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MATHEMATICAL ANALYSIS OF HIP JOINT PROSTHESIS

Damage, significantly impairing the quality of life, coxarthrosissal can meet a young age. Therefore, nowadays the prosthesis implantation is performed in the most active life period of the patient. Besides, it is known that the replacement of the prosthesis will be necessary later [1].

When acetabular cups were measured, the results were analysed and the differences were determined, I took the coordinate systems needed to describe the movement of the prosthesis.

INTRODUCTION

Arthrosis is one of the most common musculoskeletal disorders in our lifetime, almost everyone has heard about it or has met someone who was operated due to the disease.

Figure 1 – Location of prosthesis – with the help of medical imaging [2]

The wear of the hip joint involves joint pain, limping and reduced mobility.

First, in the morning, while getting up people feels as if hips became rusty, then it disappears later, and hips start "operating". After prolonged sitting or lying down there are similar complaints. The movement of the joint progressively decreases, due to the pain and the restraint of movement, the person is limping. Later the pain becomes persistent, walking ability decreases, the person has to stop more and more often, and it is difficult to put on socks and shoes [3].

Type of the hip joint prosthesis

The anatomic hip prosthesis may be cemented and cementless depending on the type of fixation used to hold the implant in place.

The so called classical cement fixation has a history of about 6 decades.

The main point of this implant family is that the plastic sleeve made of plastic with high abrasion resistance and the femur stem made of rapid hardening polyacrylate. This is the most commonly used type of prosthesis, which makes almost any prosthesis implant technically feasible [3].

Construction (Figure 2):

- socket;
- metal head - ball;
- stem of different types.

The method without cement is recommended especially for younger patients. In their case the organism itself provides the fixture of the prosthesis [3], as the bone "grows in" the surface of the prosthesis from the bonebed matching the size of the prosthesis.

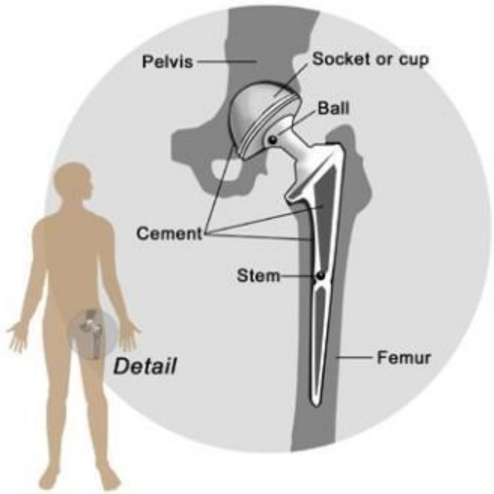

Figure2 – Parts of the prosthesis [3]

1. DEFINITION OF A COORDINATE SYSTEM FOR MODELLING

I was created a mathematical model for the hip prosthesis, it is necessary to determine the coordinate systems that make the movement of the acetabular cup and the femur head more transparent. [5, 6] We need the coordinate system of the acetabular cup as a standing coordinate system, since the socket is getting fixed in the basin bone [7]. However, the femur head can move both in a standing coordinate system and in a rotating coordinate system.

The former is the resting position, the latter is the case when the patient moves.

These coordinate systems were applied on the basis of the general mathematical model of Illés Dudás (Figure 3) [4, 8].

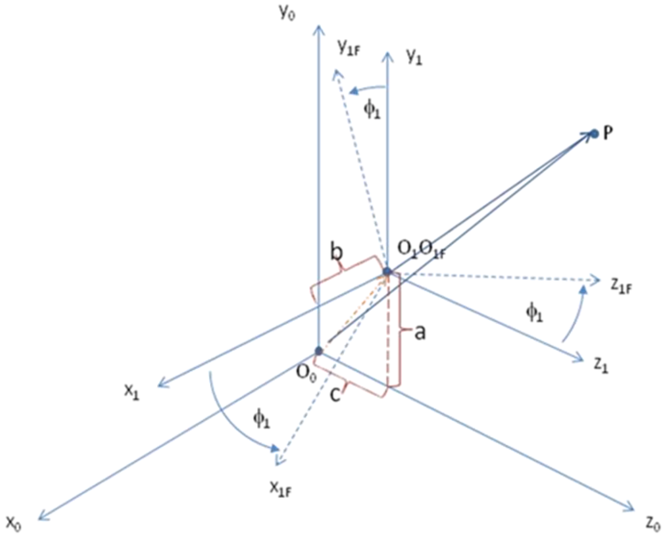

Figure 3 – Coordinate systems

where:

$K_0(x_0, y_0, z_0)$ – standing coordinate system of the acetabular cup

$K_1(x_1, y_1, z_1)$ – standing coordinate system of the femur head

$K_{1F}(x_{1F}, y_{1F}, z_{1F})$ – moving (rotating) coordinate system of the femur head

