МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

НАЦІОНАЛЬНИЙ ТЕХНІЧНИЙ УНІВЕРСИТЕТ
«ХАРКІВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ»

Lysenko L., Makhotilo K., Cherkashyna H.

OPTIMIZATION PROBLEMS OF POWER SYSTEM ECONOMIC DISPATCH

Study Guide for practical Classes
for Students of Specialty 141
«Power engineering, Electric Engineering and Electromechanics»
«Electric Power System» Department
for full-time and distance education

Рекомендовано
редакційно-видавничою
радою університету,
протокол № 1 від 25.02.21 р.

Харків
2021

UDC 621.311:681.5
O 66

Authors:
Liudmyla Lysenko, Cand. Of Sciences, Associate Professor,
Kostiantyn Makhotilo, Cand. Of Sciences, Senior Researcher,
Halyna Cherkashyna, Cand. Of Sciences, Associate Professor
National Technical University
«Kharkiv Polytechnic Institute»

Reviewers:
Anna Tischenko, Cand. Of Sciences, Associate Professor,
NTU KhPI Electric Power Stations Department
Halyna Omeliamenko, Cand. Of Sciences, Associate Professor,
NTU KhPI Electric Power Transmission Department

Lysenko L., Makhotilo K., Cherkashyna H.
O 66 **Optimization problems of power system economic dispatch** : study guide /
L. Lysenko, K. Makhotilo, H. Cherkashyna. Kharkiv : FOP Panov A.M., 2021. 130 p.
ISBN 978-617-7947-78-2

The study guide describes and illustrates with examples principles of building mathematical models of optimal power system operation problems and their solving with application of MS Excel.

The study guide is intended for students of Electric Power Engineering, Electrical Engineering and Electromechanics specialization and post-graduate students of Power Engineering specialties.

Illustrations 68. Bibl. titles: 8

UDC 621.311

ISBN 978-617-7947-78-2

INTRODUCTION

The study guide considers mathematical modeling techniques for problems of linear and nonlinear programming in the field of power engineering. Control of power system operation requires specifying economic loading of power plants under various operating conditions as well as optimal electric grid dispatch with minimum active power loss.

The first chapter describes linear programming problems of optimal fuel scheduling among power plants and economic electric grid loading.

The second chapter deals with economic dispatch problems for thermal and hydropower plants taking into account power loss in the electric grid.

The third chapter analyzes minimization of active power loss through optimal reactive power compensation in electric grids of various configurations.

The study guide considers numerous examples of various-type optimization problems of power engineering focusing on building mathematical models and searching for optimal solutions with application of Microsoft Excel Solver.

The manual contains information that may be helpful for students taking master’s and PhD programs in Electric Power Engineering.

1. LINEAR PROGRAMMING PROBLEMS IN POWER ENGINEERING

1.1. A fuel delivery scheduling problem [1]

1.1.1. Linear programming problem formulation

Linear programming (LP) deals with search for extreme of a linear function with a limited number of variables (decision variables) under linear constraints. The general mathematical formulation of an LP problem is the following.

Minimize a linear objective function

$$Z(X) = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n \Rightarrow \min \tag{1.1}$$

subject to constraints

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n (\le, =, \ge) b_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n (\le, =, \ge) b_m, \\ x_j \ge 0, \qquad j = 1, \ldots, n \end{cases} \tag{1.2}$$

where c_j, a_{ij}, and b_i are constants and x_j are decision variables.

Such problems can be solved with MS Excel Solver [2].

1.1.2. A fuel delivery scheduling problem formulation

A coal supplying company has negotiated a contract with a generating company (GenCo) on supplying coal to GenCo’s two coal-burning power plants (PP) on a regular basis. According to the contract, 18000 ton of coal must be supplied to both or either plant every 8 hours for two days. The coal costs 92 $/t, delivery expenses neglected. The heat value of the coal is 6440 kcal/kg (or 6.44 Gcal/t).

Maximum coal storage capacity available at every plant is enough for the plant’s 10-day operation with the rated capacity. The initial inventories of the plants are coal volumes for 3-day operation with the rated capacity. After two days of coal supply, the final inventories must provide the plant operation for 6 days.

Both power plants must remain on-line for the two-day period. The overall output of the plants for each 8-hour period is given in table 1.1, each plant output assumed constant during the considered period.

Table 1.1 – Assigned overall output for two coal-burning power plants

Operational period, time of the day	Load, MW	Operational period, time of the day	Load, MW
1st day		2nd day	
$1^{\underline{00}}$ – $8^{\underline{00}}$	1300	$1^{\underline{00}}$ – $8^{\underline{00}}$	1340
$9^{\underline{00}}$ – $16^{\underline{00}}$	1800	$9^{\underline{00}}$ – $16^{\underline{00}}$	1880
$17^{\underline{00}}$ – $24^{\underline{00}}$	2220	$17^{\underline{00}}$ – $24^{\underline{00}}$	2250

Power plant #1 comprises three 500 MW generating units, each characterized with the boiler heat rate, Gkal/hr,

$$H_1(P_1) = 140.6 + 2.33P_1. \tag{1.3}$$

Power plant #2 comprises six 130 MW generating units, each characterized with the boiler heat rate, Gkal/hr,

$$H_2(P_2) = 19.7 + 2.1P_2. \tag{1.4}$$

The data on minimum and maximum output and fuel consumption of the units are given in table 1.2.

Table 1.2 – Lower and upper bounds for the output of the considered power plant generating units

Unit	Min output, $P_{\min}$, MW	Max output, $P_{\max}$, MW
PP1 generating unit	150	500
PP2 generating unit	40	140

The objective is to determine the economic fuel supply schedule so as to minimize the overall operating costs of the two power plants over the period of two days.

1.1.3. Mathematical formulation of the fuel delivery scheduling problem

To solve this problem, we assume that the units of each power plant are operated at a constant rate during the considered period (six 8-hour periods). The coal delivery takes place at the beginning of every 8-hour period.

The coal consumption F(P) for every unit is found by dividing the heat rates (1.3) and (1.4) by the coal heat value.

For PP1 generating unit, $F_1(P_1)$, t/hr

$$F_1(P_1) = (140.6 + 2.33 \cdot P_1)/6.44 = 21.8323 + 0.3318 \cdot P_1. \qquad (1.5)$$

For PP2 generating unit, $F_2(P_2)$, t/hr

$$F_2(P_2) = (19.7 + 2.1 \cdot P_2)/6.44 = 2.9767 + 0.3261 \cdot P_2. \qquad (1.6)$$

The overall coal consumption for an 8-hour period is found as

$$F_{8\text{hr}}(j) = 8 \cdot [n_{\text{PP1}} \cdot F_1(j) + n_{\text{PP2}} \cdot F_2(j)], \qquad (1.7)$$

where n_{PP1} and n_{PP2} are numbers of generating units under operation at, correspondingly, PP1 and PP2;

$F_1(j)$ and $F_1(j)$ are fuel consumption by, correspondingly, PP1 generating unit and PP2 generating unit calculated with (1.5) and (1.6) at period j.

The objective function is minimization of the overall operating costs for the six 8-hour periods

$$Z = z_{\text{coal}} \cdot \sum_{j=1}^{6} [F_{8\text{hr}}(j)] \rightarrow \min, \qquad (1.8)$$

where z_{coal} is the coal price.

The optimization parameters are the number of units under operation at every plant at every 8-hour period, $n_{\text{PP1}}(j)$ and $n_{\text{PP2}}(j)$, the output of the generating units at every 8-hour period $P_1(j)$ and $P_2(j)$, and the coal volumes delivered to every power plant $D_{\text{PP1}}(j)$ and $D_{\text{PP2}}(j)$ at the beginning of every 8-hour period, $j = 1,\ldots,6$.

The constraints of the problem are the specified bounds of power plant output, power balance in the power system, the contracted volumes of coal delivery, and the required coal inventories at the plants after the contract period is over.

1. At every power plant, the output of a generating unit is given the upper and lower bounds (see table 1.2) that are assumed identical for all units at the plant:

$$\begin{aligned} 150 \le P_1 \le 500 \\ 40 \le P_2 \le 130 \end{aligned} \quad . \tag{1.9}$$

Also we assume that the generating units of one power plant produce the same power and the overall output of the power plant is found by multiplication of the number of operating units by their output:

$$\begin{aligned} P_{\mathrm{PP1}}(j) = n_{\mathrm{PP1}} \cdot P_1(j) \\ P_{\mathrm{PP2}}(j) = n_{\mathrm{PP2}} \cdot P_2(j) \end{aligned} \quad . \tag{1.10}$$

2. During each 8-hour period, the total power produced by the plants, must be equal to the scheduled load to be supplied (see table 1.1):

$$\begin{aligned} P_{\mathrm{PP1}}(1) + P_{\mathrm{PP2}}(1) = 1300 \\ P_{\mathrm{PP1}}(2) + P_{\mathrm{PP2}}(2) = 1800 \\ P_{\mathrm{PP1}}(3) + P_{\mathrm{PP2}}(3) = 2220 \\ P_{\mathrm{PP1}}(4) + P_{\mathrm{PP2}}(4) = 1340 \\ P_{\mathrm{PP1}}(5) + P_{\mathrm{PP2}}(5) = 1880 \\ P_{\mathrm{PP1}}(6) + P_{\mathrm{PP2}}(6) = 2250 \end{aligned} \quad . \tag{1.11}$$

3. Similarly, during each 8-hour period, the coal deliveries $D_1(j)$ and $D_2(j)$, $j=1,\ldots,4$, to power plant PP1 and power plant PP2, respectively, must sum to 18000 tons:

$$\begin{aligned}
&D_{\mathrm{PP1}}(1)+D_{\mathrm{PP2}}(1)=18000\\
&D_{\mathrm{PP1}}(2)+D_{\mathrm{PP2}}(2)=18000\\
&D_{\mathrm{PP1}}(3)+D_{\mathrm{PP2}}(3)=18000\\
&D_{\mathrm{PP1}}(4)+D_{\mathrm{PP2}}(4)=18000\\
&D_{\mathrm{PP1}}(5)+D_{\mathrm{PP2}}(5)=18000\\
&D_{\mathrm{PP1}}(6)+D_{\mathrm{PP2}}(6)=18000
\end{aligned}\;. \tag{1.12}$$

4. The volume of coal at every plant at the beginning of each j 8-hour period $V_{\mathrm{PP}i}(j)$ plus delivery of coal to that plant $D_{\mathrm{PP}i}(j)$ minus coal burnt at the plant during 8 hours $F_{\mathrm{8hrPP}i}(j)$ gives the volume of coal remaining at the beginning of the next, $(j+1)$, 8-hour period $V_{\mathrm{PP}i}(j+1)$:

$$\begin{aligned}
&V_{\mathrm{PP1}}(1)+D_{\mathrm{PP1}}(1)-F_{\mathrm{8hrPP1}}(1)=V_{\mathrm{PP1}}(2)\\
&V_{\mathrm{PP2}}(1)+D_{\mathrm{PP2}}(1)-F_{\mathrm{8hrPP2}}(1)=V_{\mathrm{PP2}}(2)\\
&V_{\mathrm{PP1}}(2)+D_{\mathrm{PP1}}(2)-F_{\mathrm{8hrPP1}}(2)=V_{\mathrm{PP1}}(3)\\
&V_{\mathrm{PP2}}(2)+D_{\mathrm{PP2}}(2)-F_{\mathrm{8hrPP2}}(2)=V_{\mathrm{PP2}}(3)\\
&V_{\mathrm{PP1}}(3)+D_{\mathrm{PP1}}(3)-F_{\mathrm{8hrPP1}}(3)=V_{\mathrm{PP1}}(4)\\
&V_{\mathrm{PP2}}(3)+D_{\mathrm{PP2}}(3)-F_{\mathrm{8hrPP2}}(3)=V_{\mathrm{PP2}}(4)\\
&V_{\mathrm{PP1}}(4)+D_{\mathrm{PP1}}(4)-F_{\mathrm{8hrPP1}}(4)=V_{\mathrm{PP1}}(5)\\
&V_{\mathrm{PP2}}(4)+D_{\mathrm{PP2}}(4)-F_{\mathrm{8hrPP2}}(4)=V_{\mathrm{PP2}}(5)\\
&V_{\mathrm{PP1}}(5)+D_{\mathrm{PP1}}(5)-F_{\mathrm{8hrPP1}}(5)=V_{\mathrm{PP1}}(6)\\
&V_{\mathrm{PP2}}(5)+D_{\mathrm{PP2}}(5)-F_{\mathrm{8hrPP2}}(5)=V_{\mathrm{PP2}}(6)\\
&V_{\mathrm{PP1}}(6)+D_{\mathrm{PP1}}(6)-F_{\mathrm{8hrPP1}}(6)=V_{\mathrm{PP1}}(\mathrm{end})\\
&V_{\mathrm{PP2}}(6)+D_{\mathrm{PP2}}(6)-F_{\mathrm{8hrPP2}}(6)=V_{\mathrm{PP2}}(\mathrm{end})
\end{aligned}\;. \tag{1.13}$$

The initial inventories $V_{\mathrm{PP}i}(1)$ at the plants at the beginning of the first 8-hour period are coal volumes that are available at the plants and enough for the plants operation at the rated capacity for three days, tons:

$$V_{\text{PP1}}(1) = 3 \cdot 24 \cdot 3 \cdot (21.8323 + 0.3318 \cdot 500) = 43790,$$

$$V_{\text{PP2}}(1) = 3 \cdot 24 \cdot 6 \cdot (2.9767 + 0.3261 \cdot 130) = 19599.$$

After termination of the contract, at the end of the two-day coal delivery period, the inventories at the plants must increase to provide operation of the plants at the rated capacity for at least 6 days, tons:

$$V_{\text{PP1}}(\text{end}) = 6 \cdot 24 \cdot 3 \cdot (21.8323 + 0.3318 \cdot 500) = 87580.621;$$

$$V_{\text{PP2}}(\text{end}) = 6 \cdot 24 \cdot 6 \cdot (2.9767 + 0.3261 \cdot 130) = 39197.963.$$

At the same time, the resultant inventories $V_{\text{PP1}}(\text{end})$ and $V_{\text{PP2}}(\text{end})$ cannot exceed the available coal storage capacities that provide the plants operation for 10 days at the plant. The coal storage capacity at every power plant is calculated as, tons,

$$V_{\text{PP1}}(\text{capacity}) = 10 \cdot 24 \cdot 3 \cdot (21.8323 + 0.3318 \cdot 500) = 87580.621;$$

$$V_{\text{PP2}}(\text{capacity}) = 10 \cdot 24 \cdot 6 \cdot (2.9767 + 0.3261 \cdot 130) = 39197.963.$$

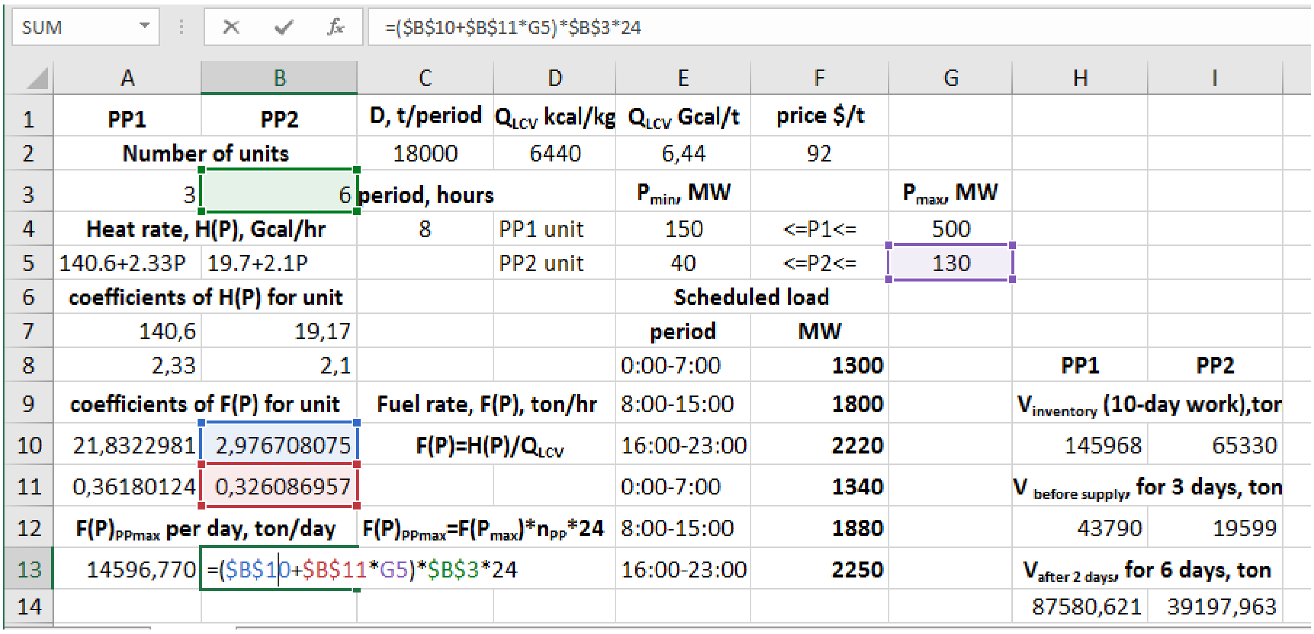
So we can write the constraints for the resultant coal inventories at the plants

$$\begin{matrix} 87580.621 \le V_{\text{PP1}}(\text{end}) \le 145968 \\ 39197.963 \le V_{\text{PP2}}(\text{end}) \le 65330 \end{matrix}. \qquad (1.14)$$

1.1.4. Solution of the fuel delivery scheduling problem

The fuel delivery scheduling problem is solved with MS Excel Solver. For this, in Excel sheet, the initial data of the problem must be entered and mathematical model (1.8) – (1.14) must be transferred with the help of Excel functions (mainly, **SUM** and **SUMPRODUCT**) and Excel expressions that are introduced manually.

The Excel screen form with the introduced initial data and preliminary calculations is shown in fig. 1.1. Each initial parameter value should be entered to a separate cell to allow writing Excel formulae rather referring to cells than inserting numerical coefficients from the keyboard. Also it is very helpful to give explanations on what parameters are introduced into which cells and to add measurement units. This facilitates searching for mistakes in case the solution is not found.



`=($B$10+$B$11*G5)*$B$3*24`

	A	B	C	D	E	F	G	H	I
1	**PP1**	**PP2**	**D, t/period**	**Q_{LCV} kcal/kg**	**Q_{LCV} Gcal/t**	**price $/t**			
2	**Number of units**		18000	6440	6,44	92			
3	3	6	**period, hours**		**P_{min}, MW**		**P_{max}, MW**		
4	**Heat rate, H(P), Gcal/hr**		8	PP1 unit	150	<=P1<=	500		
5	140.6+2.33P	19.7+2.1P		PP2 unit	40	<=P2<=	130		
6	**coefficients of H(P) for unit**				**Scheduled load**				
7	140,6	19,17			**period**	**MW**			
8	2,33	2,1			0:00-7:00	**1300**		**PP1**	**PP2**
9	**coefficients of F(P) for unit**		**Fuel rate, F(P), ton/hr**		8:00-15:00	**1800**		**$V_{inventory}$ (10-day work),tor**	
10	21,8322981	2,976708075	**$F(P)=H(P)/Q_{LCV}$**		16:00-23:00	**2220**		145968	65330
11	0,36180124	0,326086957			0:00-7:00	**1340**		**$V_{before\ supply}$, for 3 days, ton**	
12	**$F(P)_{PPmax}$ per day, ton/day**		**$F(P)_{PPmax}=F(P_{max})*n_{PP}*24$**		8:00-15:00	**1880**		43790	19599
13	14596,770	`=($B$10+$B$11*G5)*$B$3*24`			16:00-23:00	**2250**		**$V_{after\ 2\ days}$, for 6 days, ton**	
14								87580,621	39197,963

Figure 1.1 – The screen presentation of the fuel delivery scheduling problem (the pointer is in cell **B13** to show the Excel expression written to calculate PP2 maximum daily output)

Fig. 1.2 shows arrangement of the optimization parameters, the objective function and the constraints in the Excel sheet.

Cells **A16:L16** contain formula (1.7) that computes fuel consumption by the corresponding plant during the corresponding period. Objective function (1.8) is introduced in cell **M16**.

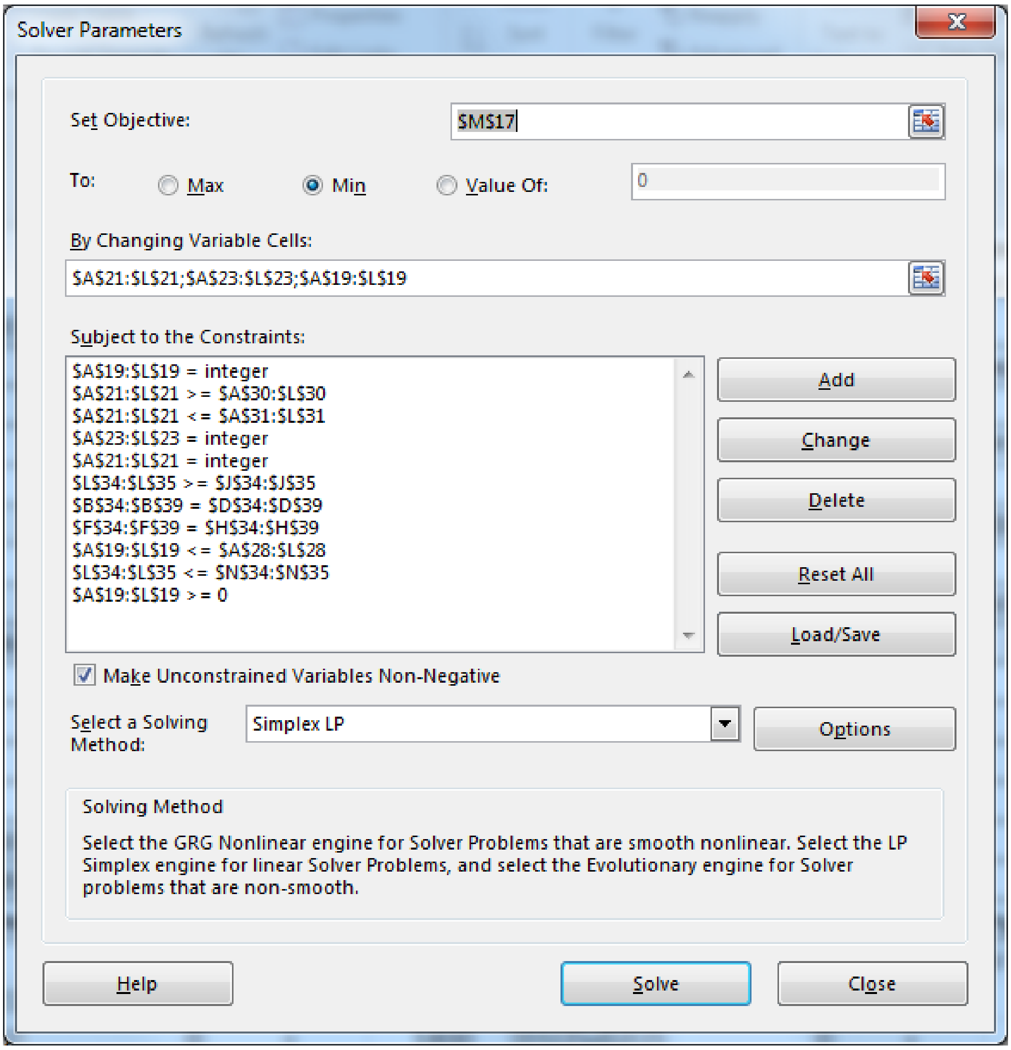
Cell **A19:L19** are assigned for the numbers of operating units at each power plant at each 8-hour period. The available number of the generating units at the plants are given in cell **A28:L28**.

Cell **A21:L21** are assigned for each plant generating unit outputs during each 8-hour period. The corresponding lower and upper bounds (1.9) for the unit generation are introduced in cells **A30:L30** and **A31:L31**, respectively.

Excel expressions for power balance equations (1.11) and coal delivery requirements (1.12) are written in cells **B34:B39** and **F34:F39**, respectively.

Expressions (1.13) describing coal volumes at the power plants at the beginning of every 8-hour period are introduced in cells **A25:N25**, cells **A25:B25** containing values of the initial inventories $V_{PP1}(1)$ and $V_{PP2}(1)$ and cells **M25:N25** containing calculation formulae for the resultant inventories $V_{PP1}(\mathrm{end})$ and $V_{PP2}(\mathrm{end})$. The constraints (1.14) on the coal volume at each power plant, $V_{PP1}(\mathrm{end})$ and $V_{PP2}(\mathrm{end})$, after the contracted supply is over are introduced, correspondingly, in cells **J34:J35** and **N34:N35**.

The Solver parameters window with the introduced settings for the considered problem is shown in fig. 1.3.



Set Objective: `$M$17`

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells:

```
$A$21:$L$21;$A$23:$L$23;$A$19:$L$19
```

Subject to the Constraints:

```
$A$19:$L$19 = integer
$A$21:$L$21 >= $A$30:$L$30
$A$21:$L$21 <= $A$31:$L$31
$A$23:$L$23 = integer
$A$21:$L$21 = integer
$L$34:$L$35 >= $J$34:$J$35
$B$34:$B$39 = $D$34:$D$39
$F$34:$F$39 = $H$34:$H$39
$A$19:$L$19 <= $A$28:$L$28
$L$34:$L$35 <= $N$34:$N$35
$A$19:$L$19 >= 0
```

☑ Make Unconstrained Variables Non-Negative

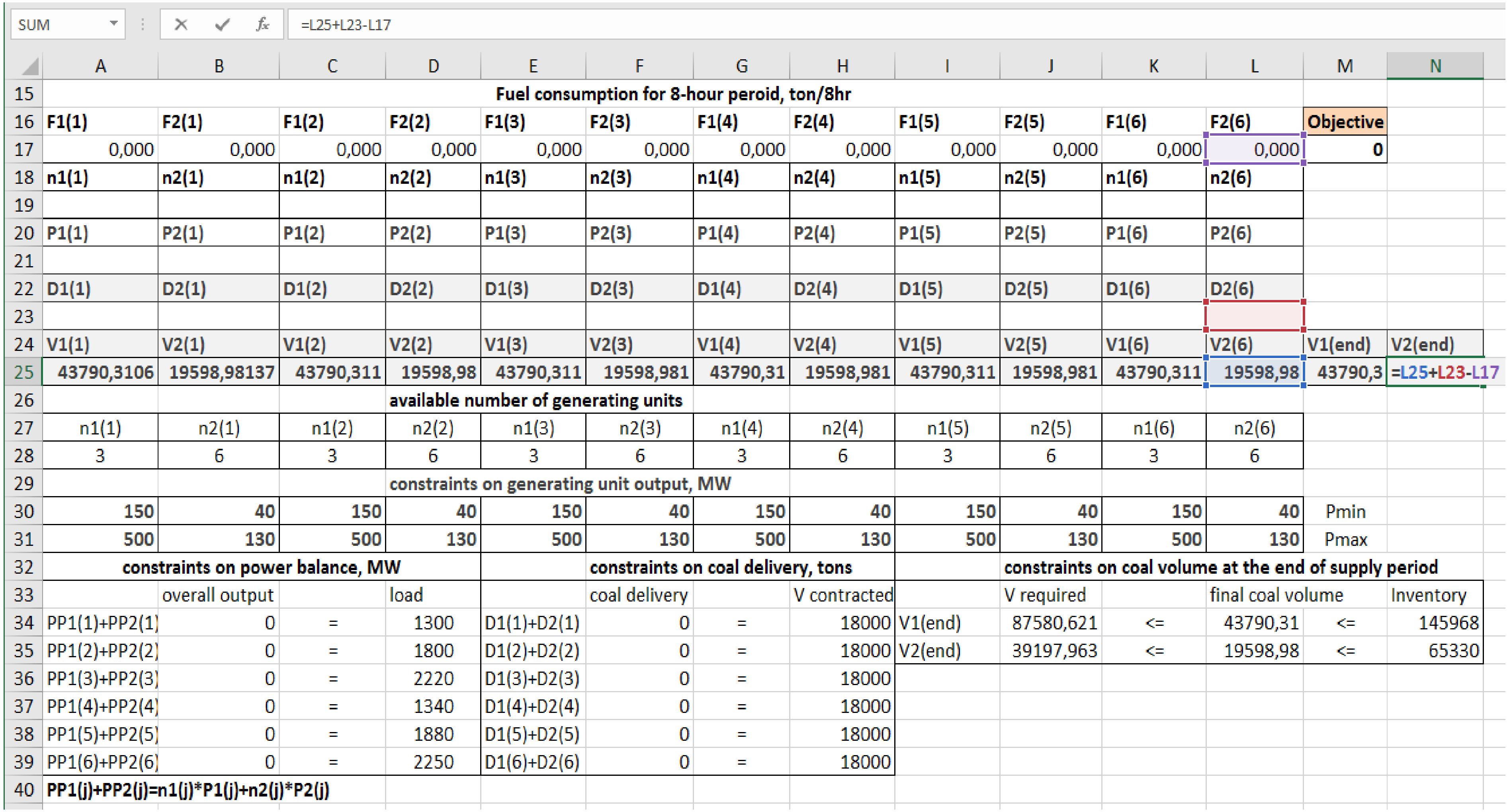
Select a Solving Method: Simplex LP

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 1.3 – The **Solver parameters** window with introduced settings

The obtained solution to the considered fuel delivery scheduling problem is shown in fig. 1.4.

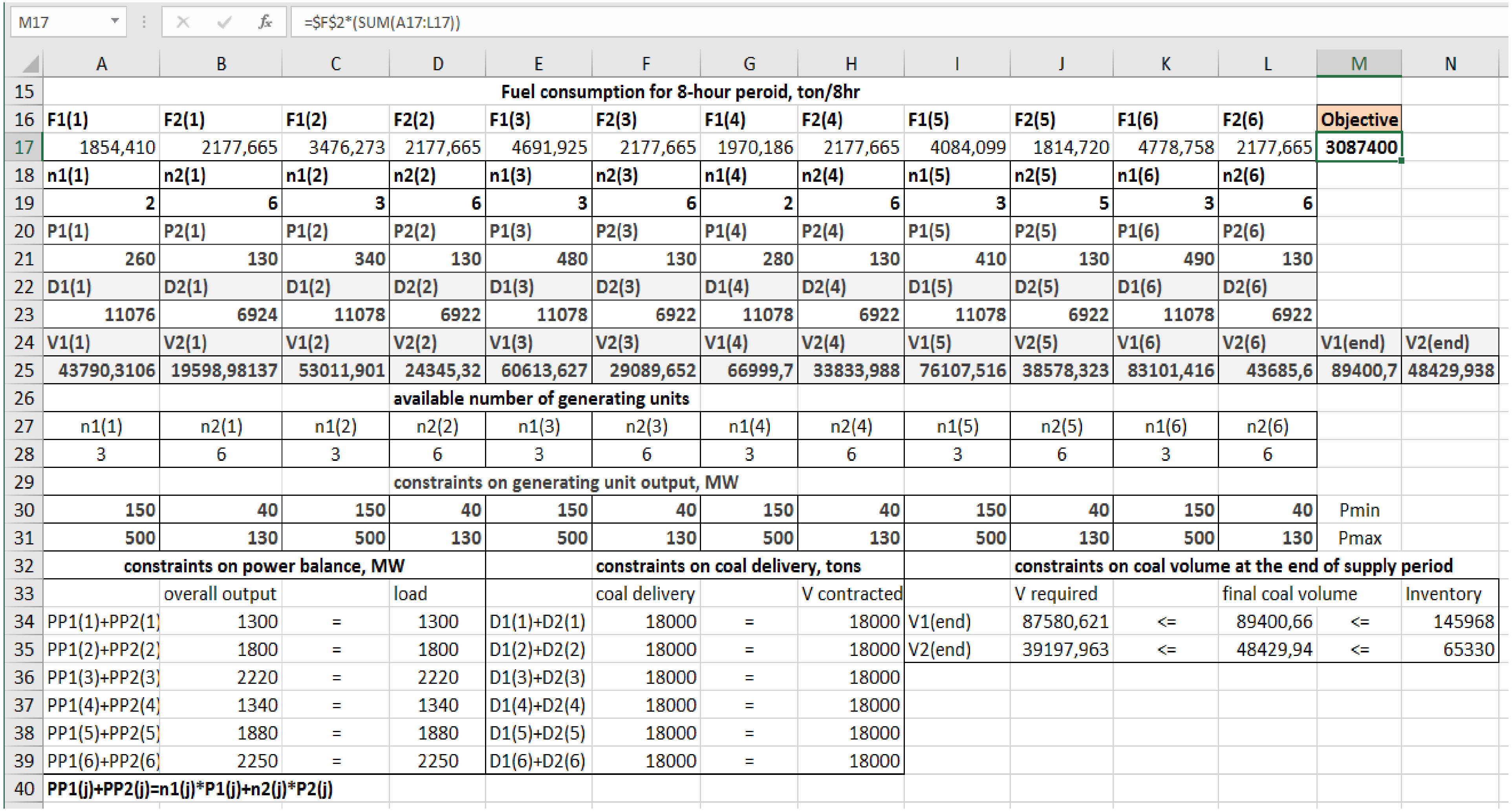


SUM

=L25+L23-L17

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
15	Fuel consumption for 8-hour peroid, ton/8hr													
16	**F1(1)**	**F2(1)**	**F1(2)**	**F2(2)**	**F1(3)**	**F2(3)**	**F1(4)**	**F2(4)**	**F1(5)**	**F2(5)**	**F1(6)**	**F2(6)**	**Objective**	
17	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	**0**	
18	**n1(1)**	**n2(1)**	**n1(2)**	**n2(2)**	**n1(3)**	**n2(3)**	**n1(4)**	**n2(4)**	**n1(5)**	**n2(5)**	**n1(6)**	**n2(6)**		
19														
20	P1(1)	P2(1)	P1(2)	P2(2)	P1(3)	P2(3)	P1(4)	P2(4)	P1(5)	P2(5)	P1(6)	P2(6)		
21														
22	D1(1)	D2(1)	D1(2)	D2(2)	D1(3)	D2(3)	D1(4)	D2(4)	D1(5)	D2(5)	D1(6)	D2(6)		
23														
24	V1(1)	V2(1)	V1(2)	V2(2)	V1(3)	V2(3)	V1(4)	V2(4)	V1(5)	V2(5)	V1(6)	V2(6)	V1(end)	V2(end)
25	43790,3106	19598,98137	43790,311	19598,98	43790,311	19598,981	43790,31	19598,981	43790,311	19598,981	43790,311	19598,98	43790,3	=L25+L23-L17
26	available number of generating units													
27	n1(1)	n2(1)	n1(2)	n2(2)	n1(3)	n2(3)	n1(4)	n2(4)	n1(5)	n2(5)	n1(6)	n2(6)		
28	3	6	3	6	3	6	3	6	3	6	3	6		
29	constraints on generating unit output, MW													
30	150	40	150	40	150	40	150	40	150	40	150	40	Pmin	
31	500	130	500	130	500	130	500	130	500	130	500	130	Pmax	
32	constraints on power balance, MW					constraints on coal delivery, tons				constraints on coal volume at the end of supply period				
33		overall output		load		coal delivery		V contracted		V required		final coal volume		Inventory
34	PP1(1)+PP2(1)	0	=	1300	D1(1)+D2(1)	0	=	18000	V1(end)	87580,621	<=	43790,31	<=	145968
35	PP1(2)+PP2(2)	0	=	1800	D1(2)+D2(2)	0	=	18000	V2(end)	39197,963	<=	19598,98	<=	65330
36	PP1(3)+PP2(3)	0	=	2220	D1(3)+D2(3)	0	=	18000						
37	PP1(4)+PP2(4)	0	=	1340	D1(4)+D2(4)	0	=	18000						
38	PP1(5)+PP2(5)	0	=	1880	D1(5)+D2(5)	0	=	18000						
39	PP1(6)+PP2(6)	0	=	2250	D1(6)+D2(6)	0	=	18000						
40	**PP1(j)+PP2(j)=n1(j)*P1(j)+n2(j)*P2(j)**													

