions of the constituent materials. Thereby material properties of FGMs are spatially dependent.

The high temperature is a usual in-service environment of FGMs. Moreover, these materials experience often an exposure to a very high temperature in a very short period of time, known as thermal shock [2]. As a result of this severe loading high thermally-induced stresses appear that likely may initiate a crack in the FGM structure. Therefore the primary interest is the analysis of temperature fields and thermally-induced stresses that have a critical relevance to fracture mechanisms in FGM structures. The goal of this research is to examine thermal and mechanical response of a FGM plate under thermal shock.

Naturally, the study of the material response under temperature loading has to be conducted using a theory of thermo-mechanical problems. In this regard constitutive models, thermo-mechanical coupling and methods used for the solution should be considered. Here we assume that a plate as a two-dimensional body is made of an isotropic graded metal-ceramic material with the volume fraction varying as a power function along the selected direction. The plate undergoes small displacements and deformations in plane strain state, and its material behavior is governed by the linear elastic law. Heat transfer in the plate is described by the Fourier conduction law. It should be mentioned that all material parameters used in the constitutive laws depend on the spatial coordinates. The latter significantly distinguishes the current analysis from the case of homogeneous materials.

A closed form solution of the problem under consideration is very complicated and is possible in few one-dimensional cases with the simplest geometry and boundary conditions and exponentially varying volume fractions. While numerical techniques permit a look at more complex tasks under various boundary and loading conditions, material variations including bi-directional FGMs, and allow performing the crack analysis of FGMs. In this respect the finite element method (FEM) is the most power tool. So, we use the

FEM within the commercial code ABAQUS for carrying out thermal, mechanical and crack propagation analyses in the FGM plate subjected to thermal shock.

First a finite element formulation of the thermo-mechanical problem of a FGM plate with a crack in a plane strain state is considered. Then, the graded finite element incorporating a gradient of material thermal and mechanical properties into the finite element model of the FGM plate is developed. We utilized a quadratic eight-node plane strain temperature-displacement element (CPE8T) available in the implicit version of ABAQUS. The element assumes a bi-quadratic displacement interpolation, but a bi-linear temperature interpolation and allows full and reduced integrations over the element area. The gradation in the material properties within the element was achieved by programming the user-defining material subroutines UMAT and UMATHT, which are called in the mechanical and thermal analyses, respectively. The both subroutines implement the gradient by means of direct sampling the properties at the Gauss points of the element owing the numerical integration over the element area.

To simulate the crack growth in the FGM plate due to thermal loading the virtual crack closure technique available in ABAQUS was adopted [3]. We supposed that crack starts to be propagating when the strain energy release rate at any point of material exceeds the material fracture toughness. In the case of several fracture modes acting at the same time, the multimodal power law was utilized in the predictions. For the sake of simplicity we assumed that there is no variation of material fracture toughness along the material gradation. It is acceptable when the crack is growing in the ceramic-rich region only.

It was obtained that the developed FE model enables to perform accurate and reliable predictions in FGM plates. The comparisons between the known analytical solutions and finite element results showed a very good agreement. The temperature and thermally-induced stress distributions within the plate under both steady state and transient regimes were considered for various gradation profiles. The calculations clearly demonstrated the great influence of spatial material properties on the level and redistributions of temperature and thermal stresses. From the thermal cracking analysis was concluded that the higher is content of metal in the graded ceramic/metal plate, the higher ability of the FGM plate to resist a thermal crack. However further investigations are required to clarify the influence of the toughness, which, in general, has to also have a gradual spatial variation.

LITERATURE

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CHARACTERIZATION OF COMPLEX MODULI FOR PLASTIC MATERIAL UNDER HARMONIC LOADING

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The problem of characterization of material response to harmonic loading is addressed. In the present research, Zairi unified constitutive model [1] is used to predict the time dependent inelastic response of amorphous glassy polymer, a polycarbonate (PC). The approach that uses the complex-value amplitude relations is preferred rather than direct numerical integration of the complete set of constitutive equation for the material. The key point of the approach adopted lays in determination of complex moduli, i.e. storage and loss moduli under harmonic loading. It is usually done by making use of equivalent linearization technique. It is shown that this technique leads to overestimation of stress amplitude. To avoid this, the modified equivalent linearization technique is used. It relies on special procedure for determination of storage modulus which based on the usage of cyclic stress–strain diagram [2]. The formulae for both storage and loss moduli are listed below