$d(b, a, c)$ – split distance between the acetabular cup and femur head (Figure 4)

φ_1 – angle value

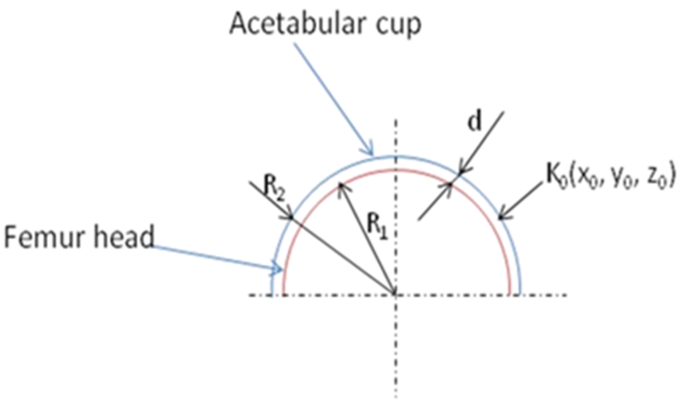

Figure 4 – Status of the acetabular cup and femur head

2. DETERMINATION OF THE TRANSFORMATION MATRICES FOR THE ANALYSIS OF THE MOVING

The two-parameter equation of the femur head:

$$\overrightarrow{r_{1F}} = \begin{bmatrix} R_1 \cdot \sin u \cdot \cos\vartheta \\ R_1 \cdot \cos u \\ R_1 \cdot \sin u \cdot \sin\vartheta \end{bmatrix} \quad (1)$$

The matrix below shows how to get from K_0 to K_1 (Figure 3):

$$M_{0,1} = \begin{bmatrix} 1 & 0 & 0 & -b \\ 0 & 1 & 0 & a \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The following matrix shows how to get from K_1 to K_{1F} (Figure 5):

$$M_{1,1F} = \begin{bmatrix} +\cos\phi_1 & +\sin\phi_1 & 0 & 0 \\ 0 & +\cos\phi_1 & +\sin\phi_1 & 0 \\ +\sin\phi_1 & 0 & +\cos\phi_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3)$$

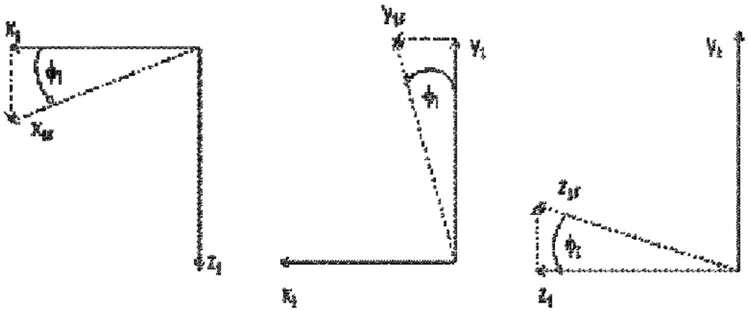

Figure 5 – Figures to the $M_{1,1F}$ matrix

The following total transformation matrix shows how to get from the standing coordinate system of the acetabular cup to the rotating coordinate system of the femur head:

$$M_{0,1F} = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & 0 & -b \\ 0 & \cos\phi_1 & \sin\phi_1 & a \\ \sin\phi_1 & 0 & \cos\phi_1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Based on these we can determinate the two parameter-equation of the acetabular cup:

$$\overrightarrow{r_{0,cup}} = M_{0,1F} \cdot \overrightarrow{r_{1,F}} = \begin{bmatrix} \cos\phi_1 \cdot \{(R_1 + d) \cdot \sin u \cdot \cos\vartheta\} + \sin\phi_1 \cdot \{(R_1 + d) \cdot \cos u\} - b \\ \cos\phi_1 \cdot \{(R_1 + d) \cdot \cos u\} + \sin\phi_1 \cdot \{(R_1 + d) \cdot \sin u \cdot \sin\vartheta\} + a \\ \sin\phi_1 \cdot \{(R_1 + d) \cdot \sin u \cdot \cos\vartheta\} + \cos\phi_1 \cdot \{(R_1 + d) \cdot \sin u \cdot \sin\vartheta\} + c \\ 1 \end{bmatrix} \quad (5)$$

3. SUMMARY

We have introduced the applications and types of hip prosthesis (acetabular cup, femur head). It can be seen that these prostheses are expensive components with very complex geometry. It is very important to choose the material to be used. Relying on the general mathematical model of Illés Dudás, I used a mathematical model to examine the function and geometry of the prosthesis. Using this model, additional wear, geometry and manufacturing tests can be carried out as a continuation of the research. The mathematical analysis I have made can be applied to any hip prosthesis geometry.

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