Figure 1.2 – Arrangement of the optimization parameters, the objective function and the constraints in the Excel sheet (the pointer is in cell **N25** to show the written Excel expression that calculates the last equation in (1.13))

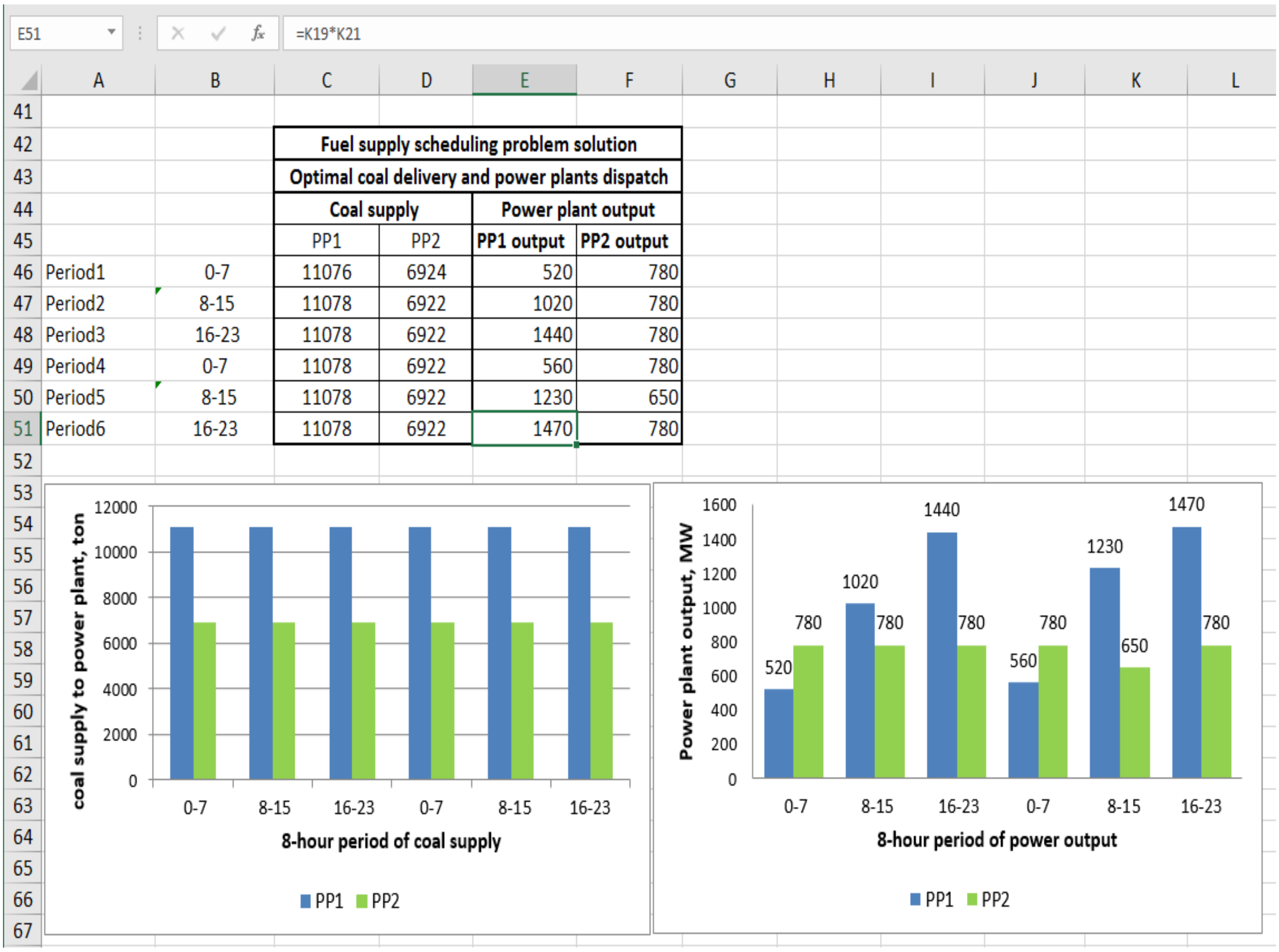


M17 | =$F$2*(SUM(A17:L17))

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
15					**Fuel consumption for 8-hour peroid, ton/8hr**									
16	**F1(1)**	**F2(1)**	**F1(2)**	**F2(2)**	**F1(3)**	**F2(3)**	**F1(4)**	**F2(4)**	**F1(5)**	**F2(5)**	**F1(6)**	**F2(6)**	**Objective**	
17	1854,410	2177,665	3476,273	2177,665	4691,925	2177,665	1970,186	2177,665	4084,099	1814,720	4778,758	2177,665	**3087400**	
18	**n1(1)**	**n2(1)**	**n1(2)**	**n2(2)**	**n1(3)**	**n2(3)**	**n1(4)**	**n2(4)**	**n1(5)**	**n2(5)**	**n1(6)**	**n2(6)**		
19	**2**	**6**	**3**	**6**	**3**	**6**	**2**	**6**	**3**	**5**	**3**	**6**		
20	**P1(1)**	**P2(1)**	**P1(2)**	**P2(2)**	**P1(3)**	**P2(3)**	**P1(4)**	**P2(4)**	**P1(5)**	**P2(5)**	**P1(6)**	**P2(6)**		
21	**260**	**130**	**340**	**130**	**480**	**130**	**280**	**130**	**410**	**130**	**490**	**130**		
22	**D1(1)**	**D2(1)**	**D1(2)**	**D2(2)**	**D1(3)**	**D2(3)**	**D1(4)**	**D2(4)**	**D1(5)**	**D2(5)**	**D1(6)**	**D2(6)**		
23	**11076**	**6924**	**11078**	**6922**	**11078**	**6922**	**11078**	**6922**	**11078**	**6922**	**11078**	**6922**		
24	**V1(1)**	**V2(1)**	**V1(2)**	**V2(2)**	**V1(3)**	**V2(3)**	**V1(4)**	**V2(4)**	**V1(5)**	**V2(5)**	**V1(6)**	**V2(6)**	**V1(end)**	**V2(end)**
25	**43790,3106**	**19598,98137**	**53011,901**	**24345,32**	**60613,627**	**29089,652**	**66999,7**	**33833,988**	**76107,516**	**38578,323**	**83101,416**	**43685,6**	**89400,7**	**48429,938**
26				**available number of generating units**										
27	n1(1)	n2(1)	n1(2)	n2(2)	n1(3)	n2(3)	n1(4)	n2(4)	n1(5)	n2(5)	n1(6)	n2(6)		
28	3	6	3	6	3	6	3	6	3	6	3	6		
29				**constraints on generating unit output, MW**										
30	**150**	**40**	**150**	**40**	**150**	**40**	**150**	**40**	**150**	**40**	**150**	**40**	Pmin	
31	**500**	**130**	**500**	**130**	**500**	**130**	**500**	**130**	**500**	**130**	**500**	**130**	Pmax	
32	**constraints on power balance, MW**					**constraints on coal delivery, tons**				**constraints on coal volume at the end of supply period**				
33		overall output		load		coal delivery		V contracted		V required		final coal volume		Inventory
34	PP1(1)+PP2(1)	1300	=	1300	D1(1)+D2(1)	18000	=	18000	V1(end)	87580,621	<=	89400,66	<=	145968
35	PP1(2)+PP2(2)	1800	=	1800	D1(2)+D2(2)	18000	=	18000	V2(end)	39197,963	<=	48429,94	<=	65330
36	PP1(3)+PP2(3)	2220	=	2220	D1(3)+D2(3)	18000	=	18000						
37	PP1(4)+PP2(4)	1340	=	1340	D1(4)+D2(4)	18000	=	18000						
38	PP1(5)+PP2(5)	1880	=	1880	D1(5)+D2(5)	18000	=	18000						
39	PP1(6)+PP2(6)	2250	=	2250	D1(6)+D2(6)	18000	=	18000						
40	**PP1(j)+PP2(j)=n1(j)*P1(j)+n2(j)*P2(j)**													

Figure 1.4 – The optimal solution to the fuel delivery scheduling problem under the adopted constraints

The graphical presentation of the results, which facilitates the interpretation of the results, is shown in fig. 1.5. According to the obtained solution, power plant 1 generates practically twice as much power as plant 2, which results in about twice as much coal consumption by plant 1 and, consequently, twice as much coal supply to this plant. The goal of the coal delivery is to increase the coal inventories at the plants to provide power plant 6-day operation at the rated capacity. This goal is reached, however, the coal volume at PP1 after the 2-day supply slightly exceeds the required level, while the resultant coal inventory at PP2 is enough for over 7-day operation.



Fuel supply scheduling problem solution

Optimal coal delivery and power plants dispatch

		Coal supply		Power plant output	
		PP1	PP2	PP1 output	PP2 output
Period1	0-7	11076	6924	520	780
Period2	8-15	11078	6922	1020	780
Period3	16-23	11078	6922	1440	780
Period4	0-7	11078	6922	560	780
Period5	8-15	11078	6922	1230	650
Period6	16-23	11078	6922	1470	780

Figure 1.5 – Diagrams of optimal coal delivery to the plants and economic load dispatch under the adopted constraints (the pointer is in cell **E25** to show the calculation of power plant output which is product (1.10) of the generating unit output and the number of operating)

1.2. Transportation Problem

1.2.1. Transportation problem formulation

A typical transportation problem (TP) is shown in fig. 1.6. It deals with *m* sources where a supply of some commodity is available and *n* destinations where the commodity is demanded. The costs of shipping from sources to destinations, c_{ij}, are known.

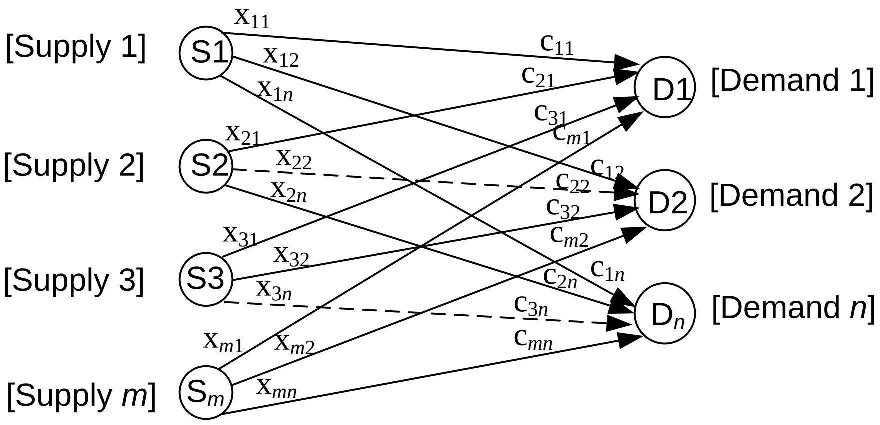


Figure 1.6 – Graphical representation of transportation problem:
Available shipping routes

The objective is to determine the amounts of the commodity x_{ij} shipped from source *i* to destination *j* under minimum total costs of shipping the commodity from all the sources to all the destinations:

$$Z(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij} \cdot x_{ij}) \to \min \tag{1.15}$$

The classic statement of the transportation problem uses a transportation matrix (transportation tableau) with the rows representing sources and columns representing destinations (table 1.3). The algorithms for solving the problem are based on this matrix representation. The amounts of the commodity x_{ij} and costs of shipping from sources to destinations c_{ij} are indicated by the entries in the matrix. If shipment is impossible between a given source and destination (dashed lines in fig. 1.6), a large cost of M is entered. This discourages the solution from using such cells. Supplies and demands are shown along the margins of the matrix.

The ordinary transportation problem has the total supply equal to the total demand and is called a **balanced** transportation problem.

Table 1.3 – Initial transportation matrix

Sources S_i	Destinations D_j			Supply
	D_1	D_2	D_n	
S_1	x_{11} c_{11}	x_{12} c_{12}	x_{1n} c_{1n}	Supply 1
S_2	x_{21} c_{21}	x_{22} c_{22}=M	x_{2n} c_{2n}	Supply 2
S_3	x_{31} c_{31}	x_{32} c_{32}	x_{3n} c_{3n}=M	Supply 3
S_m	x_{m1} c_{m1}	x_{m2} c_{m2}	x_{mn} c_{mn}	Supply *m*
Demand	Demand 1	Demand 2	Demand *n*	Balance Total supply=Total demand

The constraints of the TP are equalities:

1 –The commodity amounts shipped from a source to all the destinations must be equal to the available supply:

$$\sum_{i=1}^{m} x_{ij} = Supply\ i; \quad j = 1,\dots,n \tag{1.16}$$

2 –The commodity amounts shipped to a destination from all the sources must be equal to the demand:

$$\sum_{j=1}^{n} x_{ij} = Demand\ \ j; \quad i = 1,\dots,m \tag{1.17}$$

Two-index problems are solved in MS Excel with Solver through introduction of the initial data and mathematical model formulae into cells of Excel sheet and setting the relevant parameters in Solver, the way it is done for one-index problems.

Let us consider solving a balanced TP with MS Excel Solver.

There are four suppliers and three consumers of some commodity (fig.1.7). The available supply in each source and the demand of each consumer are shown in square brackets. The costs of unit delivery are given above the lines on the right.

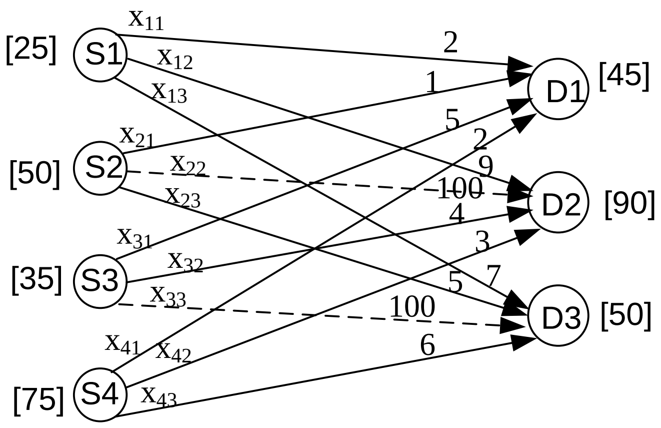


Figure 1.7 – Graphical representation of the example transportation problem

Table 1.4 represents the transportation matrix for this problem.

Table 1.4 –Transportation matrix for the transportation problem in fig. 1.6.

Sources S_i	Destinations D_j			Supply
	D_1	D_2	D_3	
S_1	x_{11} 2	x_{12} 9	x_{13} 7	Supply1
S_2	x_{21} 1	x_{22} 100	x_{23} 5	50
S_3	x_{31} 5	x_{32} 4	x_{33} 100	35
S_4	x_{41} 2	x_{42} 3	x_{43} 6	75
Demand	45	90	50	Balance 185=185

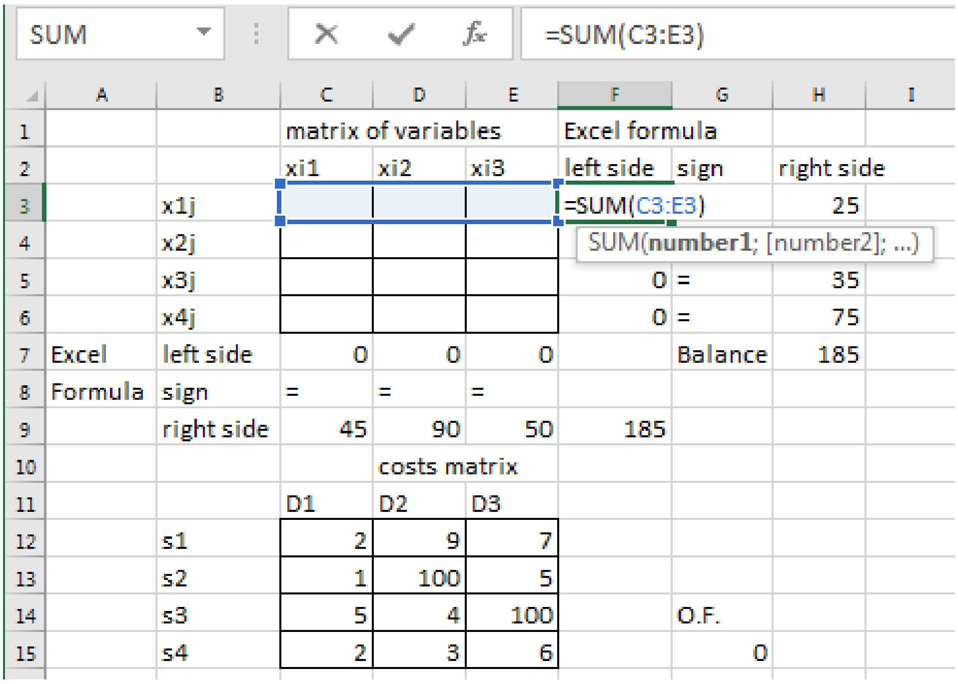
The peculiarity of the initial data introduction consists in their matrix representation. The initial transportation matrix is split into two, the first one being for the unknowns (quantities shipped) and the other – for the objecting function coefficients (costs of shipping).

The mathematical model of the example transportation problem is

$$Z(X)=2x_{11}+9x_{12}+7x_{13}+x_{21}+100x_{22}+5x_{23}+$$
$$+5x_{31}+4x_{32}+100x_{33}+2x_{41}+3x_{42}+6x_{43}\rightarrow \min$$

$$\begin{cases} x_{11}+x_{12}+x_{13}=25 \\ x_{21}+x_{22}+x_{23}=50 \\ x_{31}+x_{32}+x_{33}=35 \\ x_{41}+x_{42}+x_{43}=75 \\ x_{11}+x_{21}+x_{31}=45 \\ x_{12}+x_{22}+x_{32}=90 \\ x_{13}+x_{23}+x_{33}=50 \\ \forall x_{ij}\geq 0,\ \forall x_{ij}\geq \text{integer}, \\ i=1,2,3,4;\quad j=1,2,3 \end{cases} . \qquad (1.18)$$

The Excel screen form with the introduced initial data is shown in fig. 1.8. The Excel expressions for the constraints and the objective function of (1.18) are described in table 1.5 taking into account the Excel presentation of the problem in fig. 1.8. The solution is given in fig. 1.9. **Solver** parameters are shown in fig. 1.10.

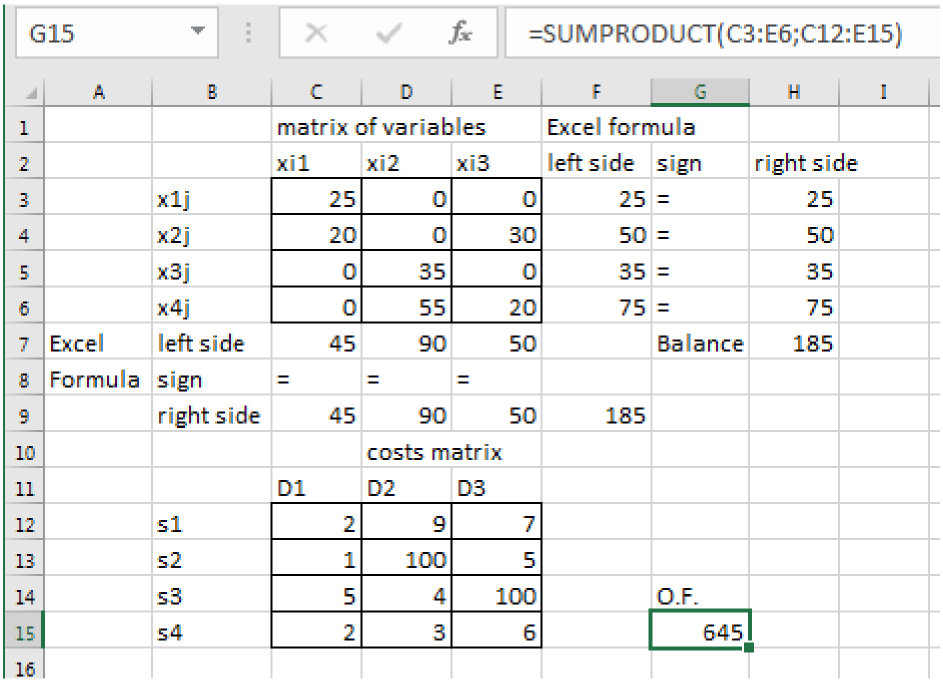


	A	B	C	D	E	F	G	H	I
1			matrix of variables			Excel formula			
2			xi1	xi2	xi3	left side	sign	right side	
3		x1j				=SUM(C3:E3)		25	
4		x2j							
5		x3j				0	=	35	
6		x4j				0	=	75	
7	Excel	left side	0	0	0		Balance	185	
8	Formula	sign	=	=	=				
9		right side	45	90	50	185			
10				costs matrix					
11			D1	D2	D3				
12		s1	2	9	7				
13		s2	1	100	5				
14		s3	5	4	100		O.F.		
15		s4	2	3	6		0		

Figure 1.8 – The screen presentation of TP (1.18) (the pointer is in cell **F3** to show application of Excel function SUM to introduction of the left side of the constrain equality $x_{11}+x_{12}+x_{13}=25$)

Table 1.5 – Excel formulae of mathematical model (1.18)

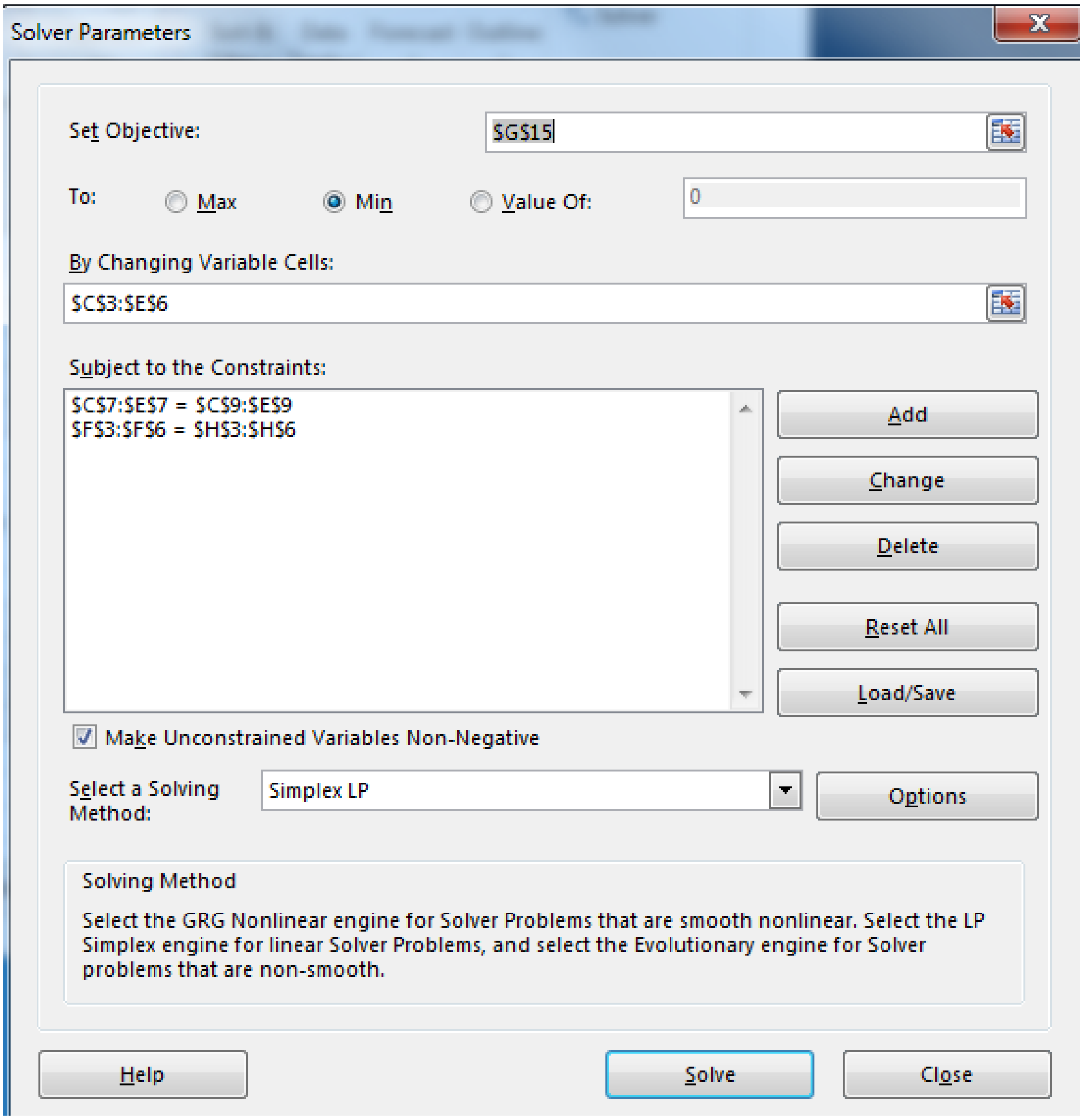
Mathematical model item and its location	Excel representation
Variables	**C3:E6**
Objective function (cell **G15**)	**=SUMPRODUCT(C3:E6;C12:E15)**
Constraints on the available supply	
Cell **F3**	**=SUM(C3:E3)**
Cell **F4**	**=SUM(C4:E4)**
Cell **F5**	**=SUM(C5:E5)**
Cell **F6**	**=SUM(C6:E6)**
Constraints on the demand	
Cell **C7**	**=SUM(C3:C6)**
Cell **D7**	**=SUM(D3:D6)**
Cell **E7**	**=SUM(E3:E6)**
Total supply	
Cell **H7**	**=SUM(H3:H6)**
Total demand	
Cell **F9**	**=SUM(C9:E9)**
Objective function	
Cell **G15**	**=SUMPRODUCT(C3:E6;C12:E15)**



G15 =SUMPRODUCT(C3:E6;C12:E15)

	A	B	C	D	E	F	G	H	I
1			matrix of variables			Excel formula			
2			xi1	xi2	xi3	left side	sign	right side	
3		x1j	25	0	0	25	=	25	
4		x2j	20	0	30	50	=	50	
5		x3j	0	35	0	35	=	35	
6		x4j	0	55	20	75	=	75	
7	Excel	left side	45	90	50		Balance	185	
8	Formula	sign	=	=	=				
9		right side	45	90	50	185			
10				costs matrix					
11			D1	D2	D3				
12		s1	2	9	7				
13		s2	1	100	5				
14		s3	5	4	100		O.F.		
15		s4	2	3	6		645		
16									

Figure 1.9 – The solution to TP (1.18) (the pointer is in cell **G15** to show the Excel formula for the objective function)



Set Objective: $G$15

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $C$3:$E$6

Subject to the Constraints:

$C$7:$E$7 = $C$9:$E$9
$F$3:$F$6 = $H$3:$H$6

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: Simplex LP

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 1.10 –Settings of **Solver parameters** window for considered Transportation Problem (1.18)

The optimal routes of the commodity shipping from the sources to the destination to minimize the total costs are shown in fig. 1.11. Above the lines, the commodity delivery volumes are given, in round brackets are costs of the commodity unit delivery.

The optimal value of the objective function Z* (that is the minimal costs of the delivery) is 645.

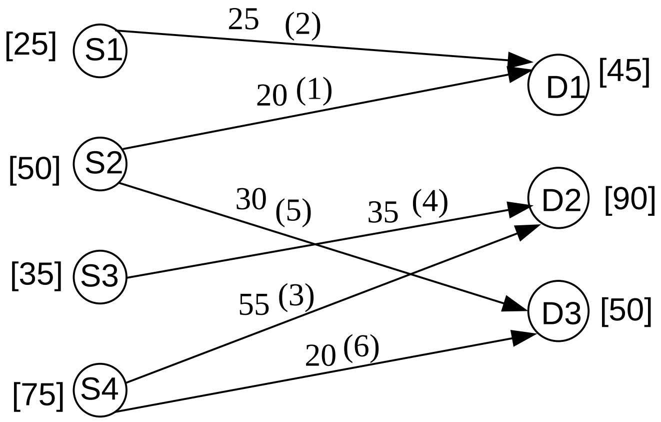


Figure 1.11 – Optimal routes of the commodity shipping for TP (1.18)

1.2.2. Transportation Problem in Power Engineering

Transportation problem can be applied to determining optimal power supply configurations, the commodity shipped being electrical power transmitted from power sources to consumers via overhead lines or cables. Power sources are power plants or power substations, while consumers are power substations or industrial/agricultural power consumers.

The objective is to find the optimal configuration of the power transmission through available grids for a given power balance or the optimal design of electricity distribution grid for given power sources and consumers.

The constraints are power balances at all the nodes of the grid.

1.2.2.1. An example problem of optimal power transmission without additional limitations [4].

In the electric grid, there are two power sources of 50 power inits (p.u.) and 30 p.u. and three load nodes of 20 p.u., 25 p.u., and 35 p.u.

Costs of power transmission are the following:

$c_{11}=1.2$ monetary units per power unit (m.u./p.u.), $c_{12}=1.8$ m.u./p.u., $c_{13}=1.5$ m.u./p.u., $c_{21}=1.6$ m.u./p.u., $c_{22}=2.3$ m.u./p.u., $c_{23}=2.1$ m.u./p.u.

The objective is the optimal configuration of power transmission under minimum total costs.

It is an ordinary balanced transportation problem. The transportation matrix is given in table 1.6.

Table 1.6 – Transportation matrix for example 1.2.2.1

Power sources $P_{i\text{gen}}$	Load nodes $P_{\text{load}j}$			Power source capacity, p.u.
	P_{load1}	P_{load2}	P_{load3}	
P_{gen1}	x_{11} 1.2	x_{12} 1.8	x_{13} 1.5	50
P_{gen2}	x_{21} 1.6	x_{22} 2.3	x_{23} 2.1	30
Consumed power, p.u.	20	25	35	Balance $80 = 80$

The mathematical model is the following.

The objective function is

$$Z(X) = 1.2x_{11} + 1.8x_{12} + 1.5x_{13} + 1.6x_{21} + 2.3x_{22} + 2.1x_{23} \to \min$$

The constraints are

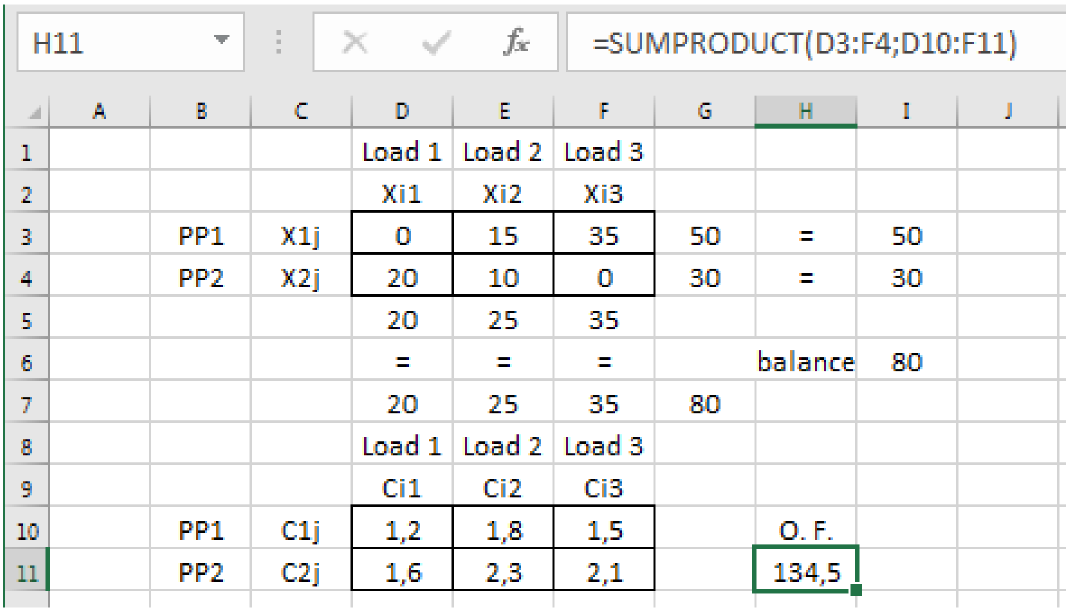
$$\begin{aligned} x_{11} + x_{12} + x_{13} &= 50 \\ x_{21} + x_{22} + x_{23} &= 30 \\ x_{11} + x_{21} &= 20 \qquad ; \\ x_{12} + x_{22} &= 25 \\ x_{13} + x_{23} &= 35 \end{aligned}$$

$$x_{11} \geq 0,\ x_{12} \geq 0,\ x_{13} \geq 0,\ x_{21} \geq 0,\ x_{22} \geq 0,\ x_{23} \geq 0.$$

Fig. 1.12 presents the optimal solution in Excel. The optimal configuration of power transmission is shown in fig. 1.13.

1.2.2.2. An example problem of optimal power transmission with constraint on transmission line capacity [4]

Quite often, power transmission capacity of an overhead line or cable from a power source to a consuming node is limited, i.e. it is impossible to transmit, via this line or cable, the amount of power higher than the transmission capacity limitation.



H11

=SUMPRODUCT(D3:F4;D10:F11)

	A	B	C	D	E	F	G	H	I	J
1				Load 1	Load 2	Load 3				
2				Xi1	Xi2	Xi3				
3		PP1	X1j	0	15	35	50	=	50	
4		PP2	X2j	20	10	0	30	=	30	
5				20	25	35				
6				=	=	=		balance	80	
7				20	25	35	80			
8				Load 1	Load 2	Load 3				
9				Ci1	Ci2	Ci3				
10		PP1	C1j	1,2	1,8	1,5		O. F.		
11		PP2	C2j	1,6	2,3	2,1		134,5		

Figure 1.12 – Optimal solution to example TP 1.2.2

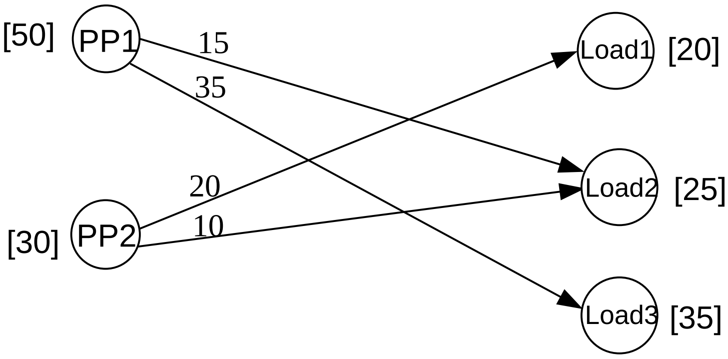


Figure 1.13 – Optimal configuration of power transmission for example TP 1.2.2.1

To solve TP for such cases, the load node which is connected to a line with limited transmission capacity from a power source is virtually split into two subnodes. The first subnode with the load equal to the power transmission limitation is connected to this line with transmission cost given. The other subnode with the remaining load (equal to the difference between the actual load and the transmission line capacity limitation) is connected to the power source via a line with a large cost of M (for example, 1000). This prevents transmission of power quantity higher than the line limitation from the source to the consumer. Power transmission costs for other lines in the grids to which this node (two virtual subnodes) is connected remain the same for both subnodes.