$$G'_N(e_0, \omega) = \left[\frac{\sigma_{aN}^{\prime 2}(e_0, \omega)}{4e_0^2} - G_N^{\prime 2}(e_0, \omega) \right]^{1/2},$$

$$\lambda'_N(e_0, \omega) = \left[\frac{\sigma_{aN}^{\prime 2}(e_0, \omega)}{4e_0^2} - \lambda_N^{\prime 2}(e_0, \omega) \right]^{1/2},$$

$$G''_N = \frac{\langle D' \rangle_N}{\omega e_0^2}, \quad \lambda''_N = \frac{G''_N}{G_0},$$

$$\langle (\cdot) \rangle_N = \frac{1}{T} \int_{T(N-1)}^{TN} (\cdot) dt, \quad T = \frac{2\pi}{\omega},$$

where $\sigma'_{aN} = \sigma'_{aN}(e_0, \omega)$ is generalized cyclic diagram, which relate the ranges of the stress intensity in the N^{th} cycle with the intensity of strain-range tensor $e_0^2 = \mathbf{e}' : \mathbf{e}' + \mathbf{e}'' : \mathbf{e}''$; \tilde{G}_N is complex shear modulus and $\tilde{\lambda}_N$ is complex plasticity factor that relates the complex amplitudes of the deviator of total strain, $\tilde{\mathbf{e}}$, inelastic strain, $\tilde{\mathbf{e}}^{\text{in}}$, and the stress deviator, $\tilde{\boldsymbol{\sigma}}'$, in the N^{th} cycle.

Obtained histories of main field variables evolution were used to find the stress–strain cyclic diagram and real as well as imaginary parts of complex shear modulus with making use of both standard and modified equivalent linearization techniques. The prediction of stress amplitude obtained in the frame of the former scheme overestimates the actual value for

more than 10% while the latter scheme gives it with desirable accuracy.

Fig. 1 illustrates the mechanical hysteresis phenomenon under cyclic loading that enable one to measure the phase shift between stress and total strain. As it was mentioned above, the actual loop can be approximated with making use of either standard or modified equivalent linearization scheme. In the figure, the actual loop (line 1) is shown along with the loops calculated in the frame of standard (line 2) and modified (line 3) equivalent linearization techniques.

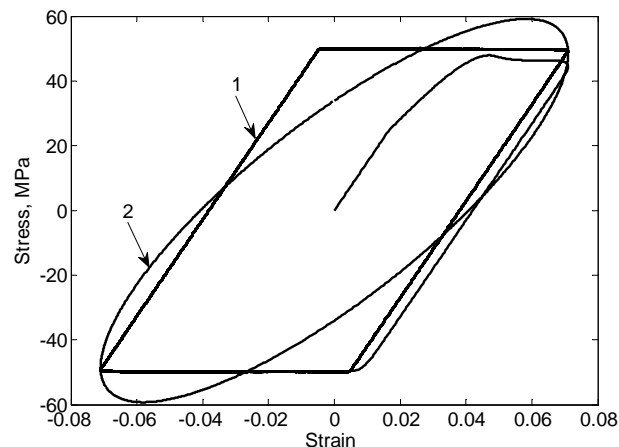


Fig.1. Hysteresis loops

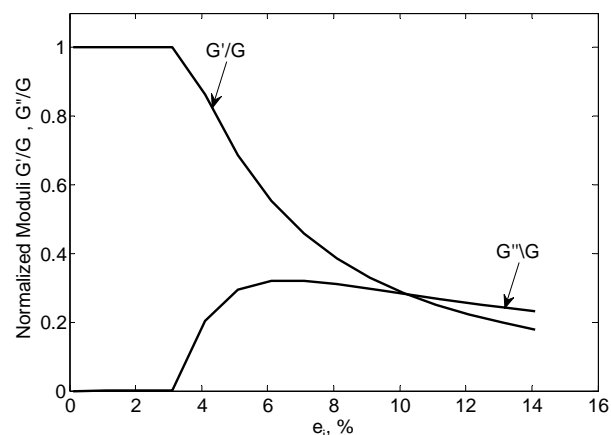


Fig.2. Normalized values of the real and imaginary parts of complex shear modulus for PC polymer

The normalized improved values of G'/G and G''/G found according to the modified scheme for frequency 1 Hz at steady-state cyclic regime and constant temperature are shown in Fig.2 for wide range of loading amplitudes. The behavior is typical for polymeric materials and is characterized by the presence of peak in the loss modulus. This diagram shows the highest losses occur at strain amplitude of about seven percent for this type of polymer.

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