Let us consider the above electric grid in which the power transmission capacity of transmission line TL1-3 connecting power source 1 and load node 3 is limited to 20 p.u. ($x_{13} < 20$) [4].

The objective is to determine the optimal configuration of power supply resulting in minimum transmission costs in the grid.

Load node 3 is split into two subnodes, Load3_1 with the load of 20 p.u (equal to power transmission capacity limitation of 20 p.u.) and Load3_2 with the load of 35 – 20 = 15 p.u..

The mathematical model for this case is the following.

The objective function

$$\begin{aligned} Z(X) = 1.2x_{11} + 1.8x_{12} + 1.5x_{13} + 1000x_{14} + \\ + 1.6x_{21} + 2.3x_{22} + 2.1x_{23} + 2.1x_{24} \to \min \end{aligned},$$

where x_{i3} is the power supply to subnode3_1 of node3,

x_{i4} is the power supply to subnode3_2 of node3.

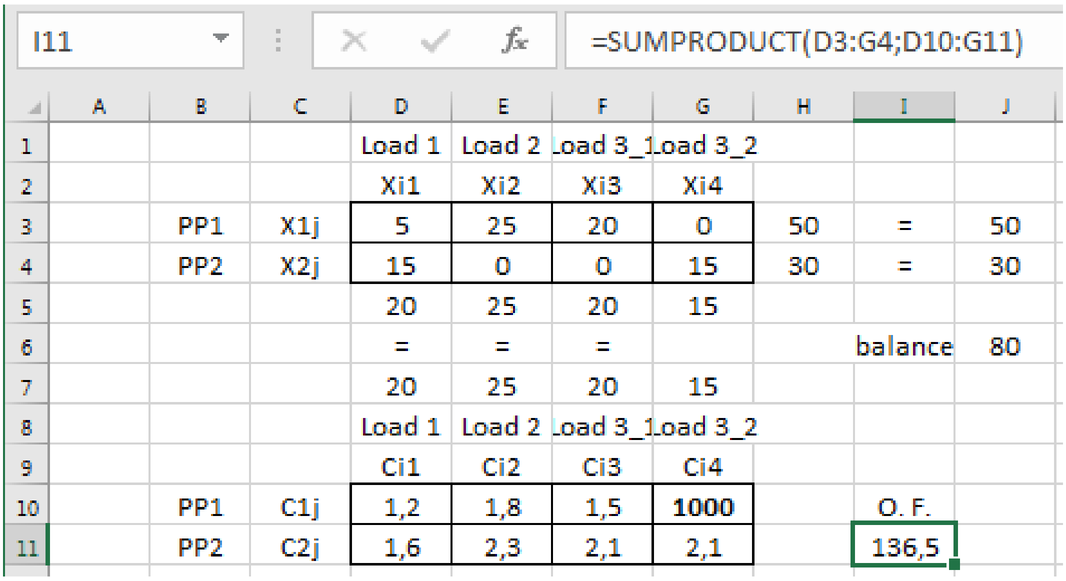
The constraints are

$$\begin{aligned} & x_{11} + x_{12} + x_{13} + x_{14} = 50 \\ & x_{21} + x_{22} + x_{23} + x_{24} = 30 \\ & x_{11} + x_{21} = 20 \\ & x_{12} + x_{22} = 25 \\ & x_{13} + x_{23} = 20 \\ & x_{14} + x_{24} = 15 \\ & x_{ij} \ge 0,\ i = 1,2;\ j = 1,2,3,4. \end{aligned}$$

Table 1.7 presents the transportation matrix for this problem (the highlighted cells show the split consumer). The optimal Excel solution to this TP and the optimal configuration of power supply under TL1-3 transmission capacity limitation of 20 p.u. are shown in fig. 1.14 and 1.15, correspondingly.

Table 1.7 – Transportation matrix for TP (1.2.2.2)

Power sources P_{igen}	Load nodes P_{loadj}				Power source capacity, p.u.
	P_{load1}	P_{load2}	P_{load3_1}	P_{load3_2}	
P_{gen1}	x_{11} 1.2	x_{12} 1.8	x_{13} 1.5	x_{14} 1000	50
P_{gen2}	x_{21} 1.6	x_{22} 2.3	x_{23} 2.1	x_{24} 2.1	30
Load power, p.u.	20	25	20	15	Balance $80 = 80$



I11 =SUMPRODUCT(D3:G4;D10:G11)

	A	B	C	D	E	F	G	H	I	J
1				Load 1	Load 2	Load 3_1	Load 3_2			
2				Xi1	Xi2	Xi3	Xi4			
3		PP1	X1j	5	25	20	0	50	=	50
4		PP2	X2j	15	0	0	15	30	=	30
5				20	25	20	15			
6				=	=	=			balance	80
7				20	25	20	15			
8				Load 1	Load 2	Load 3_1	Load 3_2			
9				Ci1	Ci2	Ci3	Ci4			
10		PP1	C1j	1,2	1,8	1,5	1000		O. F.	
11		PP2	C2j	1,6	2,3	2,1	2,1		136,5	

Figure 1.14 – Optimal solution to example TP 1.2.2.2

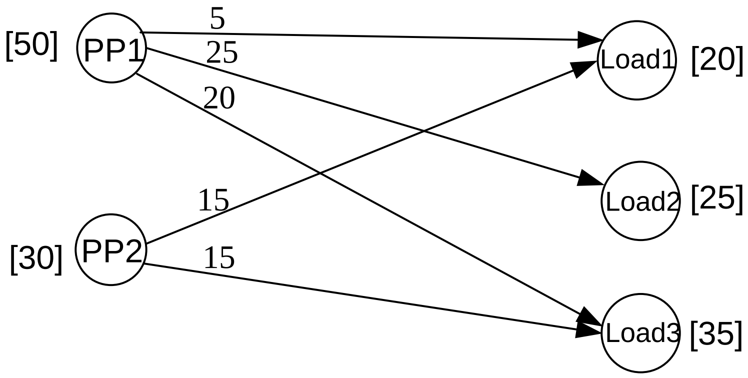


Figure 1.15 – Optimal configuration of power transmission for example TP 1.2.2.2

As we can see, the optimal configuration of power transmission changes, the total costs increasing.

1.2.2.3. An example problem of optimal power transmission with power transfer through load nodes [4]

In some cases, it is possible to supply power from a power source to a consumer (load node) through another consumer (another load node). However, there are peculiarities in solving transportation problem with power transfer.

The general algorithm of building the transportation matrix and formulating the mathematical model is the following.

1. The grid node numbering is continuous and all the nodes are considered first as power sources and then as consumers. The load nodes as power sources have zero supply and the power sources as consumers have zero demand.

2. The transportation matrix is square, (n+m)-by-(n+m), where n is number of power sources, m is number of load nodes.

3. Cost of power transfer (diagonal cells in the transportation matrix) is zero, z_{ii}=0.

4. Transfer power quantities, x_{ii} and x_{jj}, are subtracted from the node power balance equations.

5. Transportation costs between nodes are independent of the transmission direction and considered equal, z_{ij}=z_{ji}.

The mathematical model in the general form is the following.

The objective function:

$$Z(X)=\sum_{i=1}^{n+m}\sum_{j=1}^{n+m} z_{ij}x_{ij} \to \min \tag{1.19}$$

The constraints are the node power balance equations:

for the load nodes:

$$\sum_{i=1,i\neq j}^{n+m} x_{ij}-x_{jj}=P_{\text{load}j}. \tag{1.20}$$

for the power supply nodes:

$$\sum_{j=1,i\neq j}^{n+m} x_{ij}-x_{ii}=P_{\text{gen}i}. \tag{1.21}$$

In the solution, zero value of x_{ii} (diagonal cell with 0) means absence of power transfer. If x_{ii} is different from zero (diagonal cell with non-zero value), it represents the power quantity transferred through the corresponding node. The destination of the transferred power (the load node receiving the transferred power) is located in the same row as the cell with non-zero value.

Let us consider an electric grid in which there are two power supply nodes of 100 p.u. and 50 p.u. and two load nodes of 90 p.u. and 60 p.u. [4].

Costs of power transmission are

c_{11} =5 m.u./p.u., c_{12} =2 m.u./p.u., c_{21} =4 m.u./p.u., c_{22} =3 m.u./p.u.

Cost of power transfer through a load node is 2 m.u./p.u.

The transfer costs for the power sources is large, 100 m.u./p.u., to prevent power transfer through them.

The objective is to determine the optimal scheme of power transmission resulting in minimum transmission costs in the grid.

The transportation matrix of the TP is presented in table 1.8. The highlighted cells contain power quantities transferred through nodes, x_{ii}, and transfer costs, c_{ii}, (0). As you can see, the transportation matrix for the TP with power transfer is symmetrical about the main diagonal.

Table 1.8 – Transportation matrix for example TP with power transfer

All nodes as power sources	All nodes as consumers B_j				Power supply capacity, p.u.
	$P_{1gen}(1)$	$P_{2gen}(2)$	$P_{load1}(3)$	$P_{load2}(4)$	
(1) P_{1gen}	x_{11} 0	x_{12} 100	x_{13} 5	x_{14} 2	100
(2) P_{2gen}	x_{21} 100	x_{22} 0	x_{23} 4	x_{24} 3	50
(3) P_{load1}	x_{31} 5	x_{32} 4	x_{33} 0	x_{34} 2	0
(4) P_{load2}	x_{41} 2	x_{42} 3	x_{43} 2	x_{44} 0	0
Load power, p.u.	0	0	90	60	Balance 150 = 150

The mathematical model the power transfer problem is the following.

The objective function:

$$\begin{aligned} Z(X) = 0x_{11} + 100x_{12} + 5x_{13} + 2x_{14} + \\ + 100x_{21} + 0x_{22} + 4x_{23} + 3x_{24} + \\ + 5x_{31} + 4x_{32} + 0x_{33} + 2x_{34} + \\ + 2x_{41} + 3x_{42} + 2x_{43} + 0x_{44} \to \min \end{aligned}.$$

Power balance constraints:

Power supply:

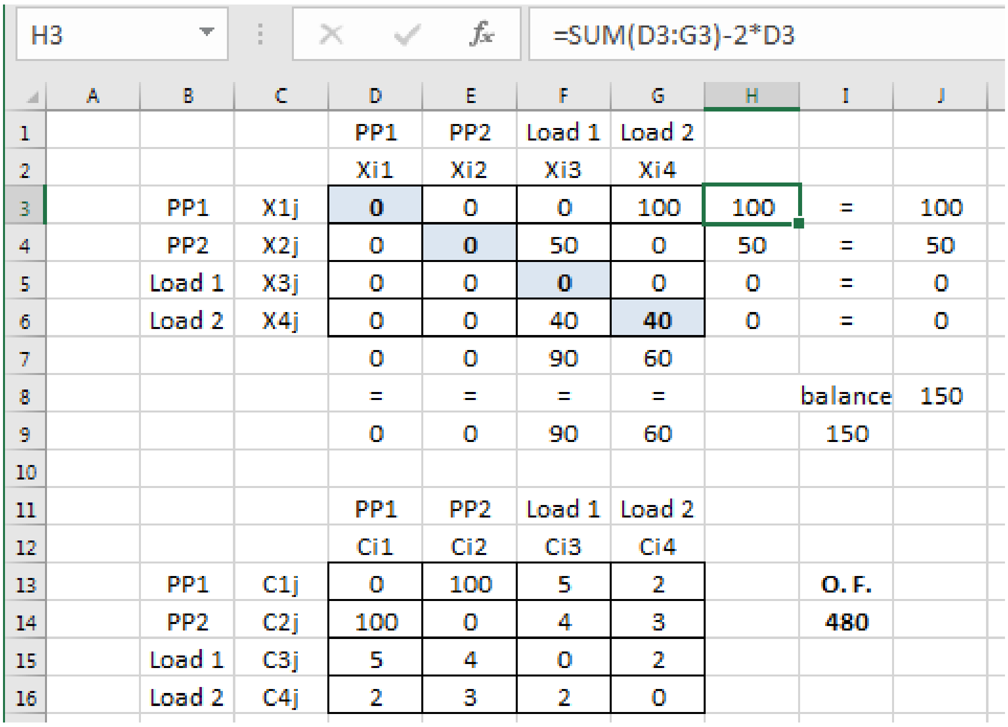
$$\begin{aligned} x_{12} + x_{13} + x_{14} - x_{11} = 100 \\ x_{21} + x_{23} + x_{24} - x_{22} = 50 \\ x_{31} + x_{32} + x_{34} - x_{33} = 0 \\ x_{41} + x_{42} + x_{43} - x_{44} = 0 \end{aligned}.$$

Power demand:

$$\begin{aligned} x_{21} + x_{31} + x_{41} - x_{11} = 0 \\ x_{12} + x_{32} + x_{42} - x_{22} = 0 \\ x_{13} + x_{23} + x_{43} - x_{33} = 90 \\ x_{14} + x_{24} + x_{34} - x_{44} = 60 \\ x_{ij} \geq 0,\ i = 1,2,3,4;\ j = 1,2,3,4. \end{aligned}$$

The optimal Excel solution to this TP and the optimal configuration of power supply with possibility of power transfer through a load node are shown in fig. 1.16 and 1.17, correspondingly.

According to the obtained result, power source 1 of 100 p.u. gives all its power to load node 2 whose demand is 60 p.u. The power volume of 40 p.u. (value in cell **G6**) is excessive and transferred to load node 1 (cell **F6**) with 90 p.u. demand. Therefore, load node 1 receives 50 p.u. from power source 1 and 40 p.u. from power source 1 through load node 2.



H3 =SUM(D3:G3)-2*D3

	A	B	C	D	E	F	G	H	I	J
1				PP1	PP2	Load 1	Load 2			
2				Xi1	Xi2	Xi3	Xi4			
3		PP1	X1j	**0**	0	0	100	100	=	100
4		PP2	X2j	0	**0**	50	0	50	=	50
5		Load 1	X3j	0	0	**0**	0	0	=	0
6		Load 2	X4j	0	0	40	**40**	0	=	0
7				0	0	90	60			
8				=	=	=	=		balance	150
9				0	0	90	60		150	
10										
11				PP1	PP2	Load 1	Load 2			
12				Ci1	Ci2	Ci3	Ci4			
13		PP1	C1j	0	100	5	2		**O. F.**	
14		PP2	C2j	100	0	4	3		**480**	
15		Load 1	C3j	5	4	0	2			
16		Load 2	C4j	2	3	2	0			

Figure 1.16 – Optimal solution to power transfer problem 1.2.2.3

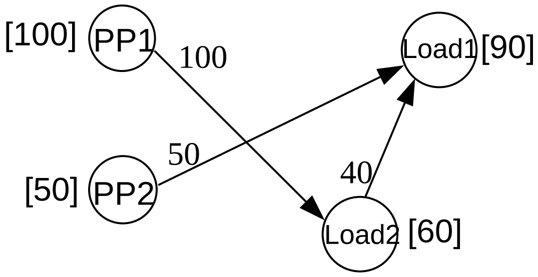


Figure 1.17 – Optimal configuration of power transmission for example TP 1.2.2.3

1.2.2.4. In case the total supply is not equal to the total demand, the transportation problem is called unbalanced.

To solve the unbalanced transportation problem, it is necessary to bring it to the form of the balanced one. It is done via adding a dummy node.

There are two situations with unbalance:

1 – <u>Supply is less than demand</u>,

$$\sum_{i=1}^{m} P_{geni} < \sum_{j=1}^{n} P_{load\,j} \tag{1.22}$$

that is, there is power deficiency in the power system (or any other commodity considered).

In this situation, the dummy power source is added with power supply equal to value of the power deficiency,

$$P_{gen\_dummy} = \sum_{j=1}^{n} P_{load\,j} - \sum_{i=1}^{m} P_{geni} \tag{1.23}$$

2 – Supply is higher than demand,

$$\sum_{i=1}^{m} P_{geni} > \sum_{j=1}^{n} P_{load\,j} \tag{1.24}$$

that is, there is power surplus in the power system (or any other commodity considered).

In this situation, the dummy load node is added with power demand equal to value of the power surplus,

$$P_{load\_dummy} = \sum_{i=1}^{m} P_{geni} - \sum_{j=1}^{n} P_{load\,j} \tag{1.25}$$

After adding a dummy node with relevant supply/demand, the transportation problem becomes balanced and is solved with usual procedure.

The interpretation of the solution is the following.

1 - For the case of power deficiency: the consumer that receives power from the dummy source in the solution lacks this quantity in reality.

1 - For the case of power surplus: the power source that supplies its power to the dummy consumer (dummy load node) in the solution retains this quantity in reality and works with reduced output (its operating output is equal to the difference between the source rated power and power supply to the dummy node).

The transportations costs for the dummy node are usually assigned zero.

When solving a TP with power transfer, the transfer costs for the dummy unit are taken large to prevent transfer through the not-existing node.

Let us consider the electric grid with two power sources of 80 power inits (p.u.) and 50 p.u. and three consumers of 50 p.u., 25 p.u., and 35 p.u.

Costs of power transmission are the following:

c_{11} =1.2 monetary units per power unit (m.u./p.u.), c_{12} =1.8 m.u./p.u., c_{13} =1.5 m.u./p.u., c_{21} =1.6 m.u./p.u., c_{22} =2.3 m.u./p.u., c_{23} =2.1 m.u./p.u.

The objective is the optimal configuration of power transmission under minimum total costs.

The total supply is 80+50=130 p.u.

The total demand is 50+25+35=110 p.u.

The total supply is higher than the total demand, therefore the TP is unbalanced.

Let us add a dummy consumer with demand equal to 130-110=20 p.u.

The transportation matrix is given in table 1.9.

Table 1.9 – Transportation matrix for the example TP with a dummy consumer

Power sources $P_{i\text{gen}}$	Load nodes $P_{\text{load}j}$				Power source capacity, p.u.
	P_{load1}	P_{load2}	P_{load3}	P_{dummy}	
P_{gen1}	x_{11} 1.2	x_{12} 1.8	x_{13} 1.5	x_{14} 0	80
P_{gen2}	x_{21} 1.6	x_{22} 2.3	x_{23} 2.1	x_{24} 0	50
Load power, p.u.	50	25	35	20	Balance $110+20=130$

Let us consider another electric grid with two power sources of 80 power inits (p.u.) and 65 p.u. and three consumers of 50 p.u., 75 p.u., and 25 p.u.

Costs of power transmission are the following:

c_{11} =1.2 monetary units per power unit (m.u./p.u.), c_{12} =1.8 m.u./p.u., c_{13} =1.5 m.u./p.u., c_{21} =1.6 m.u./p.u., c_{22} =2.3 m.u./p.u., c_{23} =2.1 m.u./p.u.

The total supply is 80 + 65 = 145 p.u.

The total demand is 50 + 75 + 25 = 150 p.u.

The total supply is lower than the total demand, therefore the TP is unbalanced.

To balance the problem, it is necessary to add a dummy power source of 150 - 145 = 5 p.u.

The transportation matrix for this transportation problem is given in table 1.10.

Table 1.10 – Transportation matrix for the example TP with a dummy power source

Power sources $P_{i\text{gen}}$	Load nodes $P_{\text{load}j}$			Power source capacity, p.u.
	P_{load1}	P_{load2}	P_{load3}	
P_{gen1}	x_{11} 1.2	x_{12} 1.8	x_{13} 1.5	80
P_{gen2}	x_{21} 1.6	x_{22} 2.3	x_{23} 2.1	65
P_{dummy}	x_{d1} 0	x_{d2} 0	x_{d3} 0	5
Load power, p.u.	50	75	25	Balance $150 = 145 + 5$

1.3. Special Types Of Transportation Problem

1.3.1. Assignment Problem

1.3.1.1. Assignment Problem is a special type of Transportation Problem with Boolean variables.

The objective is to allocate different facilities to different tasks to result in maximum profit or effectiveness or minimum costs or loss.

The assignment model is useful in solving problems such as, assignment of persons or machines to jobs, assignment of salesmen to sales territories, travelling salesman problem, etc.

The cost matrix is assignment problem is square.

If the number of facilities is the same as the number of tasks, the assignment problem is said to be balanced.

If the number of facilities is different from the number of tasks, the assignment problem is said to be unbalanced and a dummy position, either facility or destination, must be added.

The **mathematical model of Assignment Problem** is the following.

n is both the number of available facilities (existing +dummy in the case of unbalanced problem with the destinations exceeding the facilities) to allocate and the number of available destinations (existing +dummy in the case of unbalanced problem with the facilities exceeding the destinations) to assign.

$a_i = 1$is a unit of the facility A_i (i=1,…,n), for example, one worker, one transport means, one technical means, etc.

$b_j = 1$ is a unit of destinations B_j (j=1,…,n), for example, one job, one job position, one route, one construction site, etc.

c_{ij} is a quality characteristic of facility A_i allocation to destinations B_j. For example, effectiveness of i-th worker to j-th job; time required by i-th transport means to get to j-th site, time response of i-th device to variation of j-th parameter, etc. For allocation of a dummy facility of a real destination or a real facility to a dummy destination, the cost is taken equal to 0.

The unknowns of Assignment Problem are

– x_{ij} is the result of facility allocation to destination.

$$x_{ij} = \begin{cases} 1 \ \ if \ \ i-th \ \ facility \ is \ allocated \ \ to \ \ j-th \ destination \\ 0 \ \ if \ \ i-th \ \ facility \ is \ not \ allocated \ \ to \ \ j-th \ destination \end{cases}. \tag{1.26}$$

The objective function is

– $Z(X)$ – the overall quality characteristic of all the facilities allocation (maximum effectiveness/minimum loss):

$$Z(X) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \rightarrow \max/\min. \tag{1.27}$$

The constraints of Assignment Problem are

– Allocation of one of the available facilities to one destination

$$\sum_{j=1}^{n} x_{ij} = 1, \ i = 1,\ldots,n. \tag{1.28}$$

– Assignment of one of the available destination to one facility

$$\sum_{i=1}^{n} x_{ij} = 1, \ j = 1,\dots,n. \tag{1.29}$$

The general formulation of the mathematical model for Assignment Problem is

$$Z(X) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \to \max/\min$$

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = 1, \ i = 1,\dots,n \\ \sum_{i=1}^{n} x_{ij} = 1, \ j = 1,\dots,n \\ x_{ij}\begin{cases} 0, \\ 1, \end{cases} \ i = 1,..n, \ j = 1,\dots,m \end{cases} . \tag{1.30}$$

The initial cost matrix for Assignment Problem is given in table 1.11.

Table 1.11 – Initial cost matrix of Assignment Problem

Facilities A_i	Destinations B_j				Allocation
	B_1	B_2	B_j	B_n	
A_1	x_{11} c_{11}	x_{12} c_{12}	x_{1j} c_{1j}	x_{1n} c_{1n}	1
A_2	x_{21} c_{21}	x_{22} c_{22}	x_{2j} c_{2j}	x_{2n} c_{2n}	1
A_i	x_{i1} c_{i1}	x_{i2} c_{i2}	x_{ij} c_{ij}	x_{in} c_{in}	1
A_n	x_{n1} c_{n1}	x_{n2} c_{n2}	x_{nj} c_{nj}	x_{nn} c_{nn}	1
Required	1	1	1	1	Balance the number of facilities = =the number of destinations

A popular technique for solving assignment problems is Hungarian method [3]. Also, assignment problems can also be solved with MS Excel Solver.

1.3.1.2. An example of a balanced assignment problem.

A car service has four mechanics available for work on four separate jobs. All the mechanics are qualified to do these jobs but the costs of each mechanic doing each available job are different. To optimize the car service operation, one mechanic must be allocated to one job so as the total costs are minimum. The cost of assigning each mechanic to each job is given in table 1.12.

Table 1.12 – Cost of assigning each mechanic to each job

Job				
Mechanic	1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D	25	23	24	24

The initial cost matrix for the considered problem of assigning mechanics to jobs is given in table 1.13.

Table 1.13 – The initial cost matrix for assignment problem 1.3.1.1

Job					Allocation
Mechanic	1	2	3	4	
A	x_{A1} 20	x_{A2} 25	x_{A3} 22	x_{A4} 28	1
B	x_{B1} 15	x_{B2} 18	x_{B3} 23	x_{B4} 17	1
C	x_{C1} 19	x_{C2} 17	x_{C3} 21	x_{C4} 24	1
D	x_{D1} 25	x_{D2} 23	x_{D3} 24	x_{D4} 24	1
Required	1	1	1	1	Balance 4=4

The mathematical model is the following.

The objective is minimization of the total costs of doing the jobs

$$\begin{aligned} Z(X) = {} & 20x_{A1} + 25x_{A2} + 22x_{A3} + 28x_{A4} + 15x_{B1} + 18x_{B2} + 23x_{B3} + 17x_{B4} + \\ & + 19x_{C1} + 17x_{C2} + 21x_{C3} + 24x_{C4} + 25x_{D1} + 23x_{D2} + 24x_{D3} + 24x_{D4} \to \min \end{aligned}.$$

The constraints are

$$\begin{cases} x_{A1} + x_{A2} + x_{A3} + x_{A4} = 1 \\ x_{B1} + x_{B2} + x_{B3} + x_{B4} = 1 \\ x_{C1} + x_{C2} + x_{C3} + x_{C4} = 1 \\ x_{D1} + x_{D2} + x_{D3} + x_{D4} = 1 \\ x_{A1} + x_{B1} + x_{C1} + x_{D1} = 1 \\ x_{A2} + x_{B2} + x_{C2} + x_{D2} = 1 \\ x_{A3} + x_{B3} + x_{C3} + x_{D3} = 1 \\ x_{A4} + x_{B4} + x_{C4} + x_{D4} = 1 \\ \forall x_{ij} = boolean, \ i = 1,2,3,4; \quad j = 1,2,3,4 \end{cases}.$$

1.3.1.3. An unbalanced assignment problem.

Temporarily, the car service has three separate jobs available for the four mechanics. To optimize the car service operation, three mechanics must be allocated to the jobs while the fourth one may be suspended without pay. The objective is to minimize the total costs.

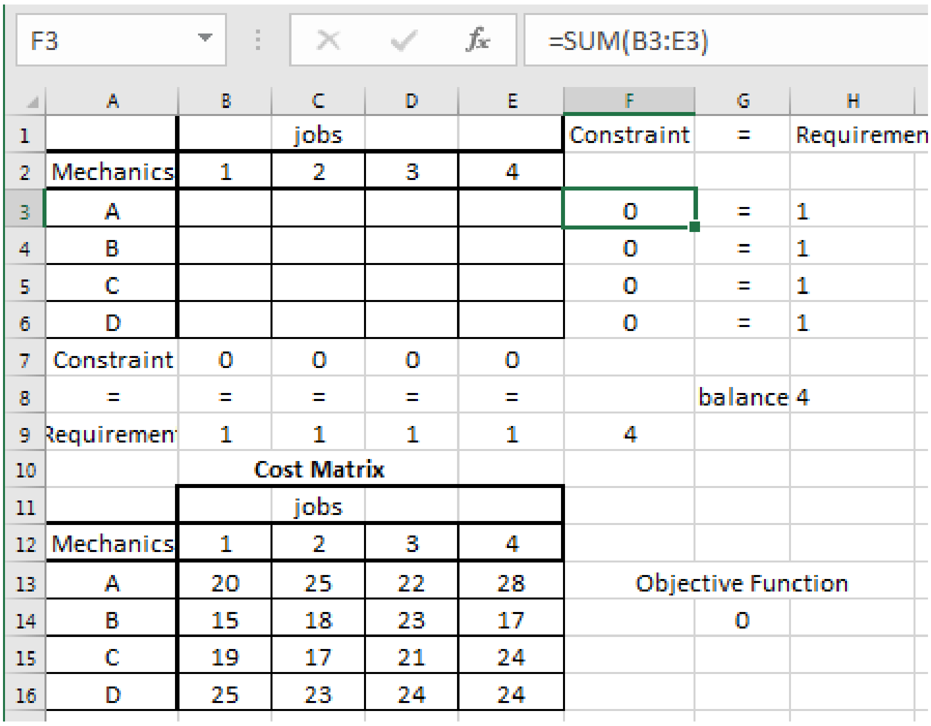
The cost of assigning each mechanic to each job is taken from table 1.13. As the number of jobs is less than the number of mechanics, a dummy job is added. The initial cost matrix for example unbalanced problem is shown in table 1.14. The cost of allocation to a dummy job is taken 0.

The mathematical model of this problem is similar to that of problem 1.3.1.1 except for the cost of the dummy job.

Table 1.14 – The initial cost matrix for the unbalanced assignment problem

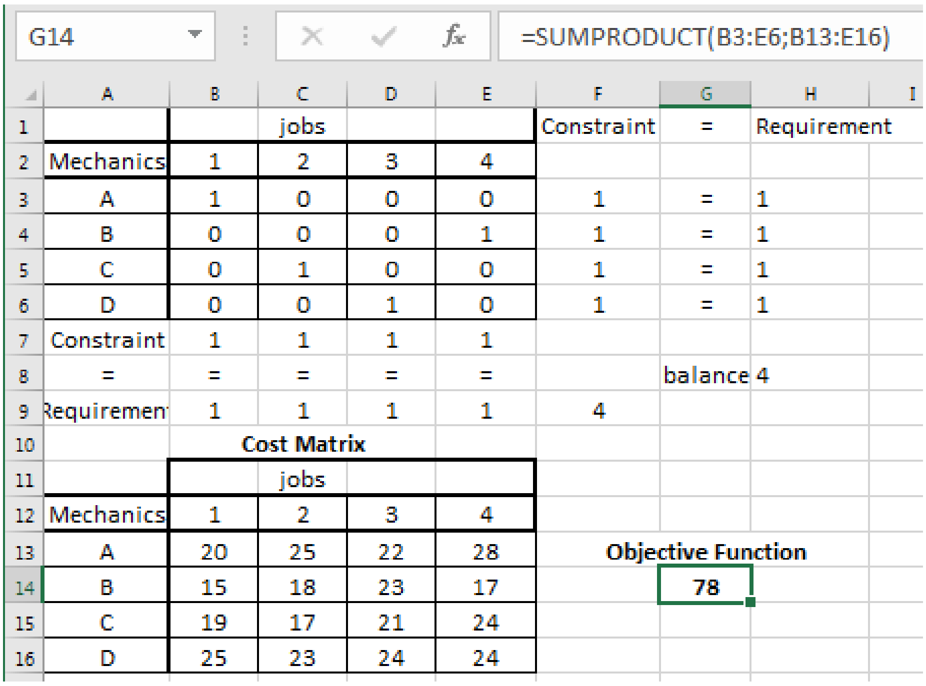
Job					Allocation
Mechanic	1	2	3	Dummy	
A	x_{A1} 20	x_{A2} 25	x_{A3} 22	x_{AD} 0	1
B	x_{B1} 15	x_{B2} 18	x_{B3} 23	x_{BD} 0	1
C	x_{C1} 19	x_{C2} 17	x_{C3} 21	x_{CD} 0	1
D	x_{D1} 25	x_{D2} 23	x_{D3} 24	x_{DD} 0	1
Required	1	1	1	1	Balance: 4=4

Solving Assignment problem in Excel is identical to solving Transportation problem in Excel. The only difference is the constraint for the unknown variables x_{ij} as Boolean (or binary). The screen presentation and the solution of balanced assignment problem 1.3.1.1 are shown in fig. 1.18 and 1.19, respectively. **Solver** parameters are given in fig. 1.20.



	A	B	C	D	E	F	G	H
1			jobs			Constraint	=	Requiremen
2	Mechanics	1	2	3	4			
3	A					0	=	1
4	B					0	=	1
5	C					0	=	1
6	D					0	=	1
7	Constraint	0	0	0	0			
8	=	=	=	=	=		balance	4
9	Requiremen	1	1	1	1	4		
10			Cost Matrix					
11			jobs					
12	Mechanics	1	2	3	4			
13	A	20	25	22	28	Objective Function		
14	B	15	18	23	17		0	
15	C	19	17	21	24			
16	D	25	23	24	24			

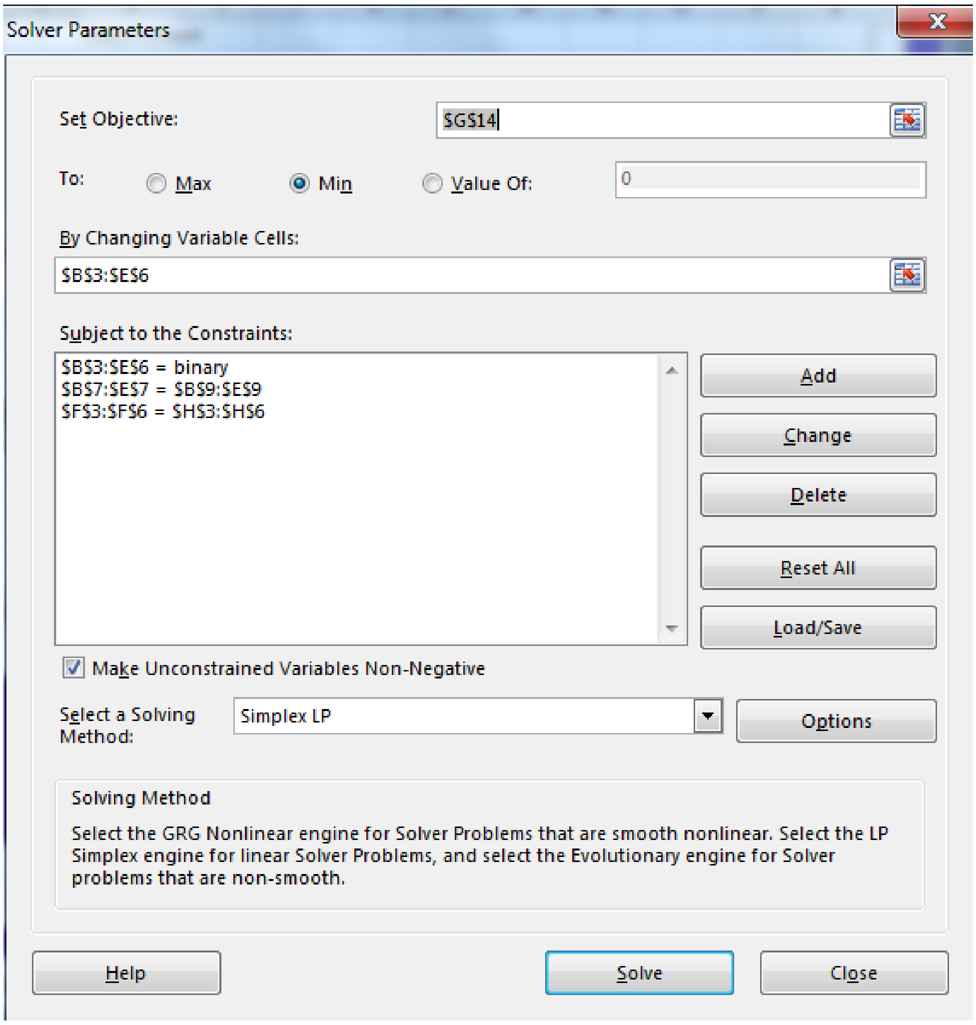
Figure 1.18 – The screen presentation of balanced assignment problem 1.3.1.1 (the pointer is in cell **F3** to show introduction of constraint on assigning mechanic A to only one of the four available jobs)



G14 =SUMPRODUCT(B3:E6;B13:E16)

	A	B	C	D	E	F	G	H
1			jobs			Constraint	=	Requirement
2	Mechanics	1	2	3	4			
3	A	1	0	0	0	1	=	1
4	B	0	0	0	1	1	=	1
5	C	0	1	0	0	1	=	1
6	D	0	0	1	0	1	=	1
7	Constraint	1	1	1	1			
8	=	=	=	=	=		balance	4
9	Requiremen	1	1	1	1	4		
10			**Cost Matrix**					
11			jobs					
12	Mechanics	1	2	3	4			
13	A	20	25	22	28		**Objective Function**	
14	B	15	18	23	17		**78**	
15	C	19	17	21	24			
16	D	25	23	24	24			

Figure 1.19 – The Excel solution of balanced assignment problem 1.3.1.1



Solver Parameters

Set Objective: $G$14

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $B$3:$E$6

Subject to the Constraints:

$B$3:$E$6 = binary
$B$7:$E$7 = $B$9:$E$9
$F$3:$F$6 = $H$3:$H$6

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: Simplex LP

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 1.20 –Settings of **Solver parameters** window for problem 1.3.1.1

The explanation of the obtained solution is the following:

Mechanic A is assigned to do job 1;

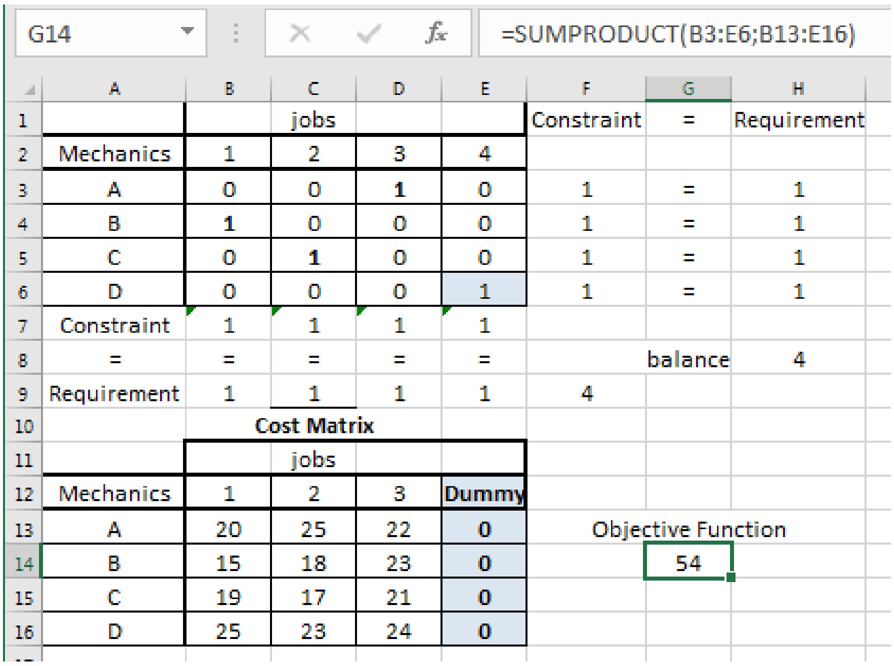
Mechanic B is assigned to do job 4;

Mechanic C is assigned to do job 2;

Mechanic D is assigned to do job 3.

With these allocations, the total cost of doing the jobs is 78.

The Excel solution to unbalanced assignment problem 1.3.1.2 is shown in fig. 1.21.



G14

=SUMPRODUCT(B3:E6;B13:E16)

	A	B	C	D	E	F	G	H
1			jobs			Constraint	=	Requirement
2	Mechanics	1	2	3	4			
3	A	0	0	1	0	1	=	1
4	B	1	0	0	0	1	=	1
5	C	0	1	0	0	1	=	1
6	D	0	0	0	1	1	=	1
7	Constraint	1	1	1	1			
8	=	=	=	=	=		balance	4
9	Requirement	1	1	1	1	4		
10			Cost Matrix					
11			jobs					
12	Mechanics	1	2	3	Dummy			
13	A	20	25	22	0	Objective Function		
14	B	15	18	23	0		54	
15	C	19	17	21	0			
16	D	25	23	24	0			

Figure 1.21 – The Excel solution of unbalanced problem 1.3.1.2 (the pointer is in cell **G14** to show introduction of the objective function with **SUMPRODUCT** function)

According to the obtained solution, there is no job for mechanic D and he may be suspended with or without pay.

1.3.2. Travelling Salesman Problem

1.3.2.1. Travelling Salesman Problem (TSP) is another special type of Transportation Problem with Boolean variables.

The objective is to determine the optimal route of visiting a few sites so as to visit all the sites only once along minimum-time or minimum-mileage path and return to the first site of the route from where he started:

$$Z(X) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \to \min \tag{1.31}$$

TSP is also solved with Hungarian method in which

c_{ij} is the distance (or cost or time) between site *i* and site *j*. If there is no direct route between two sites, the distance (or cost or time) is assigned a high value.

x_{ij} is the tour from site *i* to site *j*,

$$x_{ij} = \begin{cases} 1 \ \ if \ \ a \ \ tour \ includes \ travelling \ \ from \ site \ i \ to \ site \ \ j \ \ (i \neq j) \\ 0 \ \ if \ \ a \ \ tour \ \ does \ not \ include \ travelling \ \ from \ site \ i \ to \ site \ \ j \ \ (i \neq j) \end{cases}$$

For *n* sites, there are (*n* - 1)! possible routes. For example, 6 sites will require considering 5! (1×2×3×4×5=120) different combinations.

MS Excel Solver significantly facilitates the computation procedure for Travelling Salesman Problem.

The specific feature of this tool application to solving TSP is use of **Alldifferent Constraint** and the **Evolutionary Method**.

The **Alldifferent Constraint** is used to ensure that the salesman will visit every site only once.

The **Evolutionary method** is used because the mathematical path to the Objective contains the **Excel Index function** which is a discontinuous function.

The cost matrix in TSP is square and symmetrical about the main diagonal.

The sites to visit are placed in the same order both in the rows and in the columns.

The diagonal cells are zero.

Off diagonal cells contain distance (costs or time of travelling) between the corresponding sites.

The unknowns are the number of the site visited in the route.

The objective function is the sum of the costs of travelling (or distance or time) from site to site in the whole route.

1.3.2.2. Let us consider solving the following TSP in MS Excel.

A grid company has six power facilities (PF) that require in-service inspection. Find the shortest path to travel from power facility to power facility visiting each site once. The distance (miles) between the sites is shown in figure 1.22. The distance matrix for this problem is given in table 1.15. The Excel presentation of the distance matrix is shown in fig. 1.23.

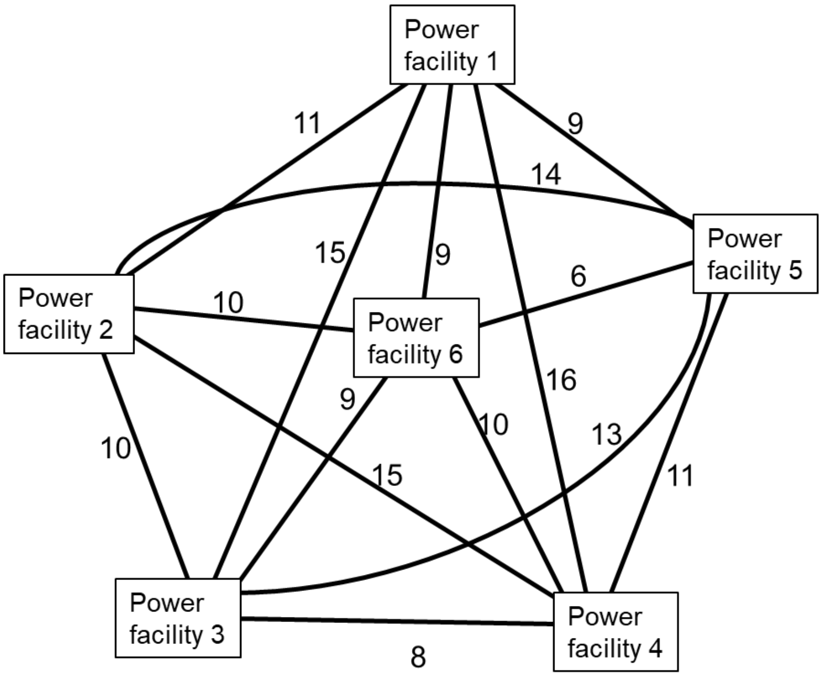
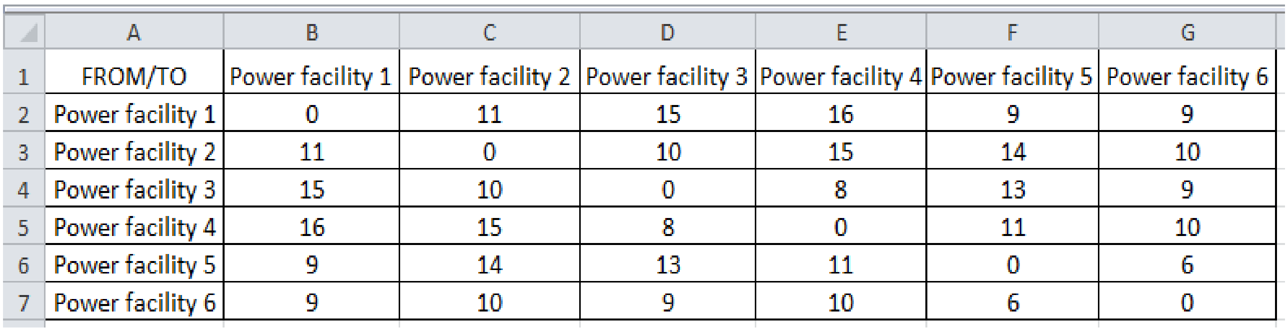


Figure 1.22 – Distance (miles) between the power facilities to inspect

Table 1.15 – The distance matrix for the example TSP

FROM/TO	Power facility 1	Power facility 2	Power facility 3	Power facility 4	Power facility 5	Power facility 6
Power facility 1	**0**	11	15	16	9	9
Power facility 2	11	**0**	10	15	14	10
Power facility 3	15	10	**0**	8	13	9
Power facility 4	16	15	8	**0**	11	10
Power facility 5	9	14	13	11	**0**	6
Power facility 6	9	10	9	10	6	**0**



	A	B	C	D	E	F	G
1	FROM/TO	Power facility 1	Power facility 2	Power facility 3	Power facility 4	Power facility 5	Power facility 6
2	Power facility 1	0	11	15	16	9	9
3	Power facility 2	11	0	10	15	14	10
4	Power facility 3	15	10	0	8	13	9
5	Power facility 4	16	15	8	0	11	10
6	Power facility 5	9	14	13	11	0	6
7	Power facility 6	9	10	9	10	6	0

Figure 1.23 – Presentation of the distance matrix in Excel

Cells **B9:G9** are assigned for the decision variables: each variable is the number of the power facility in the route.

The power facilities are designated in the Excel model not by their names but by the row that they appear in the distance matrix.

Power facility 1 appears in the 1st row of the distance matrix; therefore, power facility 1 is designated with a “1.”

Power facility 2 appears in the 2nd row in the distance matrix and is therefore assigned a designation of “2.”

Power facility 3 appears in the 3rd row and is designated “3.”

Power facility 4 appears in the 4th row and is designated “4.”

Power facility 5 appears in the 5th row and is designated “5.”

Power facility 6 appears in the 6th row and is designated “6.”

Therefore, as the initial entries, the numbers from 1 to 6 are introduced, correspondingly, to cells **B9:G9**.

This set of 6 decision variables is collectively subject to the **Alldifferent Constraint**. As a result, each one of these six decision variables will be assigned an integer between 1 and 6. None of the six decision variables in this set can be assigned the same number.

Cell **H9=B9** to show the closed route loop.

In cells **B10:G10**, the distances between the consecutive power facility sites are calculated by **INDEX** functions (table 1.16).

The **INDEX** function has the following syntax:

=INDEX(range, row number, column number)

According to the Excel presentation of the distance matrix shown in fig. 1.23, the **range** in applied **INDEX** function is cells **B2:G7**. This cell range holds the distances in the distance matrix.

The row number corresponds to the current power facility inspected. The row number is the current PF site's row number in the distance matrix.

The column number corresponds to the next power facility to visit and inspect. The column number is the next PF site's column number in the distance matrix.

It is required to introduce expression = **INDEX($B$2:$G$7;B9;C9)** into cell **B10** and then copy it to cells **C10:G10**.

Table 1.16 – Excel expressions for calculating distance between the consecutive power facility sites on the route

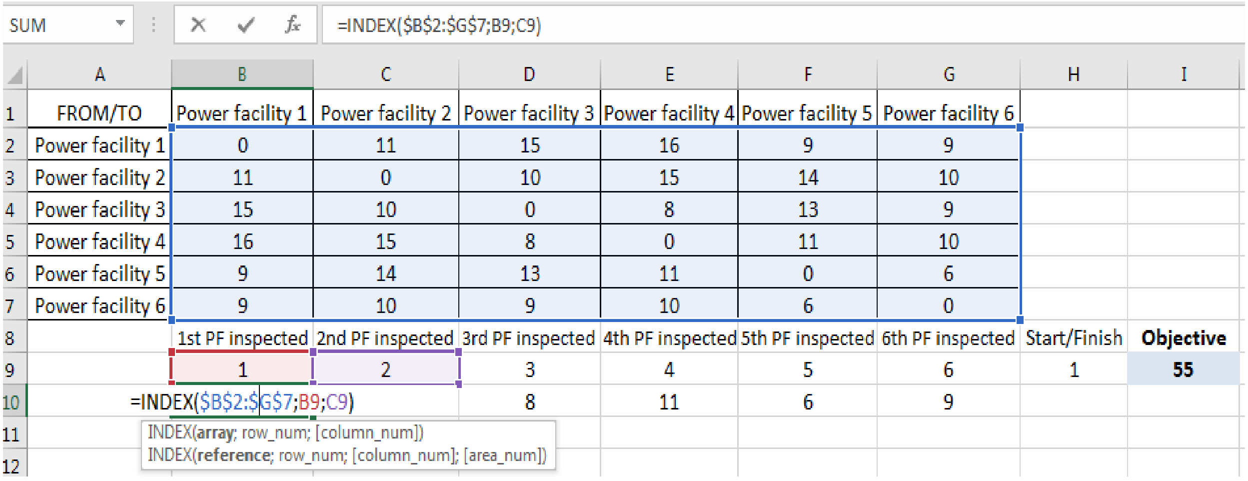
Cell	Excel expression
B10	**=INDEX($B$2:$G$7;B9;C9)**
C10	**=INDEX($B$2:$G$7;C9;D9)**
D10	**=INDEX($B$2:$G$7;D9;E9)**
E10	**=INDEX($B$2:$G$7;E9;F9)**
F10	**=INDEX($B$2:$G$7;F9;G9)**
G10	**=INDEX($B$2:$G$7;G9;H9)**
I9	**=SUM(B10:G10)**

It is required to determine the order of power facilities in the route of the inspection so as to minimize the total mileage of the route through all of the power facilities and back to the first or, in other words, to minimize the sum of distances covered between the consecutive power facility sites.

The objective function is the sum of all the distances in the route according to (1.31). It is introduced in cell **I9**.

The initial Excel model before the calculation is shown in fig. 1.24 – 1.25.

The **Solver parameters** window with introduced settings is given in fig. 1.26.



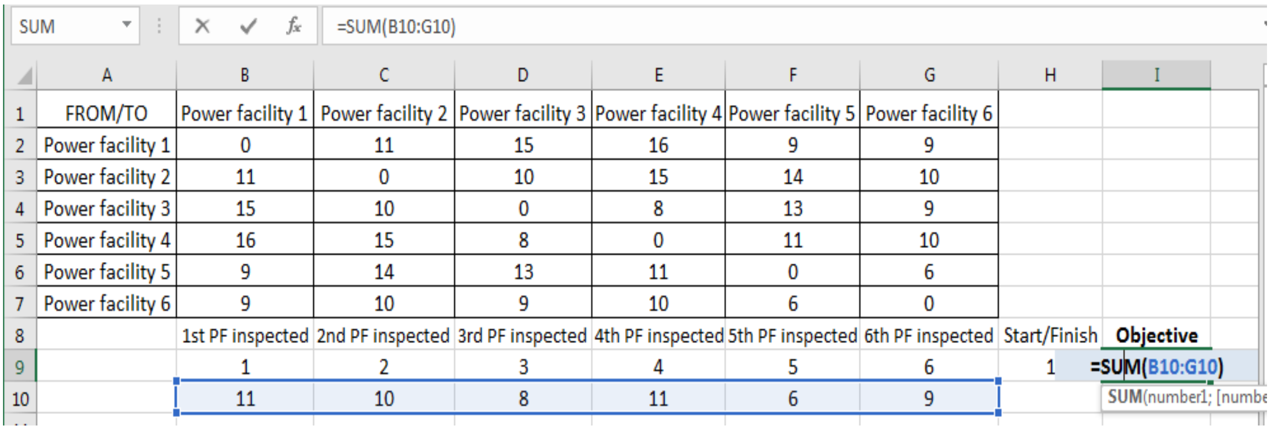
SUM

=INDEX($B$2:$G$7;B9;C9)

	A	B	C	D	E	F	G	H	I
1	FROM/TO	Power facility 1	Power facility 2	Power facility 3	Power facility 4	Power facility 5	Power facility 6		
2	Power facility 1	0	11	15	16	9	9		
3	Power facility 2	11	0	10	15	14	10		
4	Power facility 3	15	10	0	8	13	9		
5	Power facility 4	16	15	8	0	11	10		
6	Power facility 5	9	14	13	11	0	6		
7	Power facility 6	9	10	9	10	6	0		
8		1st PF inspected	2nd PF inspected	3rd PF inspected	4th PF inspected	5th PF inspected	6th PF inspected	Start/Finish	**Objective**
9		1	2	3	4	5	6	1	**55**
10	=INDEX($B$2:$G$7;B9;C9)			8	11	6	9		
11									
12									

INDEX(**array**; row_num; [column_num])
INDEX(**reference**; row_num; [column_num]; [area_num])

Figure 1.24 – Introduction of **INDEX** function (the pointer is in cell **B10** to show application of **INDEX** function to calculating the distance between the consecutive sites)



SUM

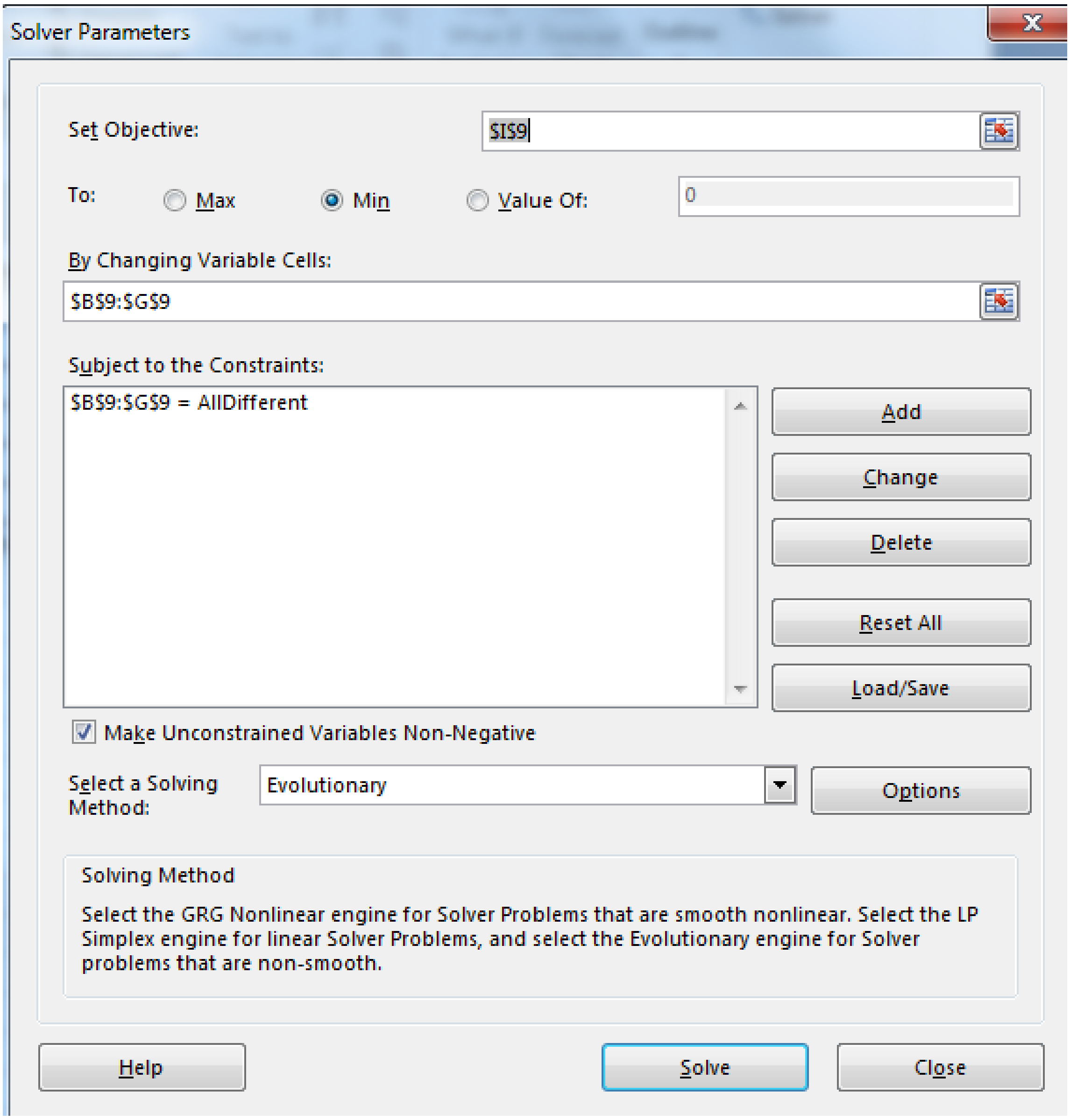
=SUM(B10:G10)

	A	B	C	D	E	F	G	H	I
1	FROM/TO	Power facility 1	Power facility 2	Power facility 3	Power facility 4	Power facility 5	Power facility 6		
2	Power facility 1	0	11	15	16	9	9		
3	Power facility 2	11	0	10	15	14	10		
4	Power facility 3	15	10	0	8	13	9		
5	Power facility 4	16	15	8	0	11	10		
6	Power facility 5	9	14	13	11	0	6		
7	Power facility 6	9	10	9	10	6	0		
8		1st PF inspected	2nd PF inspected	3rd PF inspected	4th PF inspected	5th PF inspected	6th PF inspected	Start/Finish	**Objective**
9		1	2	3	4	5	6	1	**=SUM(B10:G10)**
10		11	10	8	11	6	9		SUM(number1; [numbe

Figure 1.25 – Introduction of the objective function (the pointer is in cell **I9** to show calculation of the total mileage of the route)

Each power facility's unique row number in the distance matrix must be assigned to only 1 of the 6 decision variable cells. Therefore, the **Alldifferent Constraint** must be applied to all of the decision variable cells (cells **B9** to **G9**) simultaneously as a group. As a result of the **Alldifferent constraint**, these 6 cells will hold the integers 1 to 6. No two cells in this group will be assigned the same number. This ensures that each power facility will be visited only once and that all power facilities will be visited.

To solve Travelling Salesman Problem with MS Solver, neither Simplex LP nor GRG nonlinear methods can be chosen because discontinuous Excel function **INDEX** is used to calculate distance between the consecutive sites on the route.



Solver Parameters

Set Objective: $I$9

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $B$9:$G$9

Subject to the Constraints:
$B$9:$G$9 = AllDifferent

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: Evolutionary

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 1.26 – The **Solver parameters** window for the considered TSP

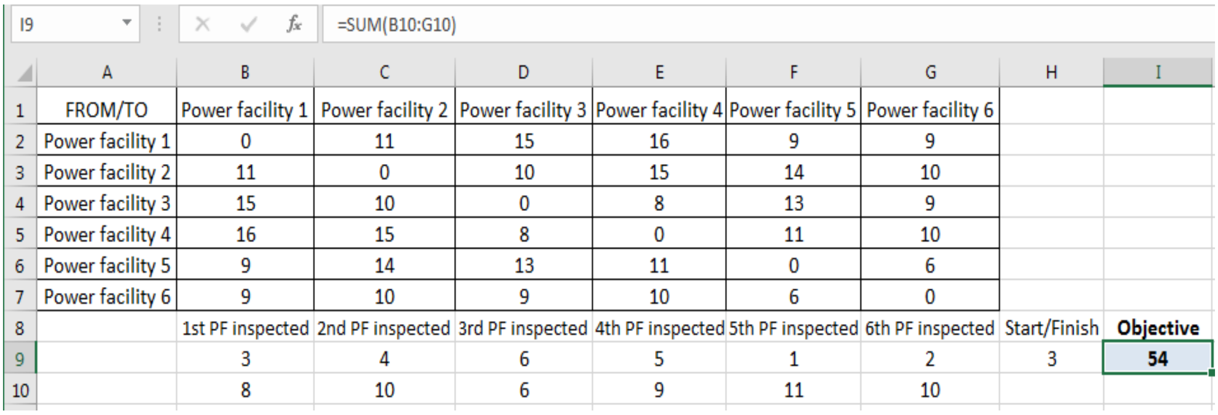
If the mathematical path to the objective contains any cells holding non-smooth or discontinuous formulas, the **Evolutionary** method is applied.

Common non-smooth Excel functions are **MIN**, **MAX**, and **ABS**.

Common discontinuous Excel functions are **INDEX**, **HLOOKUP**, **VLOOKUP**, **LOOKUP**, **INT**, **ROUND**, **COUNT**, **CEILING**, **FLOOR**, **IF**, **CHOOSE**, **NOT AND**, **OR**, **GREATER THAN**, **LESS THAN**, and **EQUAL TO**.

In **Options** window of Solver, maximum time must be set to 30 seconds. It means that after 30 seconds of calculation the Solver offers “to stop” or “to continue”. The choice is “to stop”.

The solution to example TSP 1.3.2.2 is shown in fig. 1.27.



I9 =SUM(B10:G10)

	A	B	C	D	E	F	G	H	I
1	FROM/TO	Power facility 1	Power facility 2	Power facility 3	Power facility 4	Power facility 5	Power facility 6		
2	Power facility 1	0	11	15	16	9	9		
3	Power facility 2	11	0	10	15	14	10		
4	Power facility 3	15	10	0	8	13	9		
5	Power facility 4	16	15	8	0	11	10		
6	Power facility 5	9	14	13	11	0	6		
7	Power facility 6	9	10	9	10	6	0		
8		1st PF inspected	2nd PF inspected	3rd PF inspected	4th PF inspected	5th PF inspected	6th PF inspected	Start/Finish	**Objective**
9		3	4	6	5	1	2	3	**54**
10		8	10	6	9	11	10		

Figure 1.27 – The optimal solution to example TSP 1.3.2.2

Analysis of the obtained solution is the following.

The 54-mile-long optimal route that covers all six power facility sites is

Power facility 3 → Power facility 4 → Power facility 6 → Power facility 5 → Power facility 1 → Power facility 2 → Power facility 3.

The optimal route with visiting each PF site only once is shown in fig. 1.28.

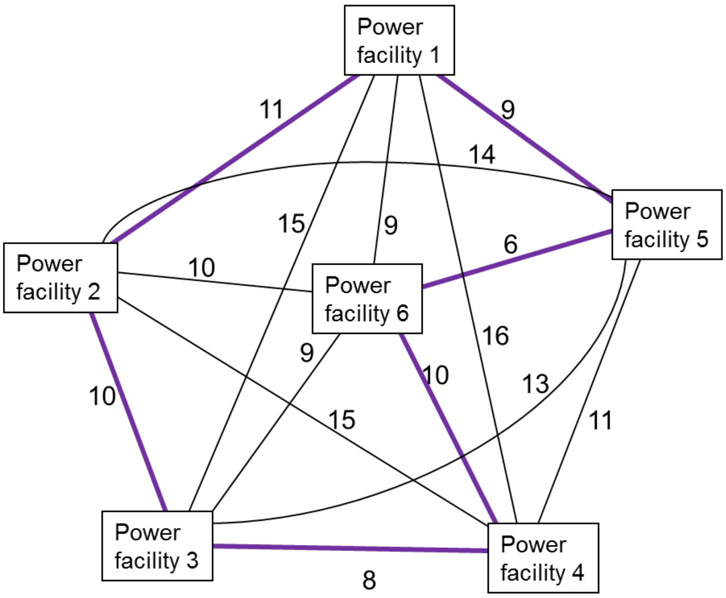


Figure 1.28 – The optimal route for in-service inspection of power facilities

The team of electricians can start from any power facility and travel along the obtained route. For example, if they start in **Power facility 1**, they should go to **Pow-**

er facility 5, then to **Power facility 6**, from **Power facility 6** to **Power facility 4**, then to **Power facility 3**, from **Power facility 3** to **Power facility 2**, and from **Power facility 2** they return to **Power facility 1**.

Or they can start in the opposite direction, from **Power facility 1 → Power facility 2 → Power facility 3 → Power facility 4 → Power facility 6 → Power facility 5** and back to **Power facility 1**.

1.4. Assignments

1.4.1. Assignment on Fuel Delivery Scheduling Problem

A coal supplying company has negotiated a contract with a generating company (GenCo) on supplying fuel to GenCo's power plants. The GenCo structure, the fuel delivery details, and fuel inventory requirements are given in tables 1.17 – 1.20.

Find the optimal coal delivery schedule for the GenCo plants according to the variant. Analyze the obtained solution.

Table 1.17 – Initial data on coal delivery, storage capacity and coalinventories

№ var	Coal	$D(j)$, ton/period	Storage capacity, ton	Initial inventory, ton	Final inventory, ton
1	G	11100	$10 \times 24 \times F_{\text{PPmax}}$	$4 \times 24 \times F_{\text{PPmax}}$	$5 \times 24 \times F_{\text{PPmax}}$
2	A	15000	$10 \times 24 \times F_{\text{PPmax}}$	$3 \times 24 \times F_{\text{PPmax}}$	$4 \times 24 \times F_{\text{PPmax}}$
3	Z	12800	$10 \times 24 \times F_{\text{PPmax}}$	$5 \times 24 \times F_{\text{PPmax}}$	$6 \times 24 \times F_{\text{PPmax}}$
4	L	10700	$10 \times 24 \times F_{\text{PPmax}}$	$6 \times 24 \times F_{\text{PPmax}}$	$5 \times 24 \times F_{\text{PPmax}}$
5	A	7400	$10 \times 24 \times F_{\text{PPmax}}$	$5 \times 24 \times F_{\text{PPmax}}$	$4 \times 24 \times F_{\text{PPmax}}$
6	OS	8600	$10 \times 24 \times F_{\text{PPmax}}$	$6 \times 24 \times F_{\text{PPmax}}$	$5 \times 24 \times F_{\text{PPmax}}$
7	T	14500	$10 \times 24 \times F_{\text{PPmax}}$	$4 \times 24 \times F_{\text{PPmax}}$	$5 \times 24 \times F_{\text{PPmax}}$
8	K	13200	$10 \times 24 \times F_{\text{PPmax}}$	$3 \times 24 \times F_{\text{PPmax}}$	$4 \times 24 \times F_{\text{PPmax}}$

Таблица 1.18 – Lower heat value of coal

Lower heat value of coal	Fuel						
	Coal A	Coal L	Coal G	Coal Z	Coal K	Coal OS	Coal T
Q_{LHV}, kcal/kg	6880	5600	5900	6100	6300	6350	6440

Table 1.19 – Variants of GenCo structure and load schedule

№ var	PP1	PP2	PP3	Load schedule			
1	4x№15	3x№11	6x№18	Day 1	MW	Day 2	MW
				0^{00} – 7^{00}	1190	0^{00} – 7^{00}	1060
				8^{00} – 15^{00}	1580	8^{00} – 15^{00}	1240
				16^{00} – 23^{00}	1720	16^{00}–23^{00}	1680
2	8x№19	2x№1	4x№12	Day 1	MW	Day 2	MW
				0^{00} – 11^{00}	910	0^{00} – 11^{00}	1140
				12^{00} – 23^{00}	1 70	12^{00}–23^{00}	1320
3	3x№3	5x№13	6x№21	Day 1	MW	Day 2	MW
				0^{00} – 11^{00}	1660	0^{00} – 11^{00}	1750
				12^{00} – 23^{00}	2280	12^{00}–23^{00}	2410
4	6x№19	2x№10	8x№17	Day 1	MW	Day 2	MW
				0^{00} – 11^{00}	430	0^{00} – 11^{00}	520
				12^{00} – 23^{00}	690	12^{00}–23^{00}	710
5	2x№6	6x№14	3x№8	Day 1	MW	Day 2	MW
				0^{00} – 11^{00}	980	0^{00} – 11^{00}	1140
				12^{00} – 23^{00}	1410	12^{00}–23^{00}	1360
6	8x№16	2x№2	10x№20	Day 1	MW	Day 2	MW
				0^{00} – 7^{00}	540	0^{00} – 7^{00}	510
				8^{00} – 15^{00}	900	8^{00} – 15^{00}	920
				16^{00} – 23^{00}	930	16^{00}–23^{00}	960
7	3x№5	5x№16	2x№7	Day 1	MW	Day 2	MW
				0^{00} – 7^{00}	1200	0^{00} – 7^{00}	13 0
				8^{00} – 15^{00}	1800	8^{00} – 15^{00}	188
				16^{00} – 23^{00}	2220	16^{00}–23^{00}	2250
8	2x№9	7x№18	5x№13	Day 1	MW	Day 2	MW
				0^{00} – 7^{00}	950	0^{00} – 7^{00}	1020
				8^{00} – 15^{00}	1480	8^{00} – 15^{00}	1560
				16^{00} – 23^{00}	1740	16^{00}–23^{00}	1800

Table 1.20– Turbine heat rates

№	Turbine type	Heat rate, Gcal/hr	Output bounds, MW
1.	К-500-65/3000 ХТГЗ-II	$Q(P) = 140.6 + 2.33P$	$300<=P<=500$
2.	К-500-65/3000 ХТГЗ-II	$Q(P) = 93.06 + 2.428P$	$300<=P<=500$
3.	К-300-240 ЛМЗ-I	$Q(P) = 19.71 + 1.908P$	$140<=P<=300$
4.	К-300-240 ЛМЗ-II	$Q(P) = 18.54 + 1.950P$	$150<=P<=300$
5.	К-300-240 ЛМЗ-III	$Q(P) = 17.9 + 1.846P$	$140<=P<=300$
6.	К-300-240 ЛМЗ-IV	$Q(P) = 17.52 + 1.870P$	$150<=P<=300$
7.	К-300-240 ЛМЗ-V	$Q(P) = 34.79 + 1.846P$	$140<=P<=300$
8.	К-300-240 ЛМЗ-VI	$Q(P) = 31.84 + 1.897P$	$140<=P<=300$
9.	К-300-240 ЛМЗ-VII	$Q(P) = 31.00 + 1.796P$	$140<=P<=300$
10.	К-300-240 ЛМЗ-VIII	$Q(P) = 30.65 + 1.819P$	$145<=P<=315$
11.	К-300-240 ЛМЗ-IX	$Q(P) = 40.63 + 1.782P$	$140<=P<=300$
12.	ПТ-135/165-130/15 ТМЗ	$Q(P) = 30.71 + 2.01P$	$65<=P<=135$
13.	ПТ-135/165-130/15 ТМЗ	$Q(P) = 19.17 + 2.1P$	$55<=P<=130$
14.	ПТ-80/100-130/13 ЛМЗ-I	$Q(P) = 15.6 + 2.04P$	$40<=P<=80$
15.	ПТ-80/100-130/13 ЛМЗ-II	$Q(P) = 13.2 + 2.10P$	$44<=P<=85$
16.	Т-50-130 ТМЗ -I	$Q(P) = 10.3 + 1.985P+0.195(P-Pu)$	$20<=P<=55$ $Pu=45.44$
17.	Т-50-130 ТМЗ -II	$Q(P) = 10.0 + 1.987P+0.376(P-Pu)$	$20<=P<=50$ $Pu=45.3$
18.	К-50-90-2 ЛМЗ -I	$Q(P) = 11.55 + 2.077P+0.149(P-Pu)$	$20<=P<=55$ $Pu=34.55$
19.	К-50-90-2 ЛМЗ -II	$Q(P) = 8.44 + 2.145P+0.144(P-Pu)$	$20<=P<=50$ $Pu=34.92$
20.	К-50-90-2 ЛМЗ -III	$Q(P) = 12.49 + 2.081P+0.145(P-Pu)$	$25<=P<=55$ $Pu=34.10$
21.	К-50-90-3 ЛМЗ -I	$Q(P) = 11.3 + 2.004P+0.232(P-Pu)$	$20<=P<=55$ $Pu=36.2$

1.4.2. Assignment on Transportation Problem

There are a few generation nodes (power plants) and several load nodes (consumers) in the electric grid (table 1.21). The transmission costs from each generation node to each load node (m.u./p.u.) as well as the nodes capacities in power units (p.u.) are given in table 1.22.

Some transmission lines are under repair or not available. For some lines, power transmission capacity is limited to a specified level.

The number of generation nodes and load nodes must be chosen according to the assignment variant (table 1.21).

Table 1.21 – Data on the grid configuration according to the variants

Variant	Power plants	Consumers	Maximum transmission capacity	Transmission not available
1	1,2,3	1,2,6,7,8	S_{16}=95, S_{38}=80	2×2, 3×6
2	2,3,4,5	1,2,5,6	S_{31}=76, S_{46}=105	2×6, 5×1
3	1,2,4	1,2,3,6,7	S_{43}=55, S_{38}=82	1×7, 4×3
4	1,2,3,4	3,4,5,6	S_{35}=84, S_{38}=63	3×3, 4×6
5	1,2,5	2,3,4,5,8	S_{15}=89, S_{58}=96	1×8, 5×2
6	1,2,3,5	1,2,5,6	S_{35}=88, S_{26}=65	5×5, 2×1
7	2,3,4	2,3,4,6,7	S_{34}=95, S_{42}=101	3×3, 2×7
8	1,2,3,5	1,2,4,6	S_{32}=75, S_{14}=84	1×6, 5×1

In “Maximum transmission capacity” column, transmission lines with limited transmission capacity are given. For example, S_{33}=50 means that transmission line capacity from power plant 3 to consumer 3 is 50 p.u. and power volume supplied from power plant 3 to consumer 3 cannot exceed 50 p.u. (in the solution, x_{33} must be ≤ 50).

In “Transmission not available” column, transmission lines which are not available between generation node *N* and load node *M* are given. For example, «2×3» means that there is no transmission line from power plant 2 to consumer 3 (in the solution, x_{23} must be =0).

Table 1.22–Generation nodes capacities, p.u.; load nodes demands, p.u.; transmission cost, m.u./p.u.

Generation facilities	Consumers								**Generation capacity, p.u.**
	Cons1	Cons 2	Cons 3	Cons 4	Cons 5	Cons 6	Cons 7	Cons 8	
Power plant 1	0.041	0.034	0.045	0.064	0.041	0.046	0.036	0.025	**360**
Power plant 2	0.047	0.025	0.022	0.021	0.023	0.022	0.021	0.022	**350**
Power plant 3	0.035	0.024	0.021	0.033	0.019	0.021	0.025	0.029	**270**
Power plant 4	0.040	0.040	0.038	0.039	0.031	0.037	0.029	0.033	**300**
Power plant 5	0.031	0.026	0.019	0.047	0.022	0.019	0.023	0.021	**250**
Load, p.u.	**162**	**138**	**171**	**127**	**196**	**153**	**215**	**184**	

With application of MS Excel Solver, determine the power plants output and optimal power transmission configuration to satisfy the consumers' electricity demand with minimum total transmission costs. Solve the problem for two conditions:

1 – without power transfer between the consuming nodes;

2 – with power transfer between the consuming nodes, the transfer cost of 0.002 $/p.u.

Analyze the obtained solutions.

Give the graphic presentations of the optimal power transfer configurations for both conditions. In the figures, show the rated capacity and the actual output of the power plants and the demand and actual consumption at the load nodes.

1.4.3. Assignment on Assignment Problem

Find optimal solutions with MS Excel Solver to the following assignment problems according to the variants.

One truck is available at each of the stations S-I$_i$ and one truck is required at each of the stations S-II$_j$. The distances between the various stations, miles, are given according to the variant (tables 1.23 – 1.28). What truck should be dispatched to which station and how should they be dispatched so as to minimize the total mileage covered?

Table 1.23 – Variant 1: Matrix of distances between the stations, miles

	Stations S-II$_i$			
Stations S-I$_j$	**I**	**II**	**III**	**IV**
1	27	31	27	35
2	28	30	29	32
3	37	29	32	31
4	32	34	28	29
5	34	33	31	30
6	30	34	33	28

Table 1.24 – Variant 2: Matrix of distances between the stations, miles

	Stations S-II$_i$			
Stations S-I$_j$	**I**	**II**	**III**	**IV**
1	27	31	27	35
2	28	30	29	32
3	37	29	32	31
4	32	34	28	29
5	34	33	31	30

Table 1.25 – Variant 3: Matrix of distances between the stations, miles

	Stations S-II$_i$					
Stations S-I$_j$	**I**	**II**	**III**	**IV**	**V**	**VI**
1	41	72	39	52	25	51
2	22	29	49	65	81	50
3	27	39	60	51	32	32
4	45	50	48	52	65	43
5	29	40	39	26	30	33
6	82	40	40	60	51	30
7	38	47	51	39	48	29

Table 1.26 – Variant 4: Matrix of distances between the stations, miles

	Stations S-II$_i$			
Stations S-I$_j$	**I**	**II**	**III**	**IV**
1	27	31	-	35
2	-	30	29	32
3	37	29	32	31
4	32	-	28	29
5	34	33	31	-
6	30	34	-	28

Table 1.27 – Variant 5: Matrix of distances between the stations, miles

	Stations S-II$_i$					
Stations S-I$_j$	**I**	**II**	**III**	**IV**	**V**	**VI**
1	41	72	39	52	25	-
2	-	29	49	65	81	50
3	27	39	-	51	32	32
4	45	50	48	52	-	43
5	29	40	39	26	30	33
6	82	40	40	-	51	30
7	38	-	51	39	48	29

Table 1.28 – Variant 6: Matrix of distances between the stations, miles

	Stations S-II$_i$				
Stations S-I$_j$	**I**	**II**	**III**	**IV**	**V**
1	41	-	39	52	25
2	22	29	49	65	-
3	27	39	-	51	32
4	45	50	48	52	65
5	29	40	39	26	30
6	-	40	40	60	51
7	38	47	51	39	48

1.4.4. Assignment on Travelling Salesman Problem

There are several power substations the electrician must visit, starting from the utility office 0, to read the equipment operation data. The variants of the route with distance (km) between the substations are given in table 1.29. Find the optimal route minimizing the total distance to travel and show it in the graph.

Table 1.29 – Job efficiency and hourly charges of the typists

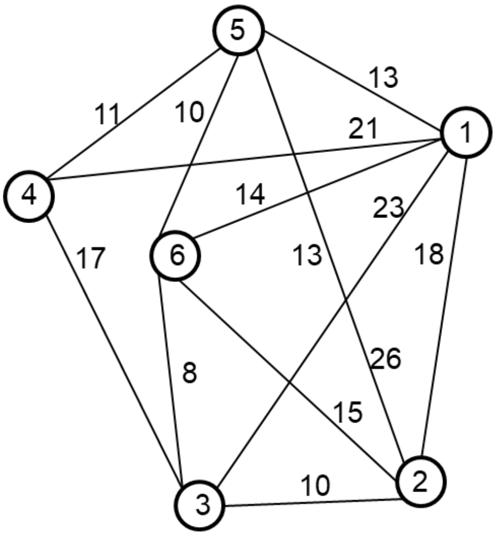
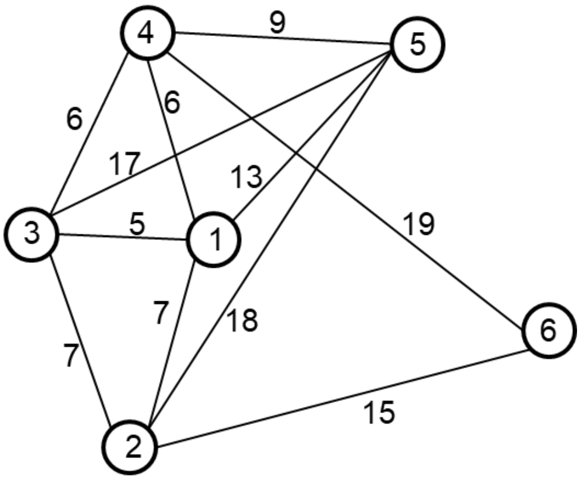
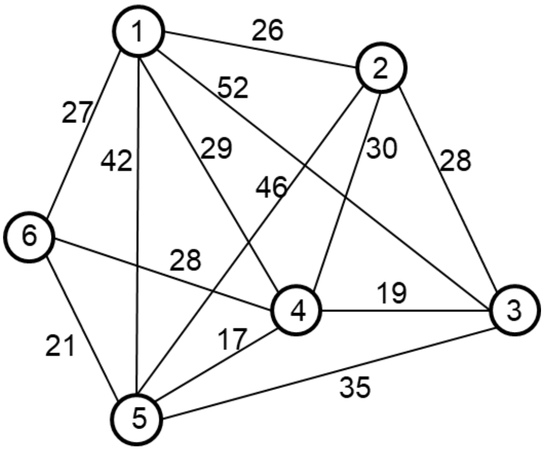
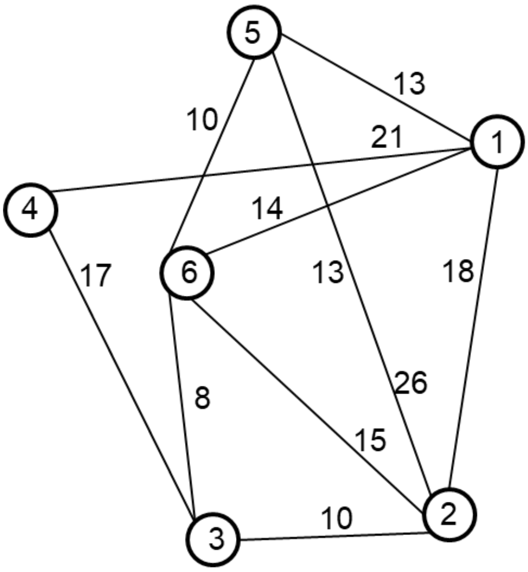
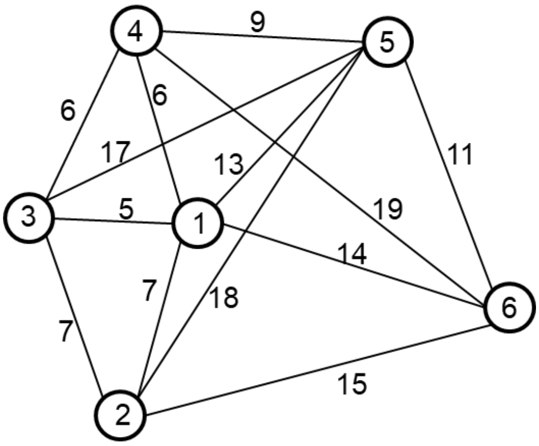
Variant	Route
1	2
1	
2	

Table 1.29 continued

1	2
3	
4	
5	

2. NON-LINEAR PROGRAMMING PROBLEMS IN POWER ENGINEERING. ECONOMIC DISPATCH OF ELECTRIC POWER SYSTEMS

2.1. Economic dispatch of generating units at a thermal power plant

2.1.1. Condition of economic dispatch of generating units at a thermal power plant

One of the most important and widespread problems in power system operation optimization is economic dispatch of power units for certain power demands.

The typical objective in optimum generation scheduling is minimization of generation costs under meeting the power requirements of the system over a definite period of time.

Mathematical model of this problem is based on input-output characteristics of thermal power units taking into account power requirements such as power plant output limitations, power demand, power loss and some other.

The input to a thermal generating unit is fuel F consumed by the unit boiler and measured in MJ/hr. The output is power generated P_G measured in MW.

The input-output characteristic of a thermal power unit is nonlinear (fig. 2.1) and can be described with a quadratic function

$$F(P_G) = a + bP_G + cP_G^2 \tag{2.1}$$

where $F(P)$ is fuel consumption by the generating unit; P_G is the unit output, a, b, and c are the input-output characteristic constants that depend on the boiler-turbine-generator design parameters and usually specified during the unit testing before the commissioning or after the unit overhaul. Constant a characterizes the fuel consumption by the unit boiler at the unit start without power generation (in no-load operation).

The constants of the input-output characteristic of a generating unit can also be specified with application of statistical data for the unit on fuel consumed for definite power outputs.

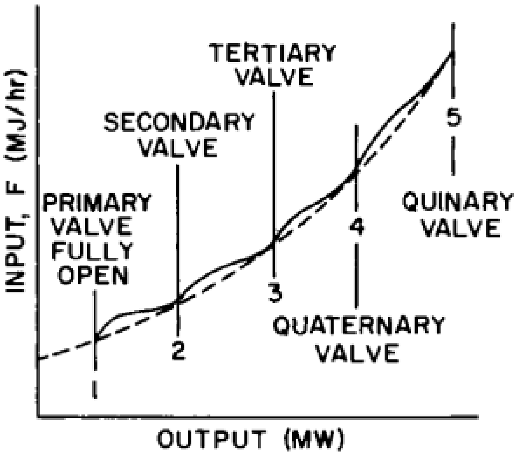


Figure 2.1 – Typical input-output curve F-P for a thermal generating unit [5]

From the generating unit input-output characteristic, it is seen that the unit output is bounded below and above (by minimum and maximum capacity)

$$P_{G\min} \le P_G \le P_{G\max} \,. \tag{2.2}$$

As a rule, minimum output P_{Gmin} is function of the unit boiler, turbine, and generator technical conditions and operation constraints. Maximum output P_{Gmax} is determined by the rated capacity of the unit.

The economic dispatch of power generation is determined under rigid requirement for active power balance in the power system which consists in equality of the total power system generation $P_{\text{G\_total}}$ to the total power system demand $P_{\text{D\_total}}$ taking into account power loss in the grids

$$P_{G\_total} = P_{D\_total} + P_{loss}. \tag{2.3}$$

Let us first consider a thermal power plant (TPP) comprising *n* generating units (fig. 2.2).

For *n*-unit power plant, the total fuel consumption is a sum of fuel consumption by each unit

$$F_{TPP} = \sum_{i=1}^{n} F_i(P_{Gi}) = F_1(P_{G1}) + F_2(P_{G2}) + \ldots + F_n(P_{Gn}) \,. \tag{2.4}$$

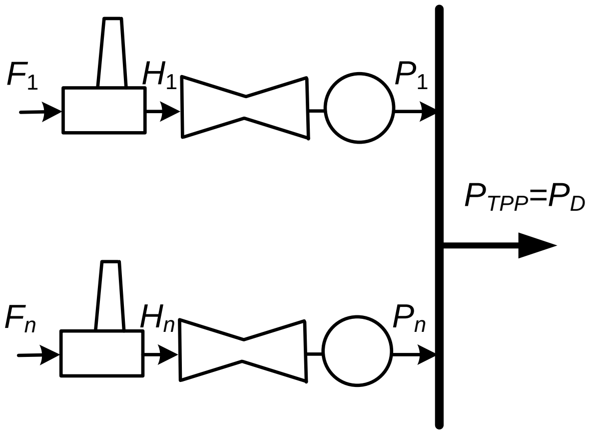


Figure 2.2 – A thermal power plant with *n* generating units

As the economic dispatch of power units consists in minimization the generation costs, which mainly depend on consumed fuel costs, we may choose minimization of fuel consumption as the objective function for this optimization problem

$$Z = \sum_{i=1}^{n} F(P_G)_i \rightarrow \min . \qquad (2.5)$$

The essential constraint in the problem is meeting the power system requirement for the power plant output: to keep active power generation in compliance with the generation demand P_D specified by the power system operator.

Typically, active power balance also includes power loss in the grids but for the economic dispatch of generating units in one power plant, power loss is negligible. Also we omit the unit output limitations (2.2) to define the condition for optimal generation dispatch at a thermal power plant.

Therefore, active power balance for the TPP is

$$P_{TPP} = P_{G1} + P_{G2} + \ldots + P_{Gn} = P_D$$

or

$$P_D - P_{G1} - P_{G2} - \ldots - P_{Gn} = 0 . \qquad (2.6)$$

Thus, the mathematical model for the problem of economic dispatch of generating units at a thermal power plant is

$$\begin{aligned}
&Z = F_1(P_{G1}) + F_2(P_{G2}) + \ldots + F_n(P_{Gn}) \to \min \\
&P_D - P_{G1} - P_{G2} - \ldots - P_{Gn} = 0 \qquad . \\
&P_{Gi} \geq 0, \quad i = 1,\ldots,n
\end{aligned} \tag{2.7}$$

The condition of optimal generating units dispatch can be found with the help of Lagrange multiplier technique.

Lagrange function L contains the objective function (2.5) and constraint (2.6) multiplied by a Lagrange multiplier:

$$L = F_1(P_{G1}) + F_2(P_{G2}) + \ldots + F_n(P_{Gn}) + \lambda \cdot (P_D - P_{G1} - P_{G2} - \ldots - P_{Gn}) \to \min \tag{2.8}$$

Taking into account (2.1), we can rewrite Lagrange function as

$$\begin{aligned}
L = (a_1 + b_1 P_{G1} + c_1 P_{G1}^2) + (a_2 + b_2 P_{G2} + c_2 P_{G2}^2) + \ldots + (a_n + b_n P_{Gn} + c_n P_{Gn}^2) + \\
+ \lambda \cdot (P_D - P_{G1} - P_{G2} - \ldots - P_{Gn}) \to \min
\end{aligned} \quad . \tag{2.9}$$

Minimum of Lagrange function (2.9) can be found by determining its first derivatives with respect to all the variables (generating units output P_{Gi}, $i = 1,..n$, and Lagrange multiplier λ) and setting them to 0.

$$\begin{aligned}
&\frac{\partial L}{\partial P_{G1}} = \frac{dF_1}{dP_{G1}} - \lambda = b_1 + 2 \cdot c_1 P_{G1} - \lambda = 0 \\
&\frac{\partial L}{\partial P_{G2}} = \frac{dF_2}{dP_{G2}} - \lambda = b_2 + 2 \cdot c_2 P_{G2} - \lambda = 0 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad . \\
&\frac{\partial L}{\partial P_{Gn}} = \frac{dF_n}{dP_{Gn}} - \lambda = b_n + 2 \cdot c_n P_{Gn} - \lambda = 0 \\
&\frac{\partial L}{\partial \lambda} = P_D - P_{G1} - P_{G2} - \ldots - P_{Gn} = 0
\end{aligned} \tag{2.10}$$

We obtain a system of (n+1) equations with (n+1) variables. After transposing the other component, λ, in the first n equations of (2.10) to the right side

$$\begin{aligned} &\frac{dF_1}{dP_{G1}} = \lambda \\ &\frac{dF_2}{dP_{G2}} = \lambda \quad , \\ &\ldots\ldots\ldots\ldots\ldots \\ &\frac{dF_n}{dP_{Gn}} = \lambda \end{aligned} \tag{2.11}$$

we can conclude that, as the right side in equations (2.11) is the same (the same Lagrange multiplier λ), the left sides are equal:

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = \ldots = \frac{dF_n}{dP_{Gn}} = idem\,. \tag{2.12}$$

Taking into account that the first derivative of input-output characteristic $F(P_G)$, MJ/hr, with respect to the power output $\frac{dF}{dP_G}$ specifies the incremental fuel rate $b + 2\cdot cP_G$, $\frac{dF}{dP_{Gi}} = b_i + 2\cdot c_i P_{Gi}$, we can write

$$b_1 + 2\cdot c_1 P_{G1} = b_2 + 2\cdot c_2 P_{G2} = \ldots = b_n + 2\cdot c_n P_{Gn} = idem\,. \tag{2.13}$$

Therefore, the Lagrange multiplier λ in this problem shows increase in the fuel consumption with increase in the generator output.

Expression (2.12), or (2.13), is a condition of optimal power dispatch for generating units in one power plant.

From condition (2.12), it follows that **the total fuel consumption in a thermal power plant will be minimal when the incremental fuel rates of all its operating units are equal**.

Usually, power unit output is limited (2.2), and the limitation must be taken into account in the mathematical model:

$$\begin{aligned}
&Z = F_1(P_{G1}) + F_2(P_{G2}) + \ldots + F_n(P_{Gn}) \to \min \\
&P_D - P_{G1} - P_{G2} - \ldots - P_{Gn} = 0 \\
&\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = \ldots = \frac{dF_n}{dP_{Gn}} \\
&P_{Gi\min} \le P_{Gi} \le P_{Gi\max}, \quad i = 1,\ldots,n
\end{aligned} \qquad . \qquad (2.14)$$

In many cases, imposed limitations on power unit output (2.2) may result in impossibility to meet condition (2.12) and, therefore, to obtain a solution. Then the procedure of solving such a problem is the following.

First, the problem is solved neglecting power unit output limitations (2.2) to find optimal incremental fuel rate λ.

After that, the problem is solved again with power unit limitations but without condition (2.12), and analysis is made to reveal deviation from the optimal incremental fuel rate and dependence of the assigned power output on the incremental fuel rate for every unit: the higher the incremental fuel rate, the lower the unit output.

2.1.2. Problem of economic dispatch for three power units connected to a single bus

A thermal power plant comprises three generating units connected to a single bus serving a given electrical load of 850 MW (fig. 2.3).

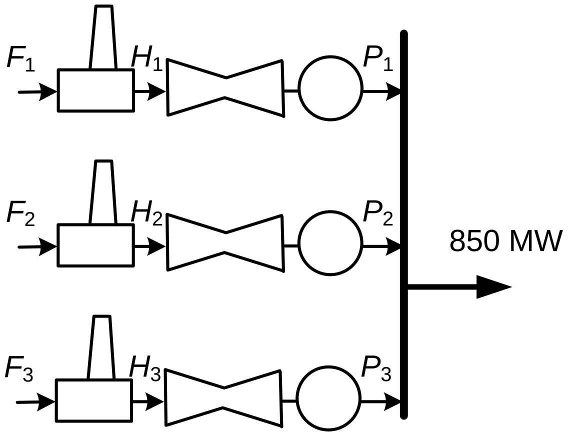


Figure 2.3 – A three-unit TPP with connected electrical load of 850 MW

The heat rates of the units are, correspondingly,

$$H_1(P_{G1}) = 510 + 7.2P_{G1} + 0.00142P_{G1}^2 \text{ [GJ/hr]};$$

$$H_2(P_{G2}) = 310 + 7.85P_{G2} + 0.00194P_{G2}^2 \text{ [GJ/hr]};$$

$$H_3(P_{G3}) = 78 + 7.97P_{G3} + 0.001482P_{G3}^2 \text{ [GJ/hr]};$$

The heat value of the fuel Q_{LHV} is 25.92 GJ/t.

The unit power output limitations, MW, are

$$\begin{aligned} &150 \le P_{G1} \le 600 \\ &100 \le P_{G2} \le 400\,. \\ &50 \le P_{G1} \le 200 \end{aligned}$$

Condition of the optimal dispatch (2.13) for the units is

$$7.2 + 2 \cdot 0.00142P_{G1} = 7.85 + 2 \cdot 0.00194P_{G2} = 7.97 + 2 \cdot 0.001482P_{G3}\,.$$

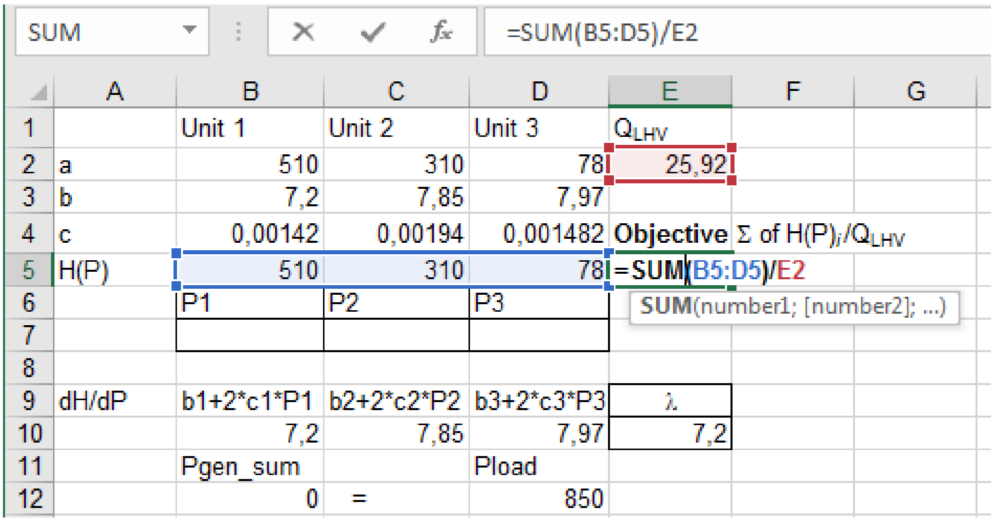
The mathematical model of the problem is

$$\begin{gathered} Z = \frac{1}{Q_{LHV}}[H_1(P_{G1}) + H_2(P_{G2}) + H_3(P_{G_3})] \to \min \\ P_{G1} + P_{G2} + P_{G3} = P_D = 850 \\ 150 \le P_{G1} \le 600 \\ 100 \le P_{G2} \le 400 \\ 50 \le P_{G1} \le 200 \\ 7.2 + 2 \cdot 0.00142P_{G1} = 7.85 + 2 \cdot 0.00194P_{G2} = 7.97 + 2 \cdot 0.001482P_{G3} \end{gathered} \quad . \tag{2.15}$$

First we will consider the economic dispatch problem for the plant ignoring the generator minimum and maximum power limits. The mathematical model under this condition is

$$\begin{gathered} Z = \frac{1}{25.92}[F_1(P_{G1}) + F_2(P_{G2}) + F_3(P_{G_3})] \to \min \\ P_{G1} + P_{G2} + P_{G3} = 850 \\ 7.2 + 2 \cdot 0.00142P_{G1} = 7.85 + 2 \cdot 0.00194P_{G2} = 7.97 + 2 \cdot 0.001482P_{G3} \\ P_{Gi} \ge 0, \quad i = 1,\dots,n \end{gathered} \quad . \tag{2.16}$$

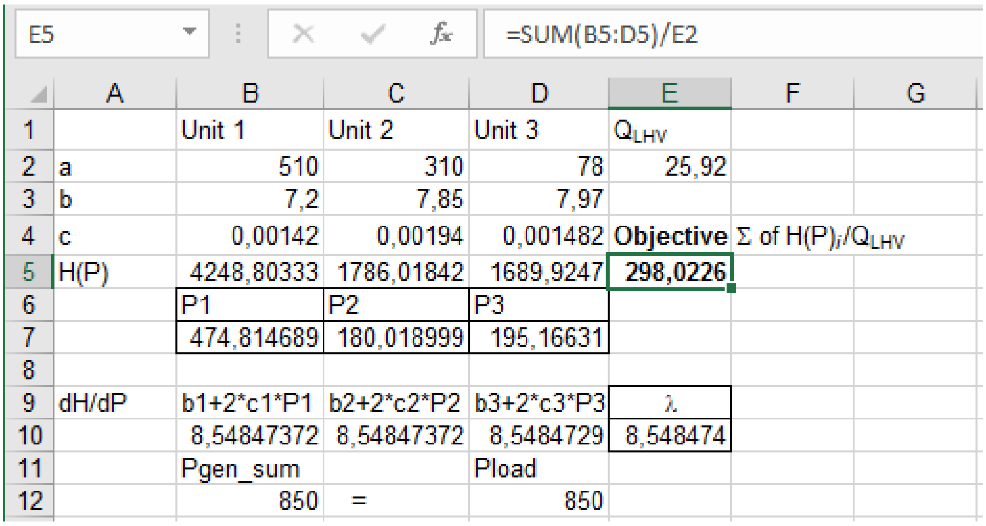
The Excel presentation and the optimal solution to problem (2.16) are given in fig. 2.4 and 2.5, respectively.



=SUM(B5:D5)/E2

	A	B	C	D	E	F	G
1		Unit 1	Unit 2	Unit 3	Q_{LHV}		
2	a	510	310	78	25,92		
3	b	7,2	7,85	7,97			
4	c	0,00142	0,00194	0,001482	**Objective**	Σ of $H(P)_i/Q_{LHV}$	
5	H(P)	510	310	78	=SUM(B5:D5)/E2		
6		P1	P2	P3			
7							
8							
9	dH/dP	b1+2*c1*P1	b2+2*c2*P2	b3+2*c3*P3	λ		
10		7,2	7,85	7,97	7,2		
11		Pgen_sum		Pload			
12		0	=	850			

Figure 2.4 – Excel presentation of problem (2.16) (the pointer is in cell E5 to show introduction of the objective function with SUM function)

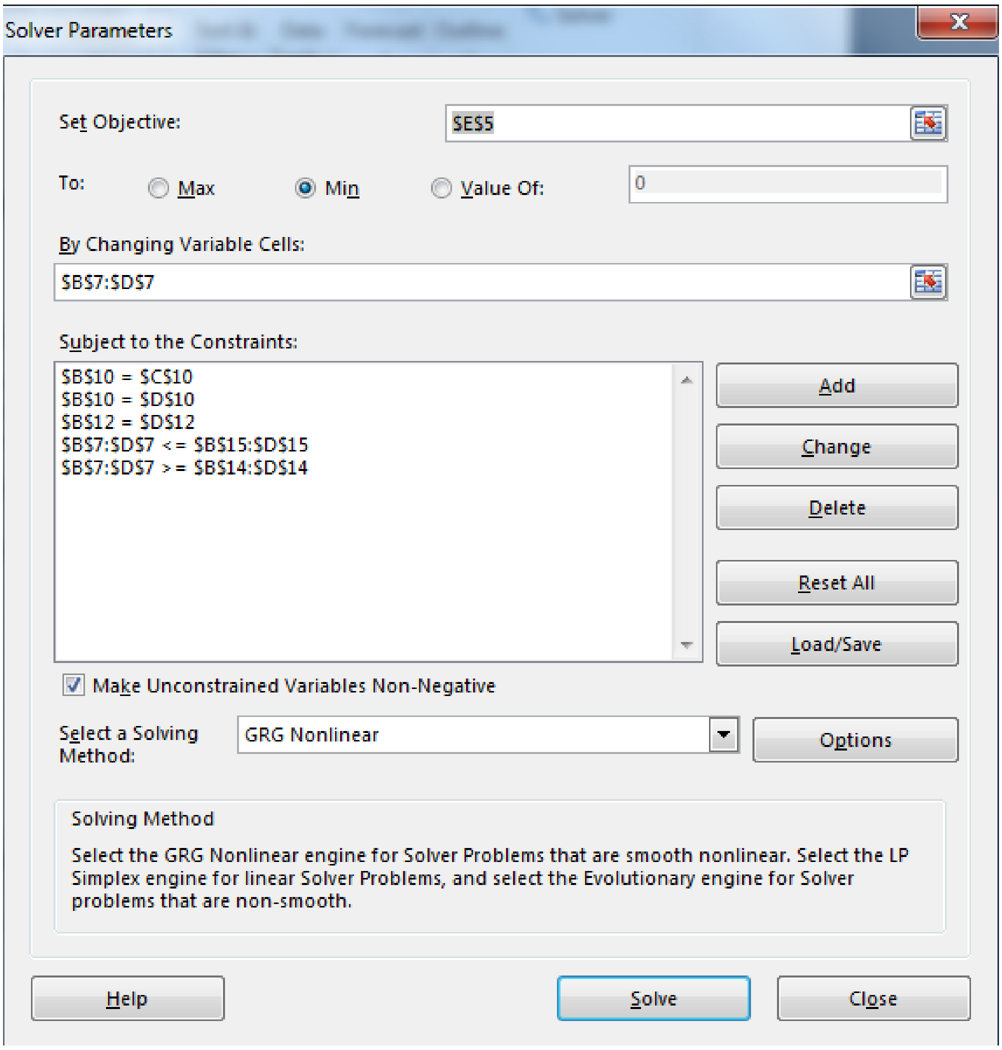


=SUM(B5:D5)/E2

	A	B	C	D	E	F	G
1		Unit 1	Unit 2	Unit 3	Q_{LHV}		
2	a	510	310	78	25,92		
3	b	7,2	7,85	7,97			
4	c	0,00142	0,00194	0,001482	**Objective**	Σ of $H(P)_i/Q_{LHV}$	
5	H(P)	4248,80333	1786,01842	1689,9247	**298,0226**		
6		P1	P2	P3			
7		474,814689	180,018999	195,16631			
8							
9	dH/dP	b1+2*c1*P1	b2+2*c2*P2	b3+2*c3*P3	λ		
10		8,54847372	8,54847372	8,5484729	8,548474		
11		Pgen_sum		Pload			
12		850	=	850			

Figure 2.5 – Excel solution to problem (2.16) (without considering power output bounds)

Figure 2.6 presents the settings of Solver parameters window for initial problem (2.15) that considers the all the constraints on economic dispatch of the generating units. The Excel solution to problem (2.15) is shown in fig. 2.7. The graphic illustration of the optimal solution obtained is presented in fig. 2.8.



Set Objective: $E$5

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $B$7:$D$7

Subject to the Constraints:

$B$10 = $C$10
$B$10 = $D$10
$B$12 = $D$12
$B$7:$D$7 <= $B$15:$D$15
$B$7:$D$7 >= $B$14:$D$14

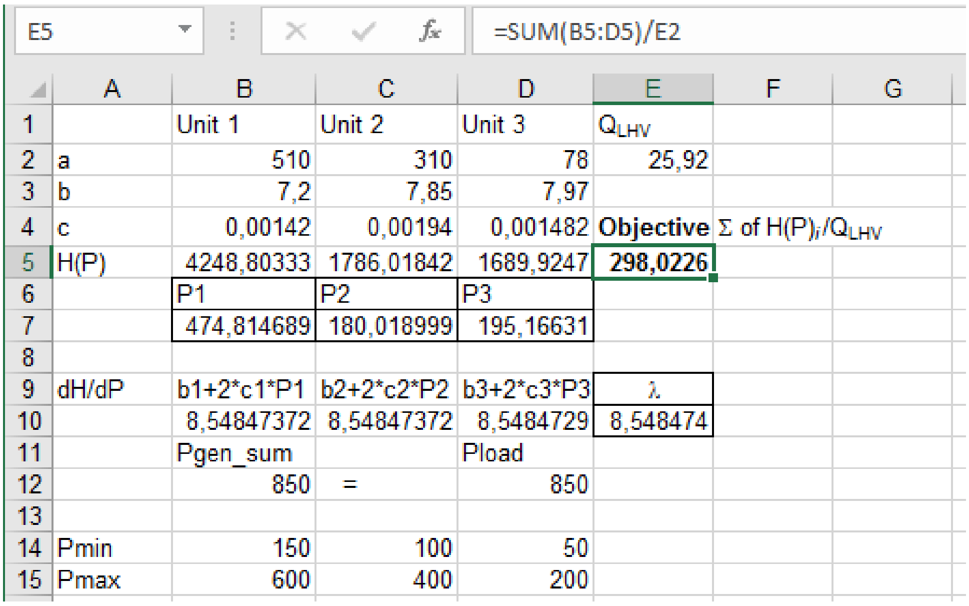
☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 2.6 – Settings of **Solver parameters** window for problem (2.15)



E5 =SUM(B5:D5)/E2

	A	B	C	D	E	F	G
1		Unit 1	Unit 2	Unit 3	Q_{LHV}		
2	a	510	310	78	25,92		
3	b	7,2	7,85	7,97			
4	c	0,00142	0,00194	0,001482	**Objective**	Σ of $H(P)_i/Q_{LHV}$	
5	H(P)	4248,80333	1786,01842	1689,9247	**298,0226**		
6		P1	P2	P3			
7		474,814689	180,018999	195,16631			
8							
9	dH/dP	b1+2*c1*P1	b2+2*c2*P2	b3+2*c3*P3	λ		
10		8,54847372	8,54847372	8,5484729	8,548474		
11		Pgen_sum		Pload			
12		850	=	850			
13							
14	Pmin	150	100	50			
15	Pmax	600	400	200			

Figure 2.7 – Excel solution to problem (2.15) with all considered constraints imposed on optimal operation of generating units

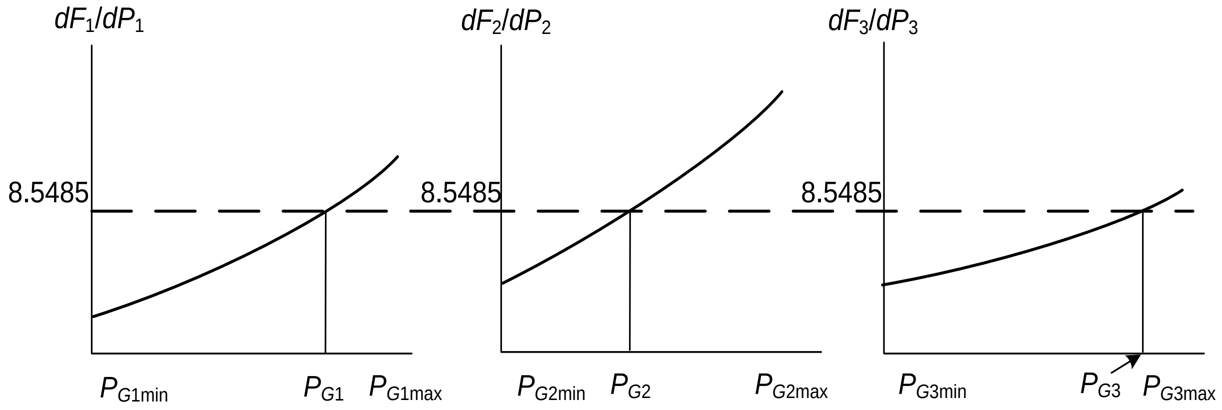


Figure 2.8 – Graphic illustration of the solution to problem (2.15)

Analysis of the obtained solution. As we can see, the optimal incremental heat rate is 8.5485, that is heat rate increases by 8.5485 GJ when the unit output increases 1 MW. As for active power dispatching, unit 1 output and unit 3 output are closer to the upper limits, especially unit 3 output, which is practically maximum, as compared with unit 2 output which is closer to the lower limit. This means that units 1 and 3 are more economical and efficient.

Comparing the obtained solutions shown in fig. 2.5 and 2.7, we can conclude that imposing the unit power output bounds does not prevent obtaining optimal solution with meeting condition (2.13). The incremental fuel rates for all the three blocks are equal, and the total fuel consumption is minimal under condition of covering electricity load of 850 MW.

2.1.3. Problem of economic dispatch for generating units in a thermal power plant consuming different fuel

A thermal power plant comprises two generating units connected to the same bus serving electrical load of 50 MW. Each unit can consume fuel oil and natural gas in any proportion. Natural gas consumption is limited to 10 f.u./hr. The objective is to find optimal dispatch of the unit so as to minimize fuel oil consumption. The fuel rates of the units are, t.o.e./hr, correspondingly,

$$F_{1oil}(P_{G1}) = 1.4609 + 0.15186 P_{G1} + 0.00145 P_{G1}^2;$$

$$F_{1gas}(P_{G1}) = 1.5742 + 0.1631 P_{G1} + 0.001358 P_{G1}^2;$$

$$F_{2oil}(P_{G2}) = 0.8008 + 0.2031P_{G2} + 0.000916P_{G2}^2;$$

$$F_{2gas}(P_{G2}) = 0.7266 + 0.2256P_{G2} + 0.000778P_{G2}^2;$$

where F_{oil} and F_{gas} are, correspondingly, fuel oil and natural gas consumption.

The power output of each generator is limited, MW, are 18 $\le P_{G1} \le 30$ and 14 $\le P_{G2} \le 25$.

Either unit can generate power P_{Gi}^*, i=1,2, consuming only fuel oil, or only natural gas, or any combination of the fuel components $\alpha_i F_{i\,oil}(P_{Gi}^*) + (1-\alpha_i)F_{i\,gas}(P_{Gi}^*)$, where α_i is fraction of the fuel oil, $0 \le \alpha_i \le 1$, and (1-α_i) is fraction of the natural gas in the fuel proportion.

The mathematical formulation of the problem is as follows.

The **variables** are the power output of the two generators P_{G1} and P_{G2} and the fractions α_1 and α_2 showing percentage of the fuel oil consumed by unit 1 and unit 2, correspondingly.

The **objective** is minimization of the fuel oil consumption

$$Z = \alpha_1 F_{1oil}(P_{G1}) + \alpha_2 F_{2oil}(P_{G2}) \to \min.$$

The **constraints** are

– limitation of natural gas consumption

$$(1-\alpha_1)F_{1gas}(P_{G1}) + (1-\alpha_2)F_{2gas}(P_{G2}) \le 10;$$

– limitations of the power output

$$\begin{matrix} 18 \le P_{G1} \le 30 \\ 14 \le P_{G2} \le 25 \end{matrix};$$

– limits of the fuel oil fraction variation

$$\begin{matrix} 0 \le \alpha_1 \le 1 \\ 0 \le \alpha_2 \le 1 \end{matrix};$$

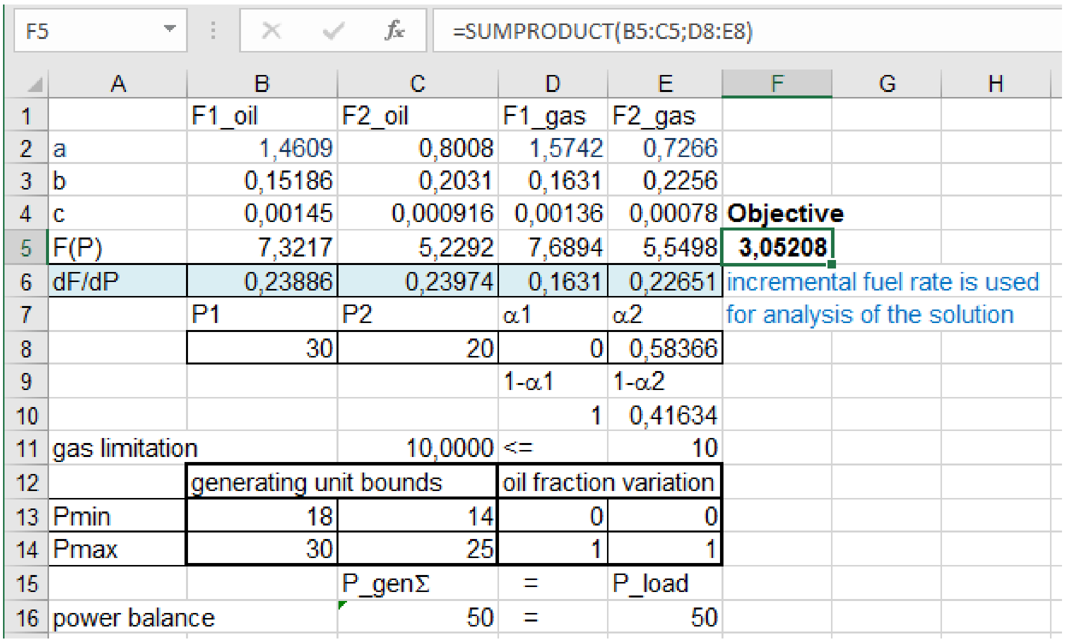
– power balance: equality of the power generated to the electrical load, MW

$$P_{G1} + P_{G2} = 50 .$$

The entire mathematical model is

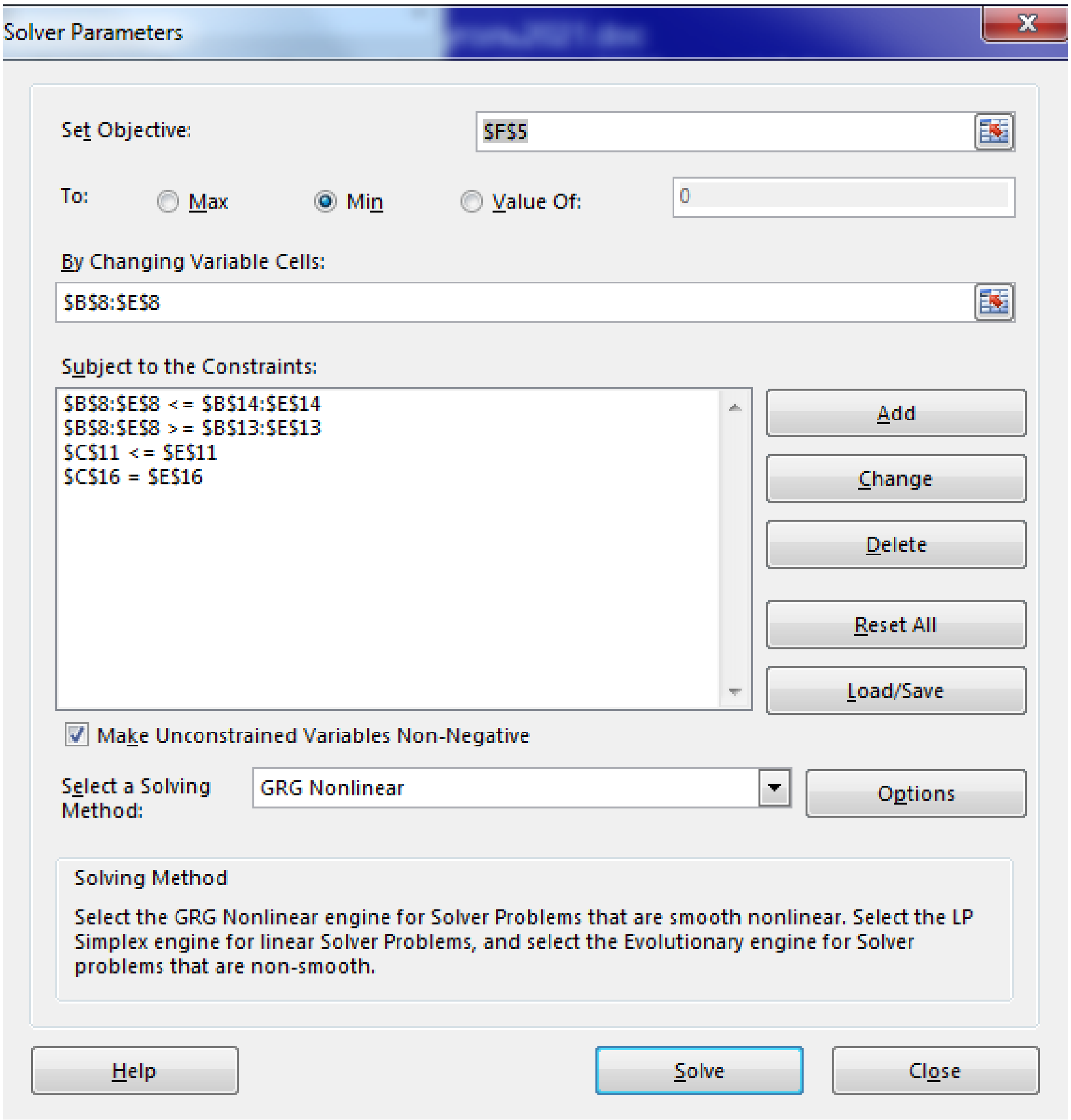
$$\begin{aligned} & Z = \alpha_1 F_{1oil}(P_{G1}) + \alpha_2 F_{2oil}(P_{G2}) \rightarrow \min \\ & (1-\alpha_1) F_{1gas}(P_{G1}) + (1-\alpha_2) F_{2gas}(P_{G2}) \le 10 \\ & 18 \le P_{G1} \le 30 \\ & 14 \le P_{G2} \le 25 \\ & 0 \le \alpha_1 \le 1 \\ & 0 \le \alpha_2 \le 1 \\ & P_{G1} + P_{G2} = 50 \end{aligned} \quad . \qquad (2.17)$$

The Excel solution to problem (2.17) is shown in fig. 2.9. The **Solver parameters** window with the introduced settings for problem (2.17) is given in fig. 2.10.



	A	B	C	D	E	F	G	H
1		F1_oil	F2_oil	F1_gas	F2_gas			
2	a	1,4609	0,8008	1,5742	0,7266			
3	b	0,15186	0,2031	0,1631	0,2256			
4	c	0,00145	0,000916	0,00136	0,00078	**Objective**		
5	F(P)	7,3217	5,2292	7,6894	5,5498	**3,05208**		
6	dF/dP	0,23886	0,23974	0,1631	0,22651	incremental fuel rate is used		
7		P1	P2	α1	α2	for analysis of the solution		
8		30	20	0	0,58366			
9				1-α1	1-α2			
10				1	0,41634			
11	gas limitation		10,0000	<=	10			
12		generating unit bounds		oil fraction variation				
13	Pmin	18	14	0	0			
14	Pmax	30	25	1	1			
15			P_genΣ	=	P_load			
16	power balance		50	=	50			

Figure 2.9 – Excel solution to problem (2.17) of optimal power and fuel dispatch of generating units



Solver Parameters

Set Objective: $F$5

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $B$8:$E$8

Subject to the Constraints:

$B$8:$E$8 <= $B$14:$E$14
$B$8:$E$8 >= $B$13:$E$13
$C$11 <= $E$11
$C$16 = $E$16

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 2.10 – Settings of **Solver parameters** window for problem (2.17)

Analysis of the obtained solution. As we can see from the solution, the optimal fuel proportions for the power units are the following.

Generating unit 1 burns only natural gas (α1=0, (1-α1) = 1) as the incremental fuel rate for gas consumption (0.1631) is lower than that for fuel oil consumption (0.23886).

Generating unit 2 burns proportion of 58.37% of fuel oil (α1=0.583661) and 41.63% of natural gas ((1-α1) = 0.416339) as the incremental fuel rates for gas consumption (0.226508) and that for fuel oil consumption (0.23974) are close, though d$F_{2\text{gas}}$/dP_2 is slightly lower than d$F_{2\text{oil}}$/dP_2, and if there were no gas consumption limitation of 10 f.u./hr, the unit 2 would burn only gas too.

The output of the 1st unit is maximum and higher than the output of the 2nd generator, which is within the limitation range, due to a lower incremental fuel rate.

2.2. Economic dispatch of all-thermal power system taking into account active power loss in the grids

2.2.1. Condition of economic dispatch of a thermal power system taking into account active power loss

A power system comprises *n* thermal power plants. The total fuel consumption in the system is the sum of fuel consumption at each thermal power plant

$$F_{\text{TPP}} = \sum_{i=1}^{n} F_{\text{TPP}i}(P_{\text{TPP}i}) = F_{\text{TPP1}}(P_{\text{TPP1}}) + F_{\text{TPP2}}(P_{\text{TPP2}}) + \ldots + F_{\text{TPP}n}(P_{\text{TPP}n}) . \qquad (2.18)$$

The objective of the economic dispatch problem for thermal power plants in a power system is the same as of the economic dispatch problem for generating units in one plant. However, the fuel costs may be different for the plants so they must be included into the objective function

$$Z = \sum_{i=1}^{n} [z_i F(P_{\text{TPP}})_{\text{TPP}i}] \rightarrow \min , \qquad (2.19)$$

where z_i is the fuel costs at the *i*-th TPP.

Besides, generation and consumption nodes of the power system are connected with electric grids in which active power generated is lost. Therefore, active power balance equation must include not only generated and consumed power but also active power loss:

$$P_{\text{Power\_sys}} = P_{\text{TPP1}} + P_{\text{TPP2}} + \ldots + P_{\text{TPP}n} = P_{\text{D}} + P_{\text{loss}}$$

or

$$P_{\text{D}} + P_{\text{loss}} - P_{\text{TPP1}} - P_{\text{TPP2}} - \ldots - P_{\text{TPP}n} = 0 . \qquad (2.20)$$

Thus, the mathematical model of the economic dispatch problem for thermal power plants at a power plant is

$$\begin{gathered}
Z = z_{\mathrm{TPP1}} \cdot F_{\mathrm{TPP1}}(P_{\mathrm{TPP1}}) + z_{\mathrm{TPP2}} \cdot F_{\mathrm{TPP2}}\ (P_{\mathrm{TPP2}}) + \ldots + \\
+ z_{\mathrm{TPP}n} \cdot F_{\mathrm{TPP}n}(P_{\mathrm{TPP}n}) \to \min \\
P_{\mathrm{D}} + P_{\mathrm{loss}} - P_{\mathrm{G1}} - P_{G2} - \ldots - P_{\mathrm{G}n} = 0 \\
P_{\mathrm{G}i} \geq 0, \quad i = 1,\ldots,n
\end{gathered} \quad . \tag{2.21}$$

The condition of optimal generating units dispatch can be found with the help of Lagrange multiplier technique.

Lagrange function *L* contains the objective function (2.5) and constraint (2.6) multiplied by a Lagrange multiplier:

$$\begin{gathered}
L = z_{\mathrm{TPP1}} \cdot F_{\mathrm{TPP1}}(P_{\mathrm{TPP1}}) + z_{\mathrm{TPP2}} \cdot F_{\mathrm{TPP2}}(P_{\mathrm{TPP2}}) + \ldots + \\
+ z_{\mathrm{TPP}n} \cdot F_{\mathrm{TPP}n}(P_{\mathrm{TPP}n}) + \lambda \cdot (P_{\mathrm{D}} + P_{\mathrm{loss}} - P_{\mathrm{G1}} - P_{\mathrm{G2}} - \ldots - P_{\mathrm{G}n}) \to \min
\end{gathered} \quad . \tag{2.22}$$

Minimum of Lagrange function (2.9) can be found by determining its first derivatives with respect to all the variables (generating units output $P_{\mathrm{G}i}$, $i = 1,..n$, and Lagrange multiplier λ) and setting them to 0.

$$\begin{gathered}
\frac{\partial L}{\partial P_{\mathrm{TPP1}}} = z_{\mathrm{TPP1}} \frac{dF_{\mathrm{TPP1}}}{dP_{\mathrm{TPP1}}} - \lambda(1 - \frac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP1}}}) = 0 \\
\frac{\partial L}{\partial P_{\mathrm{TPP2}}} = z_{\mathrm{TPP2}} \frac{dF_{\mathrm{TPP2}}}{dP_{\mathrm{TPP2}}} - \lambda(1 - \frac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP2}}}) = 0 \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\frac{\partial L}{\partial P_{\mathrm{TPP}n}} = z_{\mathrm{TPP}n} \frac{dF_{\mathrm{TPP}n}}{dP_{\mathrm{TPP}n}} - \lambda(1 - \frac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP}n}}) = 0 \\
\frac{\partial L}{\partial \lambda} = P_{\mathrm{D}} + P_{\mathrm{loss}} - P_{\mathrm{TPP1}} - P_{\mathrm{TPP2}} - \ldots - P_{\mathrm{TPP}n} = 0
\end{gathered} \quad . \tag{2.23}$$

After transposing λ in the first *n* equations of (2.23) to the right side

$$
\begin{aligned}
&z_{\mathrm{TPP1}}\frac{\dfrac{dF_{\mathrm{TPP1}}}{dP_{\mathrm{TPP1}}}}{1-\dfrac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP1}}}}=\lambda\\
&z_{\mathrm{TPP2}}\frac{\dfrac{dF_{\mathrm{TPP2}}}{dP_{\mathrm{TPP2}}}}{1-\dfrac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP2}}}}=\lambda\\
&\dots\dots\dots\dots\dots\\
&z_{\mathrm{TPP}n}\frac{\dfrac{dF_{\mathrm{TPP}n}}{dP_{\mathrm{TPP}n}}}{1-\dfrac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP}n}}}=\lambda
\end{aligned}
\quad , \tag{2.24}
$$

we obtain equalities:

$$
z_{\mathrm{TPP1}}\frac{\dfrac{dF_{\mathrm{TPP1}}}{dP_{\mathrm{TPP1}}}}{1-\dfrac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP1}}}}=z_{\mathrm{TPP2}}\frac{\dfrac{dF_{\mathrm{TPP2}}}{dP_{\mathrm{TPP2}}}}{1-\dfrac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP2}}}}=\ldots=z_{\mathrm{TPP}n}\frac{\dfrac{dF_{\mathrm{TPP}n}}{dP_{\mathrm{TPP}n}}}{1-\dfrac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP}n}}}=idem. \tag{2.25}
$$

Equalities (2.25) define the condition of optimal active power dispatch for thermal power plants at a power system.

The first derivative of input-output characteristic with respect to the power output $\frac{dF}{dP_{\mathrm{TPP}}}$ specifies the incremental fuel rate. The first derivative of power loss with respect to the power output $\frac{dP_{\mathrm{loss}}}{dP}$ specifies the incremental power loss rate.

Therefore, the Lagrange multiplier λ in this problem shows ratio of incremental fuel rate to one minus incremental power loss rate. For all power plants in a power system, this ratio multiplied by the fuel costs must be the same.

To reveal the physical meaning of the Lagrange multiplier, we consider (2.24) through finite differences

$$\lambda = z_{\text{TPP}} \frac{\dfrac{dF_{\text{TPP}}}{dP_{\text{TPP}}}}{1-\dfrac{dP_{\text{loss}}}{dP_{\text{TPP}}}} \cong z_{\text{TPP}} \frac{\dfrac{\Delta F_{\text{TPP}}}{\Delta P_{\text{TPP}}}}{1-\dfrac{\Delta P_{\text{loss}}}{\Delta P_{\text{TPP}}}} = z_{\text{TPP}} \frac{\Delta F_{\text{TPP}}}{\Delta P_{\text{TPP}} - \Delta P_{\text{loss}}} = z_{\text{TPP}} \frac{\Delta F_{\text{TPP}}}{\Delta P_{\text{D}}}. \quad (2.26)$$

Thus, we may say that the Lagrange multiplier λ shows increase in fuel costs with increase in the electricity demand by 1 MW.

2.2.2. Problem of economic dispatch for two thermal power plants taking into account active power loss in the grid

A power system comprises two thermal power plants serving one electrical load of 618 MW (fig. 2.11).

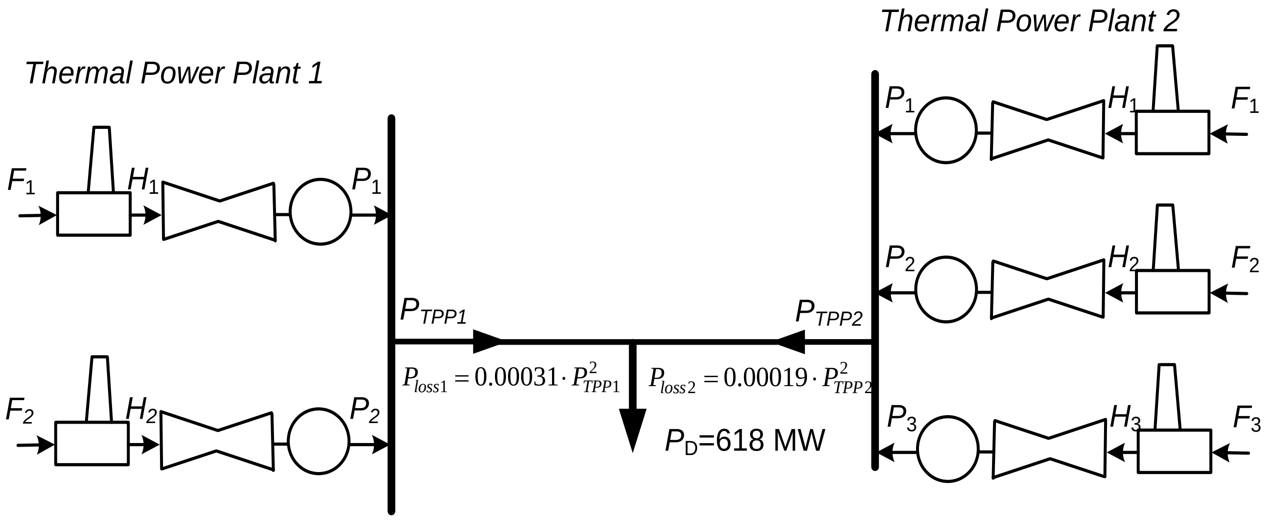


Figure 2.11 – A two-TPP power system with a connected electrical load

The heat consumption curves and power output bounds for the generating units of both plants are the following.

In TPP 1:

Heat rate of the 1st generating unit

$$H^{\mathrm{TPP1}}_{1}(P_{\mathrm{TPP1\_1}}) = 131.14 + 8.821P_{\mathrm{TPP1\_1}} + 0.1018(\mathrm{P}_{\mathrm{TPP1\_1}})^2 \text{ [GJ/h]};$$

Heat rate of the 2nd generating unit

$$H^{\mathrm{TPP1}}_{2}(P_{\mathrm{TPP1\_2}}) = 115.11 + 8.733P_{\mathrm{TPP1\_2}} + 0.0899(\mathrm{P}_{\mathrm{TPP1\_2}})^2 \text{ [GJ/h]}.$$

The heat value Q_{LHV1} of the fuel supplied to TPP1 is 24.87 GJ/t, the fuel cost z_{TPP1} is 57 $/t

Power output bounds are

$$P_{\mathrm{TPP1\_1min}} = 50 \text{ MW}, P_{\mathrm{TPP1\_1max}} = 146 \text{ MW};$$

$$P_{\mathrm{TPP1\_2min}} = 45 \text{ MW}, P_{\mathrm{TPP1\_2max}} = 150 \text{ MW}.$$

Power loss in the grid from TPP1 to the load node is $P_{\mathrm{loss1}} = 0.00031 \cdot P^2_{\mathrm{TPP1}}$. The TPP output being the sum of its units output, we can write

$$P_{\mathrm{loss1}} = 0.00031 \cdot (P^2_{\mathrm{TPP1\_1}} + P^2_{\mathrm{TPP1\_2}}).$$

In TPP 2:

Heat rate of the 1st generating unit

$$H^{\mathrm{TPP2}}_{1}(P_{\mathrm{TPP2\_1}}) = 119.5 + 8.718P_{\mathrm{TPP2\_1}} + 0.0987(\mathrm{P}_{\mathrm{TPP2\_1}})^2 \text{ [GJ/h]};$$

Heat rate of the 2nd generating unit

$$H^{\mathrm{TPP2}}_{2}(P_{\mathrm{TPP2\_2}}) = 114.01 + 8.697P_{\mathrm{TPP2\_2}} + 0.1044(\mathrm{P}_{\mathrm{TPP2\_2}})^2 \text{ [GJ/h]};$$

Heat rate of the 3rd generating unit

$$H^{\mathrm{TPP2}}_{3}(P_{\mathrm{TPP2\_3}}) = 118.35 + 9.03P_{\mathrm{TPP2\_3}} + 0.0931(\mathrm{P}_{\mathrm{TPP2\_3}})^2 \text{ [GJ/h]}.$$

The heat value Q_{LHV2} of the fuel supplied to TPP2 is 25.96 GJ/t, the fuel cost z_{TPP2} is 64$/t

Power output bounds are

$$P_{\mathrm{TPP2\_1min}} = 55 \text{ MW}, P_{\mathrm{TPP2\_1max}} = 155 \text{ MW};$$

$$P_{\text{TPP2\_2min}} = 60 \text{ MW}, P_{\text{TPP2\_2max}} = 148 \text{ MW/}$$

$$P_{\text{TPP2\_3min}} = 50 \text{ MW}, P_{\text{TPP2\_3max}} = 145 \text{ MW}.$$

Power loss in the grid from TPP2 to the load node is $P_{\text{loss2}} = 0.00019 \cdot P_{\text{TPP2}}^2$ or

$$P_{\text{loss2}} = 0.00019 \cdot (P_{\text{TPP2\_1}}^2 + P_{\text{TPP2\_2}}^2 + P_{\text{TPP2\_3}}^2)$$

It is required to find economical active power generation schedule for the dispatched demand of 618 MW.

The mathematical formulation of the problem is as follows.

The **variables** are power outputs of the units at TPP1 and TPP2:

$P_{\text{TPP1\_1}}$, $P_{\text{TPP1\_2}}$, $P_{\text{TPP2\_1}}$, $P_{\text{TPP2\_2}}$, $P_{\text{TPP2\_3}}$

The **objective** is minimization of the total generation costs in the power system of two thermal power plants:

$$\begin{aligned} Z = z_{\text{TPP1}} \frac{1}{Q_{\text{LHV1}}} [H_1^{\text{TPP1}}(P_{\text{TPP1\_1}}) + H_2^{\text{TPP1}}(P_{\text{TPP1\_2}})] + \\ + z_{\text{TPP2}} \frac{1}{Q_{\text{LHV2}}} [H_1^{\text{TPP2}}(P_{\text{TPP2\_1}}) + H_2^{\text{TPP2}}(P_{\text{TPP2\_2}}) + H_3^{\text{TPP2}}(P_{\text{TPP2\_3}})] \to \min \end{aligned} \quad (2.27)$$

The **constraints** are

– active power balance in the system

$$P_{\text{TPP1\_1}} + P_{\text{TPP1\_2}} + P_{\text{TPP2\_1}} + P_{\text{TPP2\_2}} + P_{\text{TPP2\_3}} = P_{\text{D}} + P_{\text{loss}\Sigma}$$

$P_{loss\Sigma}$ includes power loss in the transmission grid from TPP1, P_{loss1}, and power loss in the transmission grid from TPP2, P_{loss2}.

Substituting the numerical parameters, we obtain the active power balance equation for the considered power system

$$\begin{aligned} &P_{\text{TPP1\_1}} + P_{\text{TPP1\_2}} + P_{\text{TPP2\_1}} + P_{\text{TPP2\_2}} + P_{\text{TPP2\_3}} = \\ &= 618 + 0.00031 \cdot P_{\text{TPP1}}^2 + 0.00019 \cdot P_{\text{TPP2}}^2 = \\ &= 618 + 0.00031 \cdot (P_{\text{TPP1\_1}}^2 + P_{\text{TPP1\_2}}^2) + \\ &+ 0.00019 \cdot (P_{\text{TPP2\_1}}^2 + P_{\text{TPP2\_2}}^2 + P_{\text{TPP2\_3}}^2) \end{aligned} \tag{2.28}$$

– power output bounds are

$$\begin{aligned} &50 \le P_{\text{TPP1\_1}} \le 146 \qquad && 55 \le P_{\text{TPP2\_1}} \le 155 \\ &45 \le P_{\text{TPP1\_2}} \le 150 \qquad && 60 \le P_{\text{TPP2\_2}} \le 148; \\ & && 50 \le P_{\text{TPP2\_3}} \le 145 \end{aligned} \tag{2.29}$$

– the condition of optimal dispatch (1.26):

$$z_{\text{TPP1}} \frac{\dfrac{dF_{\text{TPP1}}}{dP_{\text{TPP1}}}}{1 - \dfrac{dP_{\text{loss}}}{dP_{\text{TPP1}}}} = z_{\text{TPP2}} \frac{\dfrac{dF_{\text{TPP2}}}{dP_{\text{TPP2}}}}{1 - \dfrac{dP_{\text{loss}}}{dP_{\text{TPP2}}}};$$

or, taking into account that $F(P) = \dfrac{H(P)}{Q_{LHV}}$,

$$z_{\text{TPP1}} \frac{1}{Q_{\text{LHV1}}} \cdot \frac{\dfrac{dH_1^{\text{TPP1}}}{dP_{\text{TPP1\_1}}}}{1 - 0.00031 \dfrac{dP_{\text{loss1}}}{dP_{\text{TPP1\_1}}}} = z_{\text{TPP1}} \frac{1}{Q_{\text{LHV1}}} \cdot \frac{\dfrac{dH_2^{\text{TPP1}}}{dP_{\text{TPP1\_2}}}}{1 - 0.00031 \dfrac{dP_{\text{loss1}}}{dP_{\text{TPP1\_2}}}} =$$

$$= z_{\text{TPP2}} \frac{1}{Q_{\text{LHV2}}} \cdot \frac{\dfrac{dH_1^{\text{TPP2}}}{dP_{\text{TPP2\_1}}}}{1 - 0.00019 \dfrac{dP_{\text{loss2}}}{dP_{\text{TPP2\_1}}}} = z_{\text{TPP2}} \frac{1}{Q_{\text{LHV2}}} \cdot \frac{\dfrac{dH_2^{\text{TPP2}}}{dP_{\text{TPP2\_2}}}}{1 - 0.00019 \dfrac{dP_{\text{loss2}}}{dP_{\text{TPP2\_2}}}} =$$

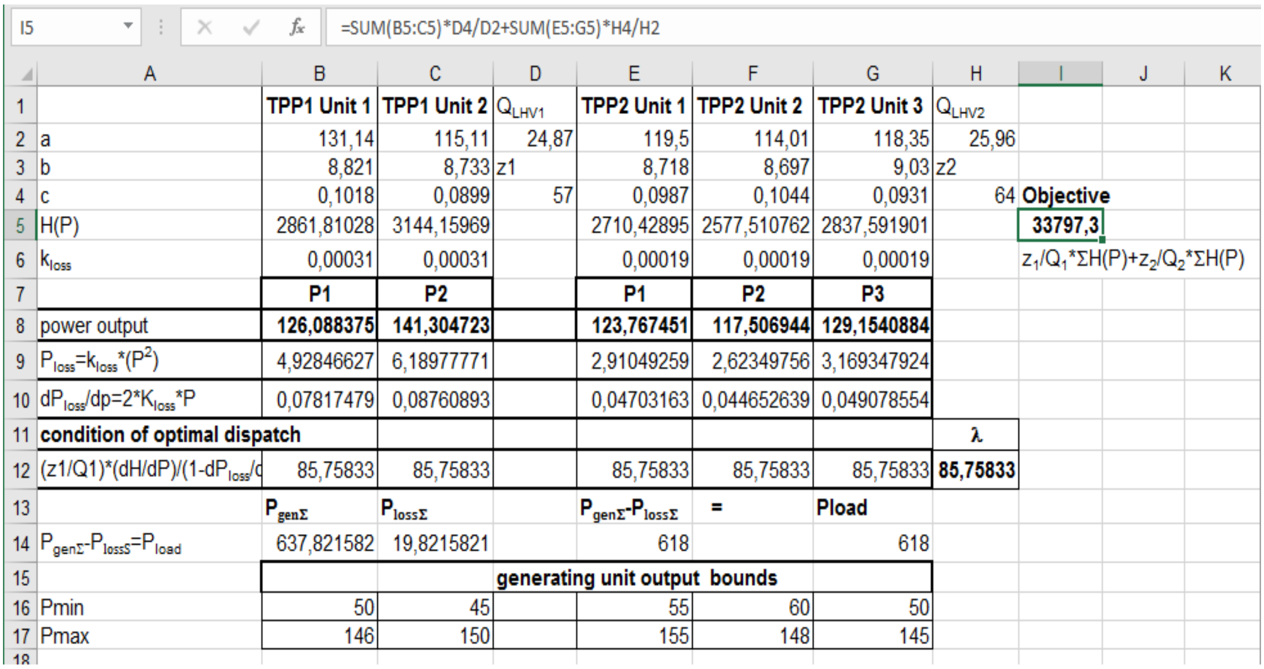
$$= z_{\text{TPP2}} \frac{1}{Q_{\text{LHV2}}} \cdot \frac{\dfrac{dH_3^{\text{TPP2}}}{dP_{\text{TPP2\_3}}}}{1 - 0.00019 \dfrac{dP_{\text{loss2}}}{dP_{\text{TPP2\_3}}}} \quad .$$

Substituting the initial data, we obtain the conditions of the optimal dispatch for the considered power system of two thermal power plants:

$$
\begin{aligned}
&57\frac{1}{24.87}\frac{8.821+2\cdot 0.1018\cdot P_{\mathrm{TPP1\_1}}}{1-2\cdot 0.00031P_{\mathrm{TPP1\_1}}}=57\frac{1}{24.87}\frac{8.733+2\cdot 0.0899\cdot P_{\mathrm{TPP1\_2}}}{1-2\cdot 0.00031P_{\mathrm{TPP1\_2}}}=\\
&=64\frac{1}{25.96}\frac{8.718+2\cdot 0.0987\cdot P_{\mathrm{TPP2\_1}}}{1-2\cdot 0.00019P_{\mathrm{TPP2\_1}}}=64\frac{1}{25.96}\frac{8.697+2\cdot 0.1044\cdot P_{\mathrm{TPP2\_2}}}{1-2\cdot 0.00019P_{\mathrm{TPP2\_2}}}=. \qquad (2.30)\\
&=64\frac{1}{25.96}\frac{9.03+2\cdot 0.0931\cdot P_{\mathrm{TPP2\_3}}}{1-2\cdot 0.00019P_{\mathrm{TPP2\_3}}}
\end{aligned}
$$

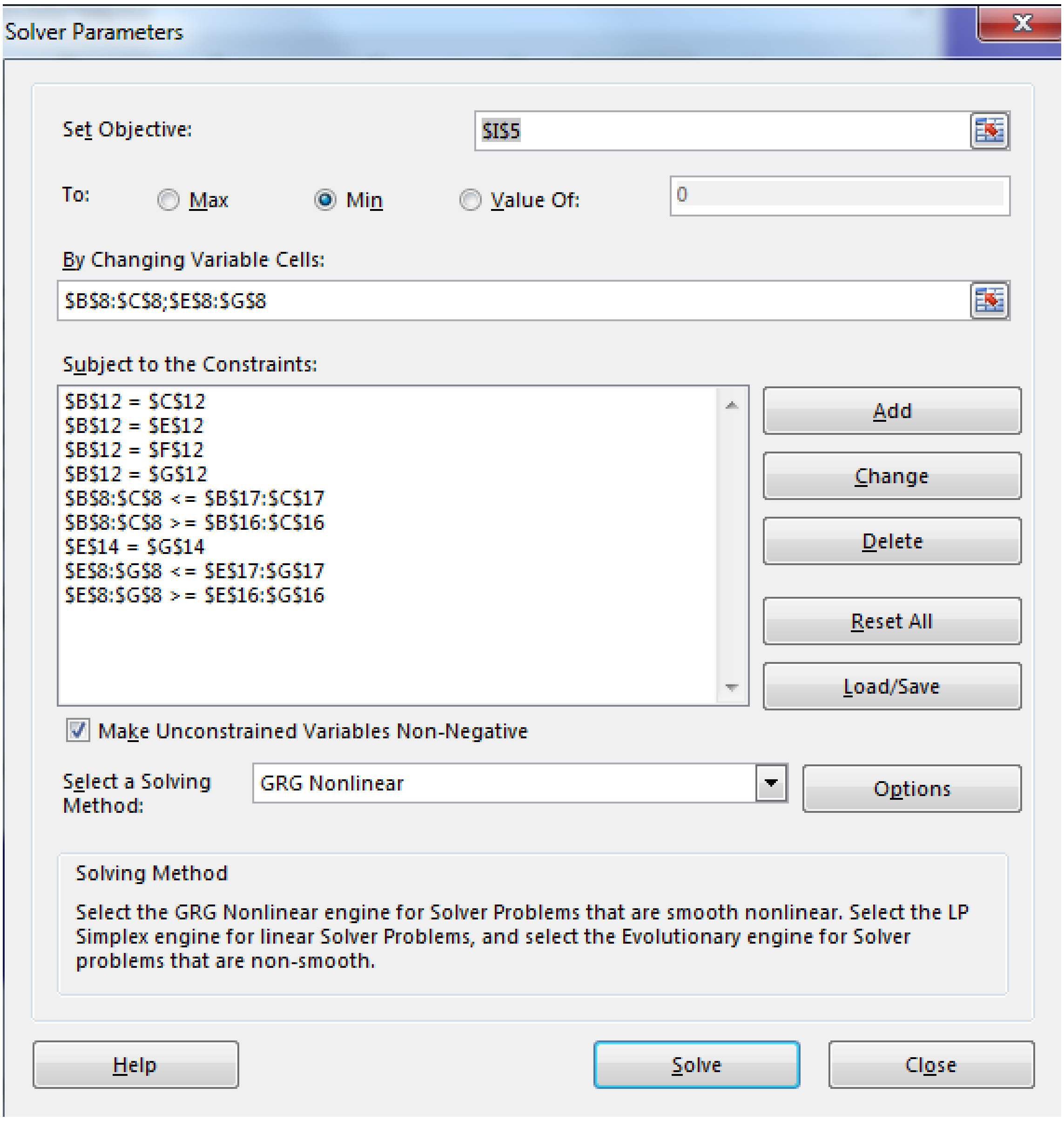
Formulae (2.27) – (2.30) describe the mathematical model of the considered economic dispatch problem.

The Excel solution to the problem is given in fig. 2.12. **Solver** parameters are shown in fig. 2.13.



	A	B	C	D	E	F	G	H	I	J	K
1		**TPP1 Unit 1**	**TPP1 Unit 2**	Q_{LHV1}	**TPP2 Unit 1**	**TPP2 Unit 2**	**TPP2 Unit 3**	Q_{LHV2}			
2	a	131,14	115,11	24,87	119,5	114,01	118,35	25,96			
3	b	8,821	8,733	z1	8,718	8,697	9,03	z2			
4	c	0,1018	0,0899	57	0,0987	0,1044	0,0931	64	**Objective**		
5	H(P)	2861,81028	3144,15969		2710,42895	2577,510762	2837,591901		**33797,3**		
6	k_{loss}	0,00031	0,00031		0,00019	0,00019	0,00019		$z_1/Q_1*\Sigma H(P)+z_2/Q_2*\Sigma H(P)$		
7		**P1**	**P2**		**P1**	**P2**	**P3**				
8	power output	**126,088375**	**141,304723**		**123,767451**	**117,506944**	**129,1540884**				
9	$P_{loss}=k_{loss}*(P^2)$	4,92846627	6,18977771		2,91049259	2,62349756	3,169347924				
10	$dP_{loss}/dp=2*K_{loss}*P$	0,07817479	0,08760893		0,04703163	0,044652639	0,049078554				
11	**condition of optimal dispatch**							**λ**			
12	(z1/Q1)*(dH/dP)/(1-dP$_{loss}$/d	85,75833	85,75833		85,75833	85,75833	85,75833	**85,75833**			
13		**$P_{gen\Sigma}$**	**$P_{loss\Sigma}$**		**$P_{gen\Sigma}-P_{loss\Sigma}$**	**=**	**Pload**				
14	$P_{gen\Sigma}-P_{lossS}=P_{load}$	637,821582	19,8215821		618		618				
15				**generating unit output bounds**							
16	Pmin	50	45		55	60	50				
17	Pmax	146	150		155	148	145				

Figure 2.12 – Excel solution to problem (2.27) – (2.30) of economic dispatch of two thermal power plants serving one load



```
Set Objective: $I$5

To: ○ Max   ● Min   ○ Value Of: 0

By Changing Variable Cells:
$B$8:$C$8;$E$8:$G$8

Subject to the Constraints:
$B$12 = $C$12
$B$12 = $E$12
$B$12 = $F$12
$B$12 = $G$12
$B$8:$C$8 <= $B$17:$C$17
$B$8:$C$8 >= $B$16:$C$16
$E$14 = $G$14
$E$8:$G$8 <= $E$17:$G$17
$E$8:$G$8 >= $E$16:$G$16

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear
```

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 2.13 – Settings of **Solver parameters** window for economic dispatch problem (2.27) – (2.30)

Analysis of the obtained solution. As we can see, the optimal incremental fuel cost rate is 85.75833, that is the fuel cost in the power system increases by 85.75833 $ when the electricity demand increases by 1 MW.

As to the optimal active power dispatching, the outputs of unit 1 at TPP1 and all the units at TPP2 are quite close. This means that operational characteristics of these units are approximately equal. The output of TPP1 unit 2 is higher, which indicates that unit 2 is more economical.

2.3. Short-range economic dispatch of a hydrothermal power system

2.3.1. Condition of economic dispatch of a hydrothermal system

Economic dispatch of a hydrothermal system is typically determined for time periods longer than an hour due to specific requirements for water reservoir conditions at each hydroplant. The rated water reservoir storage capacity, the water inflow and outflow, available amount of water in the reservoir, water reservoir utilization for other than power generation purposes, and many other factors must be taken into account when scheduling hydrogenation. The state of water reservoir depending in the season of year and weather conditions, there are two types of scheduling problems: short-range and long-range dispatch.

Short-range dispatch (from one day to one week) of a hydrothermal power system considers hourly-by-hourly scheduling of all generators in the systems with the objective to minimize fuel consumption by thermal plants. The hydroplants scheduling is determined by available hydro resources and allowed water release schedules: for scheduling period T, each hydroplant can use certain amount of water W.

Let us consider a hydrothermal power system comprising a thermal power plant (TPP) and three hydroplants (HPPs). The head at each hydroplant is assumed constant during scheduling period T. For each hydroplant, allowed water release W_i is scheduled.

The objective is to select the operating power units and load distribution among the power plants taking into account constraints imposed on the system and plant operation. The economic scheduling is determined by minimum fuel consumption during the considered scheduling period T. The time interval $\Delta\tau_t$ is an hour, $\Delta\tau_t = 1$. The scheduling period covers T hours.

The objective function is minimization of the overall fuel consumption during the considered scheduling period T:

$$Z = \sum_{t=1}^{T} \left[F_t(P_t) \cdot \Delta\tau_t \right] \to \min . \qquad (2.31)$$

where $\sum_{t=1}^{T}\left[F_t(P_{\text{TPP}t})\cdot\Delta\tau_t\right]$ is the overall fuel consumption at the thermal power plant during the entire period *T*, fuel consumption at each time interval *t* determined with (2.1), *t* being number of time interval $\Delta\tau$, $t = 1, 2, \ldots, T$, $\sum_{t=1}^{T}\Delta\tau_t = T$.

The constraints in this problem are

– power balance in the power system at each time interval *t*

$$P_{\text{Bal}t} = P_{\text{G}\Sigma t} - P_{\text{D}t} - P_{\text{loss}t} = 0; \tag{2.32}$$

where $P_{\Gamma t}$ is the overall generation at each time interval *t*, $P_{\text{G}\Sigma t} = P_{\text{TPP}t} + P_{\text{HPP1}t} + P_{\text{HPP2}t} + P_{\text{HPP3}t}$;

– water release for hydroplants: each hydroplant is allowed to use a certain amount W_{Qi} of the water resource for the entire scheduling period *T*

$$\begin{aligned} W_{\text{HPP1}} &= W_{Q1} - \sum_{t=1}^{T}\left[Q_{1t}\cdot\Delta\tau_t\right] = 0 \\ W_{\text{HPP2}} &= W_{Q2} - \sum_{t=1}^{T}\left[Q_{2t}\cdot\Delta\tau_t\right] = 0. \\ W_{\text{HPP3}} &= W_{Q3} - \sum_{t=1}^{T}\left[Q_{3t}\cdot\Delta\tau_t\right] = 0 \end{aligned} \tag{2.33}$$

where Q_{it} is the water release by HPP$_i$ at each time period $\Delta\tau_t$, i=1,2,3.

To obtain the condition of optimal scheduling of the hydrothermal power system, we compose Lagrange function that contains the objective function (2.31) and constraints (2.32) and (2.33) each multiplied by a Lagrange multiplier:

$$\begin{gathered} L = \sum_{t=1}^{T}\left[F_t(P_{\text{TPP}t})\cdot\Delta\tau_t\right] + \sum_{t=1}^{T}\left[\lambda_{\text{Bal}t}\cdot(P_{\text{G}\Sigma t} - P_{\text{D}t} - P_{\text{loss}t})\right] + \lambda_{\text{HPP1}}\cdot\left(W_{Q1} - \sum_{t=1}^{T}\left[Q_{1t}\cdot\Delta\tau_t\right]\right) + \\ + \lambda_{\text{HPP2}}\cdot\left(W_{Q2} - \sum_{t=1}^{T}\left[Q_{2t}\cdot\Delta\tau_t\right]\right) + \lambda_{\text{HPP3}}\cdot\left(W_{Q3} - \sum_{t=1}^{T}\left[Q_{3t}\cdot\Delta\tau_t\right]\right) \to \min \end{gathered}. \tag{2.34}$$

The minimum of Lagrange function at each time period *t* is found under condition of zero derivatives of Lagrange function over all its variables:

– zero derivatives over power plants output

$$-\quad\begin{aligned}
&\frac{\partial L}{\partial P_{\mathrm{TPP}t}}=\frac{dF_t}{dP_{\mathrm{TPP}t}}+\lambda_{\mathrm{Bal}t}\left(1-\frac{dP_{\mathrm{loss}t}}{dP_{\mathrm{TPP}t}}\right)=0\\
&\frac{\partial L}{\partial P_{\mathrm{HPP1}t}}=\lambda_{\mathrm{Bal}t}\left(1-\frac{dP_{\mathrm{loss}t}}{dP_{\mathrm{HPP1}t}}\right)+\lambda_{\mathrm{HPP1}t}\frac{dQ_{\mathrm{HPP1}t}}{dP_{\mathrm{HPP1}t}}=0\\
&\frac{\partial L}{\partial P_{\mathrm{HPP2}t}}=\lambda_{\mathrm{Bal}t}\left(1-\frac{dP_{\mathrm{loss}t}}{dP_{\mathrm{HPP2}t}}\right)+\lambda_{\mathrm{HPP2}t}\frac{dQ_{\mathrm{HPP2}t}}{dP_{\mathrm{HPP2}t}}=0\\
&\frac{\partial L}{\partial P_{\mathrm{HPP3}t}}=\lambda_{\mathrm{Bal}t}\left(1-\frac{dP_{\mathrm{loss}t}}{dP_{\mathrm{HPP3}t}}\right)+\lambda_{\mathrm{HPP3}t}\frac{dQ_{\mathrm{HPP3}t}}{dP_{\mathrm{HPP3}t}}=0
\end{aligned}\ . \tag{2.35}$$

– zero derivatives over Lagrange multipliers

$$\begin{aligned}
&\frac{\partial L}{\partial \lambda_{\mathrm{Bal}t}}=P_{\mathrm{G}\Sigma t}-P_{\mathrm{D}t}-P_{\mathrm{loss}t}=0\\
&\frac{\partial L}{\partial \lambda_{\mathrm{HPP1}}}=W_{Q1}-\sum_{t=1}^{T}\left[Q_{1t}\cdot\Delta\tau_t\right]=0\\
&\frac{\partial L}{\partial \lambda_{\mathrm{HPP2}}}=W_{Q2}-\sum_{t=1}^{T}\left[Q_{2t}\cdot\Delta\tau_t\right]=0\\
&\frac{\partial L}{\partial \lambda_{\mathrm{HPP3}}}=W_{Q3}-\sum_{t=1}^{T}\left[Q_{3t}\cdot\Delta\tau_t\right]=0
\end{aligned}\ . \tag{2.36}$$

Let us denote

$\beta=\frac{dF}{dP_{\mathrm{TPP}}}$ is incremental fuel rate at the thermal power plant;

$\sigma_{\mathrm{TPP}}=\frac{dP_{\mathrm{loss}}}{dP_{\mathrm{TPP}}}$ is incremental active power loss with change in TPP output;

$\sigma_{\mathrm{HPP}}=\frac{dP_{\mathrm{loss}}}{dP_{\mathrm{HPP}}}$ is incremental active power loss with change in HPP output;;

$q=\frac{dQ}{dP_{\mathrm{HPP}}}$ – is incremental water flow rate through the hydroturbine at the hydropower plant.

Then, derivatives (2.35) can be written as

$$\begin{aligned}
&\frac{\partial L}{\partial P_{\text{TPP}t}} = \beta_t + \lambda_{\text{Bal}t}(1-\sigma_{\text{TPP}t}) = 0\\
&\frac{\partial L}{\partial P_{\text{HPP1}t}} = \lambda_{\text{Bal}t}(1-\sigma_{\text{HPP1}t}) + \lambda_{\text{HPP1}t}q_{\text{HPP1}t} = 0\\
&\frac{\partial L}{\partial P_{\text{HPP2}t}} = \lambda_{\text{Bal}t}(1-\sigma_{\text{HPP2}t}) + \lambda_{\text{HPP2}t}q_{\text{HPP2}t} = 0\\
&\frac{\partial L}{\partial P_{\text{HPP3}t}} = \lambda_{\text{Bal}t}(1-\sigma_{\text{HPP3}t}) + \lambda_{\text{HPP3}t}q_{\text{HPP3}t} = 0
\end{aligned}. \qquad (2.37)$$

Derivatives (2.37) can be rewritten to extract $\lambda_{\text{Бал}t}$:

$$\begin{aligned}
&-\lambda_{\text{Bal}t} = \frac{\beta_t}{(1-\sigma_{\text{TPP}t})}\\
&-\lambda_{\text{Bal}t} = \lambda_{\text{HPP1}t}\frac{q_{\text{HPP1}t}}{(1-\sigma_{\text{HPP1}t})}\\
&-\lambda_{\text{Bal}t} = \lambda_{\text{HPP2}t}\frac{q_{\text{HPP2}t}}{(1-\sigma_{\text{HPP2}t})}\\
&-\lambda_{\text{Bal}t} = \lambda_{\text{HPP3}t}\frac{q_{\text{HPP3}t}}{(1-\sigma_{\text{HPP3}t})}
\end{aligned}. \qquad (2.38)$$

The left sides in (2.38) being the same, the right sides are identical

$$\frac{\beta_t}{(1-\sigma_{\text{TPP}t})} = \lambda_{\text{HPP1}t}\frac{q_{\text{HPP1}t}}{(1-\sigma_{\text{HPP1}t})} = \lambda_{\text{HPP2}t}\frac{q_{\text{HPP}t}}{(1-\sigma_{\text{HPP2}t})} = \lambda_{\text{HPP3}t}\frac{q_{\text{HPP3}t}}{(1-\sigma_{\text{HPP3}t})} = idem. \qquad (2.39)$$

Equality (2.39) specifies **the condition of economic dispatch of a hydro-thermal system** at every time period *t*.

All the parameters of (2.39) are function of power characteristics of the power facilities: incremental fuel β and water flow rates $q_{\text{HPP}i}$ and electric parameters of electric grids: incremental power loss σ_i. Therefore, condition (2.39) is true for any time period and subscript index *t* can be omitted

$$\frac{\beta}{(1-\sigma_{\text{TPP}})} = \lambda_{\text{HPP1}}\frac{q_{\text{HPP1}}}{(1-\sigma_{\text{HPP1}})} = \lambda_{\text{HPP2}}\frac{q_{\text{HPP}}}{(1-\sigma_{\text{HPP2}})} = \lambda_{\text{HPP3}}\frac{q_{\text{HPP3}}}{(1-\sigma_{\text{HPP3}})} = idem. \qquad (2.40)$$

According to (2.40), for economic dispatch of a hydrothermal power system, there must be constant relationship λ_{HPP} between the thermal and hydroplants at any time period (I hour). For example, the load must be dispatched between TPP and HPP1 in compliance with the following relation

$$\lambda_{\text{HPP1}} = \frac{\beta}{(1-\sigma_{\text{HPP}})} \cdot \frac{(1-\sigma_{\text{HPP1}})}{q_{\text{HPP1}}}.$$

To understand the physical meaning of λ_{HPP}, let us consider the situation of TPP and HPP located close to each other with zero power loss in the grid. The condition (2.40) reduces to $\beta = \lambda_{\text{HPP}} q$ or. $\frac{dF}{dP_{\text{TPP}}} = \lambda_{\text{HPP}} \frac{dQ}{dP_{\text{HPP}}}$. In the difference form it is

$$\frac{\Delta F}{\Delta P_{\text{TPP}}} = \lambda_{\text{HPP}} \frac{\Delta Q}{\Delta P_{\text{HPP}}} \text{ю}$$

In case the power increments at the TPP and the HTT are identical, $\Delta P_{\text{TPP}} = \Delta P_{\text{HPP}}$, we obtain

$$\lambda_{\text{TPP}} = \frac{\Delta F}{\Delta P} \cdot \left(\frac{\Delta Q}{\Delta P} \right)^{-1} = \frac{\Delta F}{\Delta Q}. \tag{2.41}$$

Expression (2.41) describes the relationship between the flow rate at HPP and fuel rate at TPP. Therefore, λ_{HPP} represents efficiency of water resource utilization in the hydrothermal power system: this multiplier shows how much fuel can be saved at the TPP if water discharge at the HPP increases.

Thus, multipliers $\lambda_{\text{HPP}i}$ link TPP operation mode and the corresponding HPP operation. Hydroplants differ in heads and flow rate, consequently, $\lambda_{\text{ГЕС}i}$ differs too.

From (2.41), it follows that the optimal scheduling will be in the situation when water resource of every HPP is used with the same efficiency throughout the scheduling period T in case the head remains constant, that is $\lambda_{\text{HPP}t} = idem$ during the entire optimization period.

2.3.2 Problem of economic dispatch for TPP and HPP under limited water discharge during scheduling period T

It is required to schedule load between a 1500 MW thermal power plant and a 1100 MW hydropower plant under limited daily volume of water discharged equal to 123,350 thous.m^3 [1].

The load is connected to the TPP bus with zero power loss in the line and has the following schedule:

(I) $01^{00} – 12^{00}$ – 1200 MW.

(II) $13^{00} – 24^{00}$ – 1500 MW.

The distance between the load and the HPP is long, so there is power loss in the transmission line from the HPP calculated as, MW,

$$P_{\text{loss}} = 0.00008 \cdot P_{\text{HPP}}^2 .$$

The TPP fuel rate is

$F(P_{\text{TPP}})$=14,13 + 0,23P_{TPP} +0,00045P_{TPP}^2, [t/hr].

The output bounds: 150 MW $\leq P_{\text{TPP}} \leq$1500 MW.

The fuel cost is c_{fuel} = 89 \$/t.

The HPP flow rate is different for different output ranges:

$Q(P_{\text{HPP}})_{(1)}$=407,055 + 6,13P_{HPP} , [thous.m^3/hr]

for output bounds 0 $\leq P_{\text{HPP}} \leq$1000 MW;

$Q(P_{\text{HPP}})_{(2)}$=6537,5 + 14,8(P_{HPP} -1000)+0,0617(P_{HPP} -1000)^2, [thous.m^3/hr]

for output bounds 1000 $< P_{\text{HPP}} \leq$1100 MW.

The inflow into the water reservoir is supposed zero.

The **mathematical model** of the problem is the following.

The **decision variable** are two variables for the TPP output in every scheduling period, P_{TPP}(I) and P_{TPP}(II), and four variables for the HPP output, two for each scheduling period: $P(I)_{\text{HPP(1)}}$, $P(I)_{\text{HPP(2)}}$, $P(II)_{\text{HPP(1)}}$, $P(II)_{\text{HPP(2)}}$, as the water discharge depends on the output bounds.

Besides, the HPP having two power characteristics versus the power output, the water discharge is function of the HPP generation and Boolean variables should be added to the decision variables to show which of the flow rates is taken into account in the first and second half-periods.

Let α stand for the P_{HPP} <=1000 MW, and (1- α) – for P_{HPP} >1000 MW.

The **objective** is fuel consumption minimization during the entire period T

$$Z = c_{\text{fuel}} \cdot \left(\sum_{i=1}^{12} \left[F(P(\text{I})_{\text{TPP}i}) \right] + \sum_{i=1}^{12} \left[F(P(\text{II})_{\text{TPP}i}) \right] \right) \to \min$$

Constraints are the following.

– Active power balance at every time period i (1 hour):

$$P_{\text{Bal}i} = P_{\text{HPP}i} + P_{\text{TPP}i} - P_{\text{D}i} - P_{\text{loss}i} = 0$$

or taking into account the selection of P_{HPP}: $P_{\text{HPP}} = \alpha \cdot P_{\text{HPP(1)}} + (1-\alpha) \cdot P_{\text{HPP(2)}}$,

$$P_{\text{Bal}i} = \alpha \cdot P_{\text{HPP(1)}i} + (1-\alpha) \cdot P_{\text{HPP(2)}i} + P_{\text{TPP}i} - P_{\text{D}i} - P_{\text{loss}i} = 0$$

– The overall water discharge within the scheduling period T:

$$Q_{\Sigma} - \sum_{i=1}^{T} \left[Q_{\text{HPP}i} \cdot \Delta t_i \right] = 0, \quad \Delta t_i = 1 \text{ hour} \;, \; \sum_{i=1}^{T} \left[\Delta t_i \right] = T$$

At the beginning of the scheduling period $Q_{\text{start}} = Q_{\text{max}}$; at the end of the scheduling period $Q_{\text{end}} = Q_{\text{min}}$.

Epy water discharge at every i hour is $Q_{\text{HPP}i} = Q_i$;

$$Q_{\min} \le Q_i \le Q_{\max}$$

The entire scheduling period T = 24 hours is divided into two 12-hour half-periods within which the load is constant and, therefore, the generation is constant too, which allows presenting the daily water discharge Q_{Σ} as sum of water discharge in the first 12 hours with constant flow rate $Q(\text{I})_{\text{HPP}}$ = const1 (as $P(\text{I})_{\text{HPP}}$ = const) and water discharge in the other 12 hours with another constant flow rate

$Q(\text{II})_{\text{HPP}}$ = const2 (as $P(\text{II})_{\text{HPP}}$=const).

Then, the equation for water discharge balance can be written as follows

$$Q_{\Sigma} - Q(\text{I})_{\Sigma} - Q(\text{II})_{\Sigma} = 0,$$

where

$$Q(\text{I})_{\Sigma} = 12 \cdot \left\lfloor \alpha(\text{I}) \cdot Q(\text{I})_{(1)} + \left(1 - \alpha(\text{I})\right) Q(\text{I})_{(2)} \right\rfloor$$

and

$$Q(\text{II})_{\Sigma} = 12 \cdot \left\lfloor \alpha(\text{II}) \cdot Q(\text{II})_{(1)} + \left(1 - \alpha(\text{II})\right) Q(\text{II})_{(2)} \right\rfloor,$$

so the equation for water discharge balance can be rewritten as

$$Q_{\Sigma} - 12 \cdot \left\lfloor \alpha(\text{I}) \cdot Q(\text{I})_{(1)} + \left(1 - \alpha(\text{I})\right) Q(\text{I})_{(2)} \right\rfloor - 12 \cdot \left\lfloor \alpha(\text{II}) \cdot Q(\text{II})_{(1)} + \left(1 - \alpha(\text{II})\right) Q(\text{II})_{(2)} \right\rfloor = 0$$

– The condition of economic dispatch: the same Lagrange multiplier λ_{HPP}at every hour within the scheduling period

$$\lambda_{\text{HPP}i} = \frac{\beta_i}{(1 - \sigma_{\text{TPP}i})} \cdot \frac{(1 - \sigma_{\text{HPP}i})}{q_{\text{HPP}i}} = \frac{\dfrac{dF}{dP_{\text{TPP}i}}}{1 - \dfrac{dP_{\text{loss}i}}{dP_{\text{TPP}i}}} \cdot \frac{1 - \dfrac{dP_{\text{loss}i}}{dP_{\text{HPP}i}}}{\dfrac{dQ}{dP_{\text{HPP}i}}} = idem$$

or

$$\frac{\dfrac{dF}{dP_{\text{TPP(I)}}}}{1 - \dfrac{dP_{\text{loss(I)}}}{dP_{\text{TPP(I)}}}} \cdot \frac{1 - \dfrac{dP_{\text{loss(I)}}}{dP_{\text{HPP(I)}}}}{\dfrac{dQ}{dP_{\text{HPP(I)}}}} = \frac{\dfrac{dF}{dP_{\text{TPP(II)}}}}{1 - \dfrac{dP_{\text{loss(II)}}}{dP_{\text{TPP(II)}}}} \cdot \frac{1 - \dfrac{dP_{\text{loss(II)}}}{dP_{\text{HPP(II)}}}}{\dfrac{dQ}{dP_{\text{HPP(II)}}}}$$

When calculating the Lagrange multiplier λ_{HPP}, the HPP flow rate $Q(P_{\text{HPP}})$ must be chosen according to the hydroplant output: $Q(P) = \alpha \cdot Q_{\text{HPP}(1)} + (1 - \alpha) \cdot Q_{\text{HPP}(2)}$.

After substituting the initial data, the mathematical model of the problem with accounting for all the constraints is the following.

$$Z = c_{\text{fuel}} \cdot \sum_{i=1}^{12} \left[F_i(P_{\text{TPP(I)}i}) \right] + c_{\text{fuel}} \cdot \sum_{i=1}^{12} \left[F_i(P_{\text{TPP(II)}i}) \right] \to \min$$

$$1200 + 0{,}00008 \cdot \left[\alpha(\text{I}) \cdot P(\text{I})_{\text{HPP(1)}} + (1 - \alpha(\text{I})) \cdot P(\text{I})_{\text{HPP(2)}i} \right] -$$
$$- \alpha(\text{I}) \cdot P(\text{I})_{\text{HPP(1)}i} - (1 - \alpha(\text{I})) \cdot P(\text{I})_{\text{HPP(2)}i} - P(\text{I})_{\text{TPP}i} = 0$$

$$1500 + 0{,}00008 \cdot \left[\alpha(\text{II}) \cdot P(\text{II})_{\text{HPP(1)}i} + (1 - \alpha(\text{II})) \cdot P(\text{II})_{\text{HPP(2)}i} \right] -$$
$$- \alpha(\text{II}) \cdot P(\text{II})_{\text{HPP(1)}i} + (1 - \alpha(\text{II})) \cdot P(\text{II})_{\text{HPP(2)}i} + P(\text{II})_{\text{TPP}i} = 0$$

$$123350 - 12 \cdot \left[\alpha(\text{I}) \cdot Q(\text{I})_{(1)} + \left(1 - \alpha(\text{I})\right) Q(\text{I})_{(2)} \right] = Q_{\Sigma}(\text{II})$$

$$Q_{\Sigma}(\text{II}) - 12 \cdot \left[\alpha(\text{II}) \cdot Q(\text{II})_{(1)} + \left(1 - \alpha(\text{II})\right) Q(\text{II})_{(2)} \right] = 0$$

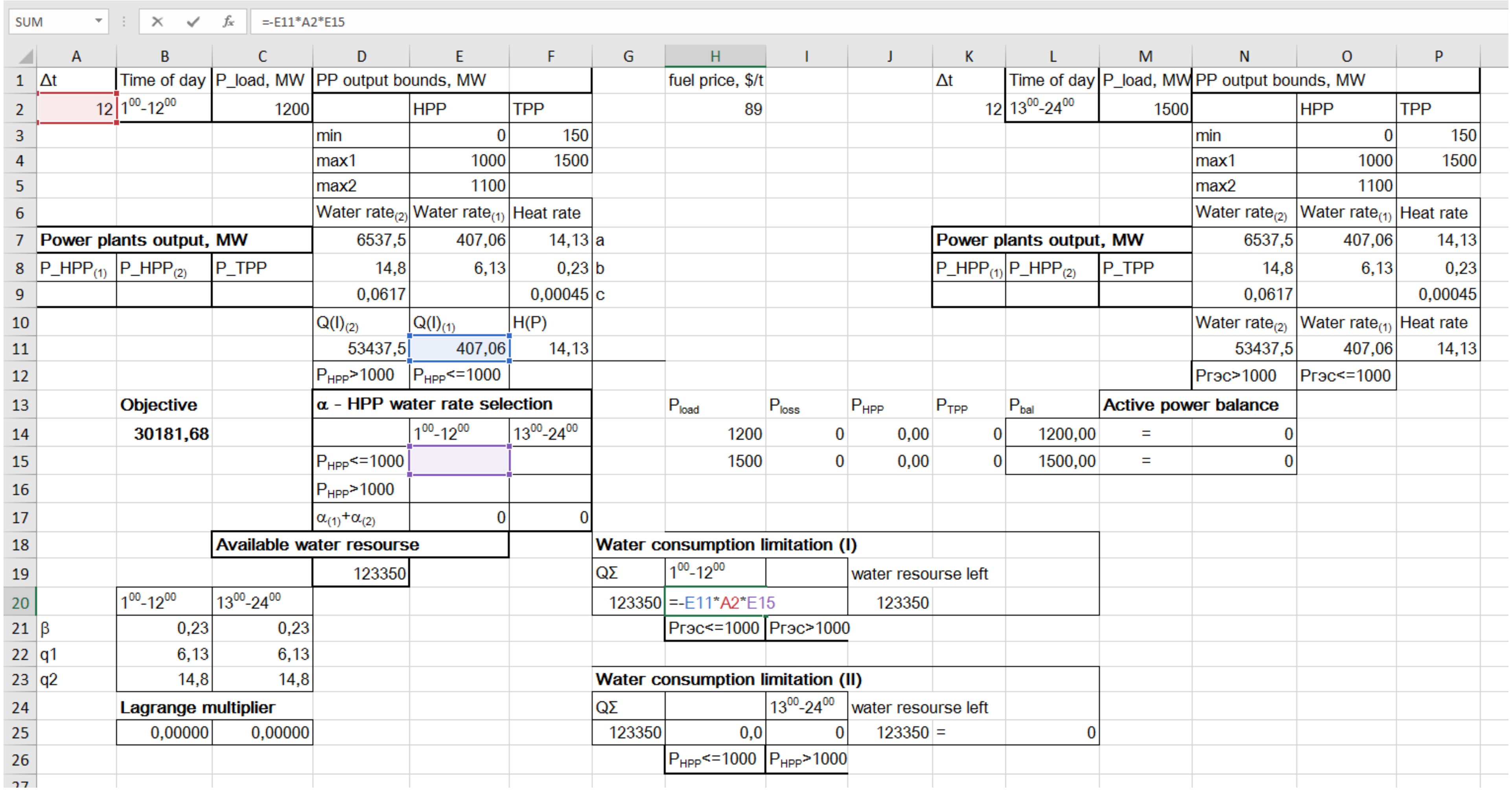
$$\frac{\dfrac{dF}{dP_{\text{TPP(I)}}}}{1 - \dfrac{dP_{\text{loss(I)}}}{dP_{\text{TPP(I)}}}} \cdot \frac{1 - \dfrac{dP_{\text{loss(I)}}}{dP_{\text{HPP(I)}}}}{\dfrac{dQ}{dP_{\text{HPP(I)}}}} = \frac{\dfrac{dF}{dP_{\text{TPP(II)}}}}{1 - \dfrac{dP_{\text{loss(II)}}}{dP_{\text{TPP(II)}}}} \cdot \frac{1 - \dfrac{dP_{\text{loss(II)}}}{dP_{\text{HPP(II)}}}}{\dfrac{dQ}{dP_{\text{HPP(II)}}}}$$

$$0 \le P(\text{I})_{\text{HPP(1)}} \le 1000$$
$$1000 < P(\text{I})_{\text{HPP(2)}} \le 1100$$

$$0 \le P(\text{II})_{\text{HPP(1)}} \le 1000$$
$$1000 < P(\text{II})_{\text{HPP(2)}} \le 1100$$

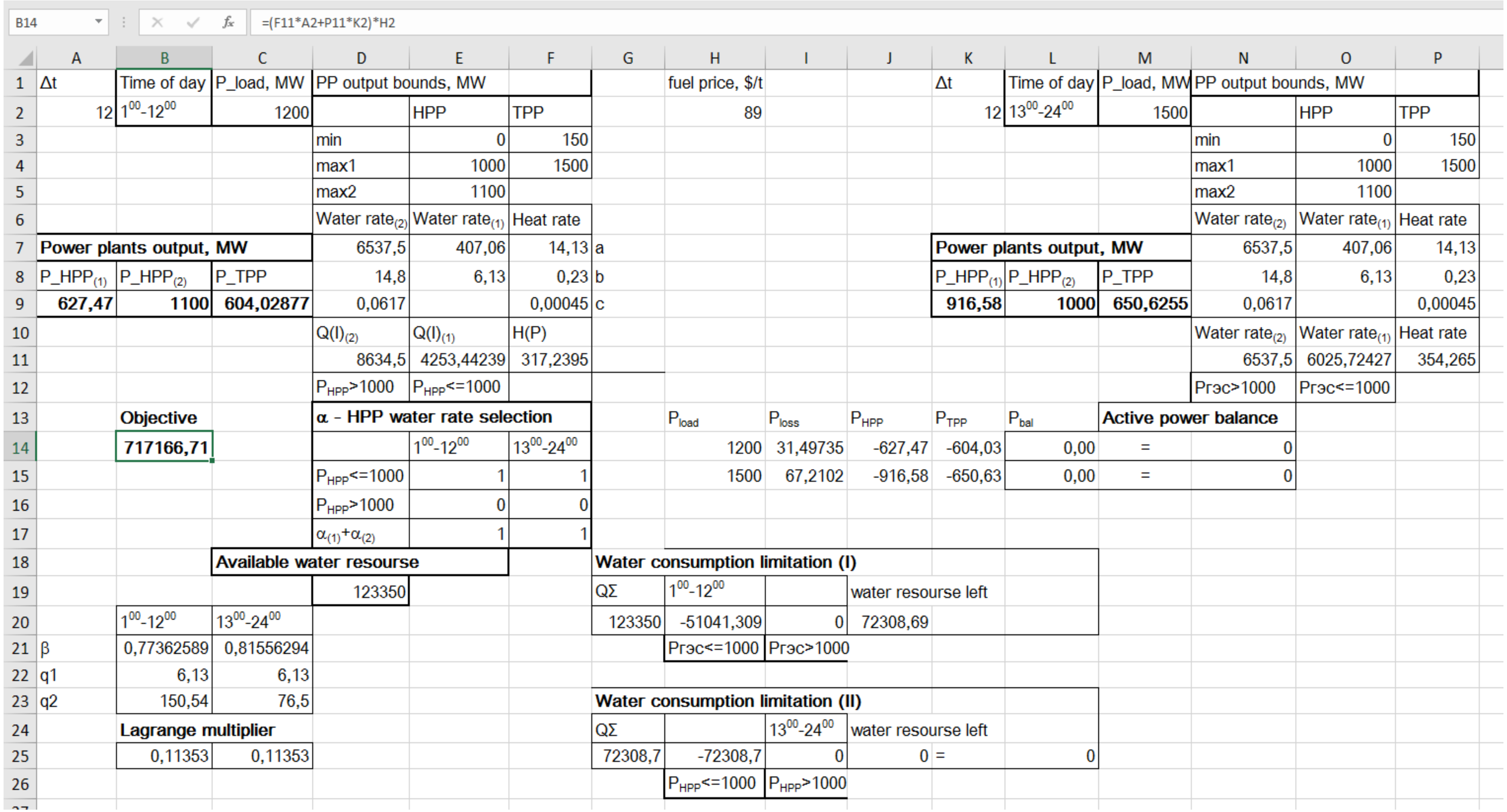
$$150 \le P(\text{I})_{\text{TPP}} \le 1500 \qquad (2.42)$$
$$150 \le P(\text{II})_{\text{TPP}} \le 1500$$

The Excel presentation of the initial data and the mathematical model is given in fig. 2.14. The solution obtained is presented in fig. 2.15. Fig. 2.16 and 2.17 show the **Solver** parameters.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Δt	Time of day	P_load, MW	PP output bounds, MW				fuel price, $/t			Δt	Time of day	P_load, MW	PP output bounds, MW		
2	12	1^{00}-12^{00}	1200		HPP	TPP		89			12	13^{00}-24^{00}	1500		HPP	TPP
3				min	0	150								min	0	150
4				max1	1000	1500								max1	1000	1500
5				max2	1100									max2	1100	
6				Water rate$_{(2)}$	Water rate$_{(1)}$	Heat rate								Water rate$_{(2)}$	Water rate$_{(1)}$	Heat rate
7	**Power plants output, MW**			6537,5	407,06	14,13	a				**Power plants output, MW**			6537,5	407,06	14,13
8	P_HPP$_{(1)}$	P_HPP$_{(2)}$	P_TPP	14,8	6,13	0,23	b				P_HPP$_{(1)}$	P_HPP$_{(2)}$	P_TPP	14,8	6,13	0,23
9				0,0617		0,00045	c							0,0617		0,00045
10				Q(I)$_{(2)}$	Q(I)$_{(1)}$	H(P)								Water rate$_{(2)}$	Water rate$_{(1)}$	Heat rate
11				53437,5	407,06	14,13								53437,5	407,06	14,13
12				P_{HPP}>1000	P_{HPP}<=1000									Prэс>1000	Prэс<=1000	
13		**Objective**		**α – HPP water rate selection**				P_{load}	P_{loss}	P_{HPP}	P_{TPP}	P_{bal}	**Active power balance**			
14		**30181,68**			1^{00}-12^{00}	13^{00}-24^{00}		1200	0	0,00	0	1200,00	=	0		
15				P_{HPP}<=1000				1500	0	0,00	0	1500,00	=	0		
16				P_{HPP}>1000												
17				$\alpha_{(1)}+\alpha_{(2)}$	0	0										
18			**Available water resourse**				**Water consumption limitation (I)**									
19				123350			QΣ	1^{00}-12^{00}		water resourse left						
20		1^{00}-12^{00}	13^{00}-24^{00}				123350	=-E11*A2*E15		123350						
21	β	0,23	0,23					Prэс<=1000	Prэс>1000							
22	q1	6,13	6,13													
23	q2	14,8	14,8													
24		**Lagrange multiplier**					**Water consumption limitation (II)**									
24							QΣ		13^{00}-24^{00}	water resourse left						
25		0,00000	0,00000				123350	0,0	0	123350	=	0				
26								P_{HPP}<=1000	P_{HPP}>1000							

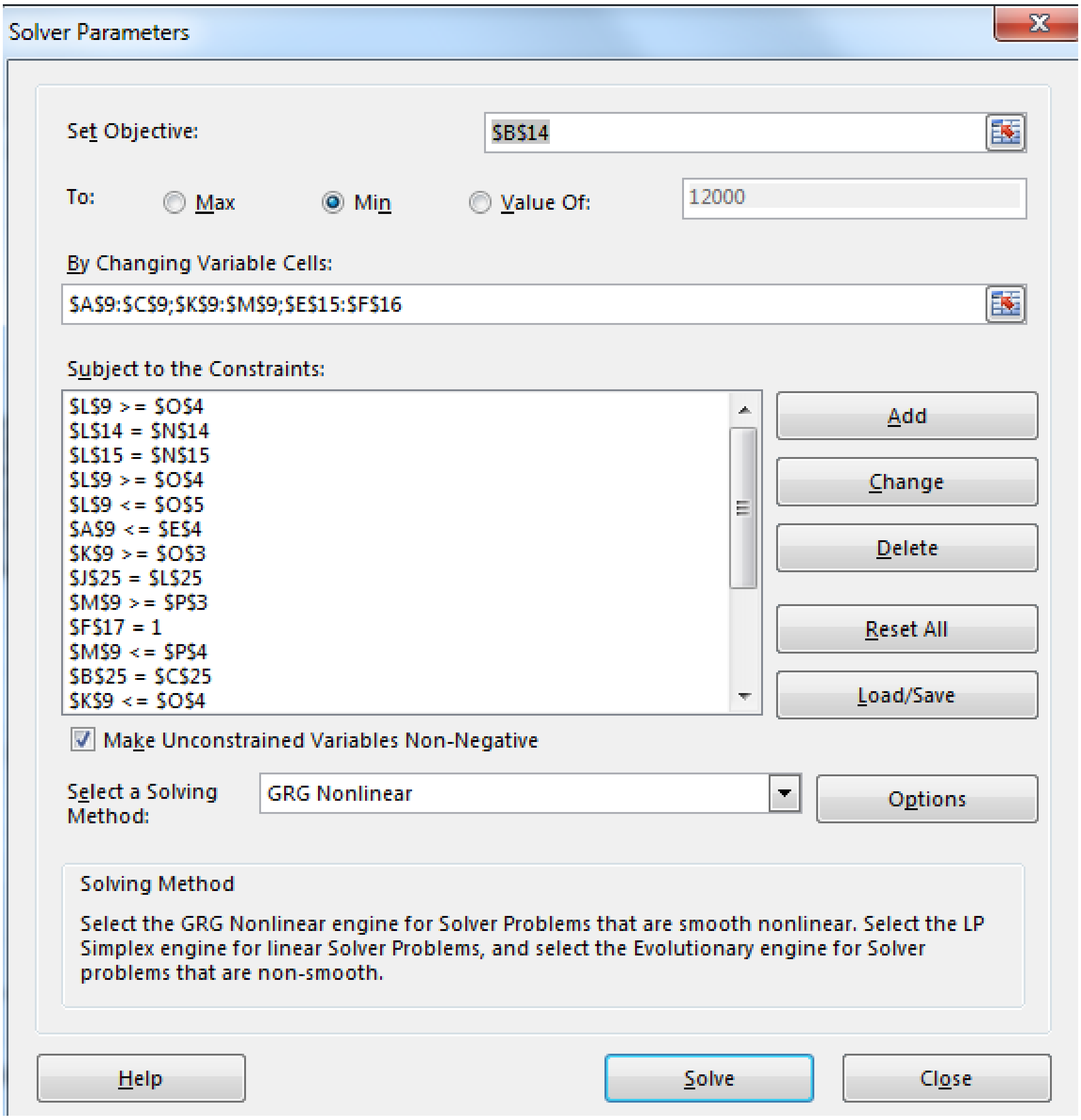
Figure 2.14 – Excel presentation of the initial data and the mathematical model (2.42) of economic dispatch problem 2.3.2 for the hydro-thermal power system under constrained water discharge from the reservoir (the pointer is in cell **H20** to present formula for calculating amount of water discharged during the first 12 hours)



B14 =(F11*A2+P11*K2)*H2

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Δt	Time of day	P_load, MW	PP output bounds, MW				fuel price, $/t			Δt	Time of day	P_load, MW	PP output bounds, MW		
2	12	1^{00}-12^{00}	1200		HPP	TPP		89			12	13^{00}-24^{00}	1500		HPP	TPP
3				min	0	150								min	0	150
4				max1	1000	1500								max1	1000	1500
5				max2	1100									max2	1100	
6				Water rate$_{(2)}$	Water rate$_{(1)}$	Heat rate								Water rate$_{(2)}$	Water rate$_{(1)}$	Heat rate
7	**Power plants output, MW**			6537,5	407,06	14,13	a				**Power plants output, MW**			6537,5	407,06	14,13
8	P_HPP$_{(1)}$	P_HPP$_{(2)}$	P_TPP	14,8	6,13	0,23	b				P_HPP$_{(1)}$	P_HPP$_{(2)}$	P_TPP	14,8	6,13	0,23
9	**627,47**	**1100**	**604,02877**	0,0617		0,00045	c				**916,58**	**1000**	**650,6255**	0,0617		0,00045
10				Q(I)$_{(2)}$	Q(I)$_{(1)}$	H(P)								Water rate$_{(2)}$	Water rate$_{(1)}$	Heat rate
11				8634,5	4253,44239	317,2395								6537,5	6025,72427	354,265
12				P_{HPP}>1000	P_{HPP}<=1000									Pгэс>1000	Pгэс<=1000	
13		**Objective**		**α – HPP water rate selection**				P_{load}	P_{loss}	P_{HPP}	P_{TPP}	P_{bal}	**Active power balance**			
14		**717166,71**			1^{00}-12^{00}	13^{00}-24^{00}		1200	31,49735	-627,47	-604,03	0,00	=	0		
15				P_{HPP}<=1000	1	1		1500	67,2102	-916,58	-650,63	0,00	=	0		
16				P_{HPP}>1000	0	0										
17				$\alpha_{(1)}+\alpha_{(2)}$	1	1										
18			**Available water resourse**				**Water consumption limitation (I)**									
19				123350			QΣ	1^{00}-12^{00}		water resourse left						
20		1^{00}-12^{00}	13^{00}-24^{00}				123350	-51041,309	0	72308,69						
21	β	0,77362589	0,81556294					Pгэс<=1000	Pгэс>1000							
22	q1	6,13	6,13													
23	q2	150,54	76,5				**Water consumption limitation (II)**									
24		**Lagrange multiplier**					QΣ		13^{00}-24^{00}	water resourse left						
25		0,11353	0,11353				72308,7	-72308,7	0	0	=	0				
26								P_{HPP}<=1000	P_{HPP}>1000							

Figure 2.15 – Excel solution to economic dispatch problem 2.3.2 for the hydrothermal power system under constrained water discharge from the reservoir



Solver Parameters

Set Objective: `$B$14`

To: ○ Max ● Min ○ Value Of: 12000

By Changing Variable Cells: `$A$9:$C$9;$K$9:$M$9;$E$15:$F$16`

Subject to the Constraints:

```
$L$9 >= $O$4
$L$14 = $N$14
$L$15 = $N$15
$L$9 >= $O$4
$L$9 <= $O$5
$A$9 <= $E$4
$K$9 >= $O$3
$J$25 = $L$25
$M$9 >= $P$3
$F$17 = 1
$M$9 <= $P$4
$B$25 = $C$25
$K$9 <= $O$4
```

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

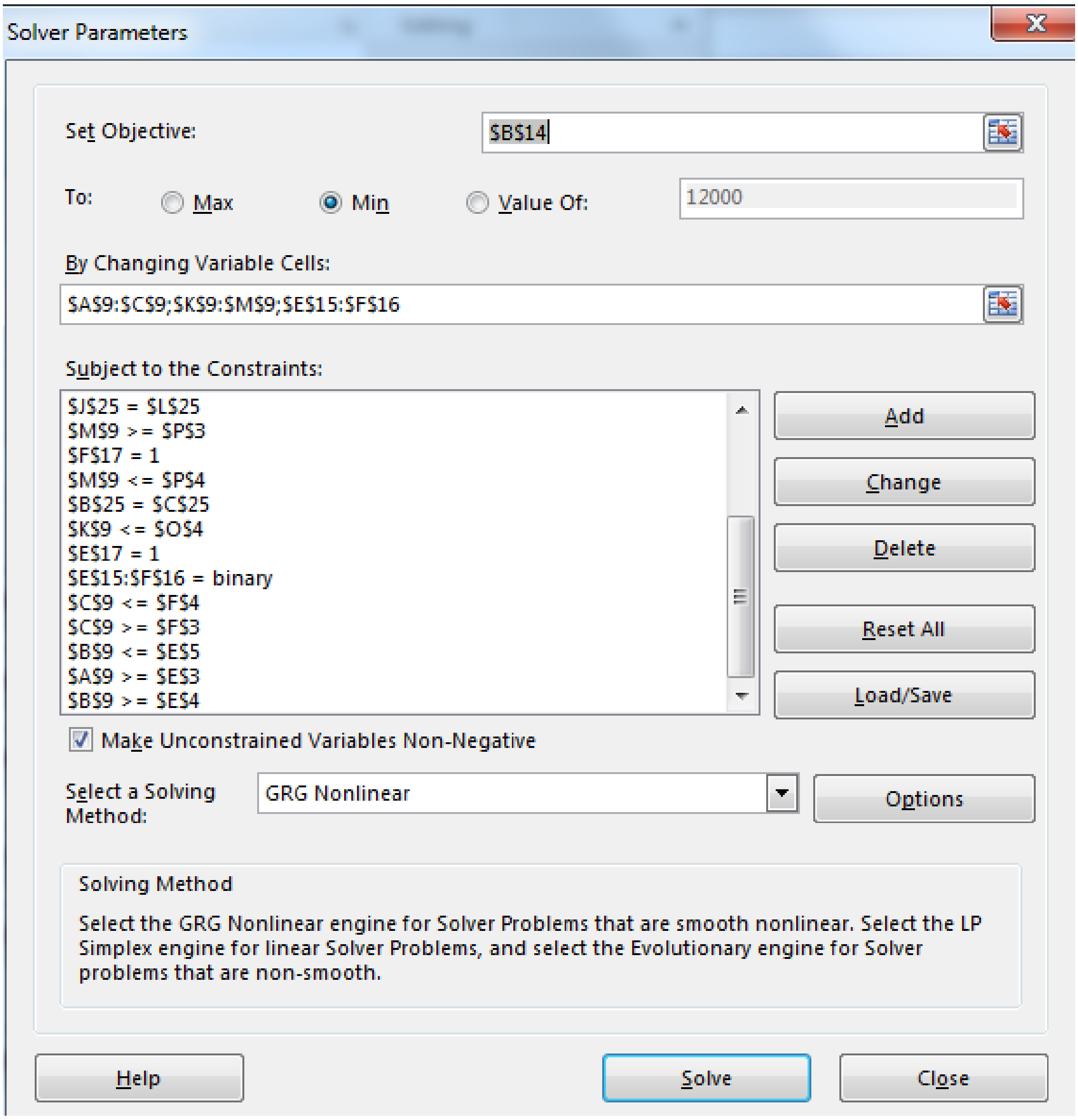
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 2.16 – Settings of **Solver parameters** window for economic dispatch problem 2.3.2

Analysis of the results. The optimal schedule of the thermal and hydroplants operation is shown in table 2.1. The constrained water discharge does not allows hydroplant operation with a higher capacity. The optimal efficiency of the water resource use is equal to the product of the fuel price and λ_{HPP}, that is 10.10417 \$/m^3.

Table 2.1 – Optimal schedule of the hydrothermal power system operation

Period of time	P_{TPP}, MW	P_{HPP}, MW	Water discharge, thous.m^3	λ_{HPP}
$01^{00} – 12^{00}$	604,03257	627,46	51040,92	0,11353
$13^{00} – 24^{00}$	650,621	916,59	72309,1	0,11353



Solver Parameters

Set Objective: $B$14

To: ○ Max ● Min ○ Value Of: 12000

By Changing Variable Cells:
$A$9:$C$9;$K$9:$M$9;$E$15:$F$16

Subject to the Constraints:
$J$25 = $L$25
$M$9 >= $P$3
$F$17 = 1
$M$9 <= $P$4
$B$25 = $C$25
$K$9 <= $O$4
$E$17 = 1
$E$15:$F$16 = binary
$C$9 <= $F$4
$C$9 >= $F$3
$B$9 <= $E$5
$A$9 >= $E$3
$B$9 >= $E$4

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 2.17 – Settings of **Solver parameters** window for economic dispatch problem (the final constrains)

2.4. Assignment

2.4.1. Assignment 1. Determine the economic dispatch for the generating units of a thermal power plant according to the variant. Find the solution taking into consideration condition (2.13) of optimal power dispatch for generating units in one power plant. Explain the solution obtained.

Variant 1

There are two 100 MW generating units and four 60 MW generating units in a thermal power plants. The TPP serves electrical load of 348 MW.

The input-output characteristics of the units, GJ/hr, are:

$H_1(P_1) = 144.01 + 7.167P_1 + 0.0833P_1^2$;

$H_2(P_2) = 150.315 + 7.132P_2 + 0.0799P_2^2$;

$H_3(P_3) = 119.5 + 6.11P_3 + 0.0397P_3^2$;

$H_4(P_4) = 119.8 + 6.17P_4 + 0.0401P_4^2$;

$H_5(P_5) = 118.35 + 6.09P_5 + 0.0399P_5^2$;

$H_6(P_6) = 121.24 + 6.06P_6 + 0.0393P_6^2$.

The heat value of the fuel Q_{LHV} is 25.81 GJ/t.

The generating units' power output bounds are:

$P_{1\min} = 35$ MW, $P_{1\max} = 102$ MW, $P_{2\min} = 34$ MW, $P_{2\max} = 105$ MW,

$P_{3\min} = 23$ MW, $P_{3\max} = 55$ MW, $P_{4\min} = 15$ MW, $P_{4\max} = 58$ MW,

$P_{5\min} = 17$ MW, $P_{5\max} = 61$ MW, $P_{6\min} = 25$ MW, $P_{6\max} = 60$ MW.

Variant 2

There are five 150 MW generating units and three 50 MW generating units in a thermal power plants. The TPP serves electrical load of 715 MW.

The input-output characteristics of the units, GJ/hr, are:

$H_1(P_1) = 320.14 + 8.821P_1 + 0.1018P_1^2$;

$H_2(P_2) = 315.11 + 8.733P_2 + 0.0899P_2^2$;

$H_3(P_3) = 319.5 + 8.718P_3 + 0.0987P_3^2$;

$H_4(P_4) = 314.01 + 8.697P_4 + 0.1044P_4^2$;

$H_5(P_5) = 318.35 + 9.03P_5 + 0.0931P_5^2$.

$H_6(P_6) = 116.24 + 5.87P_6 + 0.0373P_6^2$;

$H_7(P_7) = 117.13 + 5.90P_7 + 0.0378P_7^2$;

$H_8(P_8) = 115.02 + 5.89P_8 + 0.0375P_8^2$.

The heat value of the fuel Q_{LHV} is 26.14 GJ/t.

The generating units' power output bounds are:

$P_{1\min} = 50$ MW, $P_{1\max} = 146$ MW, $P_{2\min} = 45$ MW, $P_{2\max} = 150$ MW,

$P_{3\min} = 55$ MW, $P_{3\max} = 155$ MW, $P_{4\min} = 60$ MW, $P_{4\max} = 148$ MW,

$P_{5\min} = 50$ MW, $P_{5\max} = 145$ MW, $P_{6\min} = 13$ MW, $P_{6\max} = 52$ MW,

$P_{7\min} = 16$ MW, $P_{7\max} = 51$ MW, $P_{8\min} = 20$ MW, $P_{8\max} = 45$ MW.

Variant 3

There are three 60 MW generating units and two 100 MW generating units in a thermal power plants. The TPP serves electrical load of 341 MW.

The input-output characteristics of the units, GJ/hr, are:

$H_1(P_1) = 119.5 + 6.11P_1 + 0.0387P_1^2$;

$H_2(P_2) = 121.01 + 5.99P_2 + 0.0401P_2^2$;

$H_3(P_3) = 120.35 + 6.03P_3 + 0.0398P_3^2$;

$H_4(P_4) = 151.011 + 7.129P_4 + 0.0798P_4^2$;

$H_5(P_5) = 150.21 + 7.101P_5 + 0.0801P_5^2$.

The heat value of the fuel Q_{LHV} is 26.09 GJ/t.

The generating units' power output bounds are:

$P_{1\min}$ = 20 MW, $P_{1\max}$ = 60 MW, $P_{2\min}$ = 18 MW, $P_{2\max}$ = 59 MW,

$P_{3\min}$ = 20 MW, $P_{3\max}$ = 58 MW;

$P_{4\min}$ = 25 MW, $P_{4\max}$ = 100 MW, $P_{5\min}$ = 30 MW, $P_{5\max}$ = 100 MW.

Variant 4

There are two 50 MW generating units, three 60 MW generating units and one 100 MW generating unit in a thermal power plants. The TPP serves electrical load of 320 MW.

The input-output characteristics of the units, GJ/hr, are:

$H_1(P_1) = 118.35 + 5.93P_1 + 0.0372P_1^2$;

$H_2(P_2) = 116.24 + 5.88P_2 + 0.0376P_2^2$;

$H_3(P_3) = 125.02 + 6.17P_3 + 0.0391P_3^2$;

$H_4(P_4) = 122.61 + 6.23P_4 + 0.0393P_4^2$;

$H_5(P_5) = 123.14 + 6.21P_5 + 0.0388P_5^2$;

$H_6(P_6) = 147.88 + 7.099P_6 + 0.0789P_6^2$.

The heat value of the fuel Q_{LHV} is 25.98 GJ/t.

The generating units' power output bounds are:

$P_{1\min}$ = 16 MW, $P_{1\max}$ = 50 MW, $P_{2\min}$ = 15 MW, $P_{2\max}$ = 50 MW;

$P_{3\min}$ = 17 MW, $P_{3\max}$ = 60 MW, $P_{4\min}$ = 20 MW, $P_{4\max}$ = 59 MW,

$P_{5\min}$ = 15 MW, $P_{5\max}$ = 60 MW;

$P_{6\min}$ = 30 MW, $P_{6\max}$ = 100 MW.

Variant 5

There is one 60 MW generating unit, two 100 MW generating units and one 150 MW generating unit in a thermal power plants. The TPP serves electrical load of 285 MW.

The input-output characteristics of the units, GJ/hr, are:

$H_1(P_1) = 120.4 + 6.21P_1 + 0.0381P_1^2$;

$H_2(P_2) = 150.315 + 7.132P_2 + 0.0799P_2^2$;

$H_3(P_3) = 144.01 + 7.127P_3 + 0.0813P_3^2$;

$H_4(P_4) = 314.01 + 8.697P_4 + 0.1044P_4^2$;

The heat value of the fuel Q_{LHV} is 26.24 GJ/t.

The generating units' power output bounds are:

$P_{1\min} = 20$ MW, $P_{1\max} = 60$ MW;

$P_{2\min} = 25$ MW, $P_{2\max} = 100$ MW, $P_{3\min} = 28$ MW, $P_{3\max} = 99$ MW;

$P_{4\min} = 45$ MW, $P_{4\max} = 150$ MW.

Variant 6

There are two 100 MW generating units and three 150 MW generating units in a thermal power plants. The TPP serves electrical load of 510 MW.

The input-output characteristics of the units, GJ/hr, are:

$H_1(P_1) = 150.315 + 7.132P_1 + 0.0799P_1^2$;

$H_2(P_2) = 149.01 + 7.101P_2 + 0.0804P_2^2$;

$H_3(P_3) = 318.35 + 9.03P_3 + 0.0931P_3^2$;

$H_4(P_4) = 315.11 + 8.91P_4 + 0.0969P_4^2$;

$H_5(P_5) = 320.14 + 8.89P_5 + 0.0971P_5^2$.

The heat value of the fuel Q_{LHV} is 26.03 GJ/t.

The generating units' power output bounds are:

$P_{1\min} = 26$ MW, $P_{1\max} = 100$ MW, $P_{2\min} = 28$ MW, $P_{2\max} = 100$ MW;

$P_{3\min} = 38$ MW, $P_{3\max} = 150$ MW, $P_{4\min} = 40$ MW, $P_{4\max} = 150$ MW,

$P_{5\min} = 42$ MW, $P_{5\max} = 149$ MW.

The TPP serves electrical load of 510 MW.

2.4.2. Assignment 2. According to the variant, find the optimal fuel proportions for the generating units in a thermal power plant serving definite electric load so as to minimize natural gas consumption. The initial data are given in tables 2.1 and 2.2.

Table 2.2 – Generating units, TPP output, and fuel oil consumption limitation

Variant	№ of unit	TPP output P_{TPP}, MW	Fuel oil consumption limitation, F_{oil}, t.o.e./hr
1	1, 3, 4, 5	200	14
2	2, 4, 5	150	12
3	1, 2, 4	145	18
4	2, 3, 4, 5	210	21
5	2, 3, 5	155	16
6	1, 2, 3, 4	180	18
7	1, 4, 5	150	12
8	3, 4, 5	165	15

Table 2.3 – Fuel rates of generating units for different fuels and output bounds

Input-output characteristics of generating units	Power output limitations, MW
$F_{oil1}(P_1)=1.7089+0.1692P_1+0.00154P_1^2$	$28 \le P_1 \le 50$
$F_{gas1}(P_1)=0.7806+0.1901P_1+0.000797P_1^2$	
$F_{oil2}(P_2)=1.2041+0.0913P_2+0.00112P_2^2$	$25 \le P_2 \le 45$
$F_{gas2}(P_2)=0.7004+0.2011P_2+0.000808P_2^2$	
$F_{oil3}(P_3)=1.6118+0.1487P_3+0.00138P_3^2$	$30 \le P_3 \le 50$
$F_{gas3}(P_3)=0.9031+0.2217P_3+0.000991P_3^2$	
$F_{oil4}(P_4)=1.5911+0.1606P_4+0.00148P_4^2$	$45 \le P_4 \le 75$
$F_{gas4}(P_4)=0.8155+0.2183P_4+0.00848P_4^2$	
$F_{oil5}(P_5)=1.4004+0.1499P_5+0.00138P_5^2$	$40 \le P_5 \le 75$
$F_{gas5}(P_5)=0.7616+0.2201P_5+0.00795P_5^2$	

2.4.3. Assignment 3. Solve problem 2.3.2 in MS Excel for the load schedule according to the variant.

Variant 1

Time period	Load, MW
01^{00}-03^{00}	850
04^{00}-06^{00}	990
07^{00}-09^{00}	1380
10^{00}-12^{00}	1270
13^{00}-15^{00}	1310
16^{00}-18^{00}	1480
19^{00}-21^{00}	1640
22^{00}-24^{00}	1290

Variant 2

Time period	Load, MW
01^{00}-06^{00}	985
07^{00}-11^{00}	1475
12^{00}-16^{00}	1412
17^{00}-22^{00}	1820
23^{00}-24^{00}	1090

Variant 3

Time period	Load, MW
01^{00}-04^{00}	975
05^{00}-08^{00}	1284
09^{00}-12^{00}	1376
13^{00}-16^{00}	1342
17^{00}-20^{00}	1714
21^{00}-24^{00}	1308

Variant 4

Time period	Load, MW
01^{00}-06^{00}	1015
07^{00}-12^{00}	1405
13^{00}-18^{00}	1310
19^{00}-24^{00}	1675

Variant 5

Time period	Load, MW
01^{00}-03^{00}	808
04^{00}-05^{00}	1050
06^{00}-09^{00}	1414
10^{00}-12^{00}	1390
13^{00}-14^{00}	1800
15^{00}-17^{00}	1220
18^{00}-19^{00}	1510
20^{00}-21^{00}	1800
22^{00}-24^{00}	1220

Variant 6

Time period	Load, MW
01^{00}-05^{00}	902
06^{00}-09^{00}	1391
10^{00}-12^{00}	1316
13^{00}-15^{00}	1410
16^{00}-18^{00}	1654
19^{00}-21^{00}	1765
22^{00}-24^{00}	1148

3. NON-LINEAR PROGRAMMING PROBLEMS IN POWER ENGINEERING. OPTIMAL REACTIVE POWER COMPENSATION IN ELECTRICITY SUPPLY GRID

3.1. Necessity for reactive power compensation

Reactive power is required for electric motors and power units of equipment to work. The measuring unit of reactive power is kVAr (kiloVoltAmps reactive) or MVAr (MegaVoltAmps reactive). Reactive power deteriorates the system power factor cosφ and increases active power loss in the grid.

To improve the power factor and reduce the grid losses, sources of reactive power, for example, capacitors, are installed in the system at transmission level (high voltage), at distribution substation level (medium voltage) or at load level (low voltage), which is called reactive power compensation. The reactive power sources are called compensating devices or reactive-power compensators (VAR compensators). Reactive-power compensators are recommended to install as close to a definite consumption node as possible to avoid distributing this power at other nodes of the grid.

Let's consider a simple line of resistance R connecting a power source of voltage U and a load node consuming active and reactive power $P+jQ$ (fig. 3.1) [4].

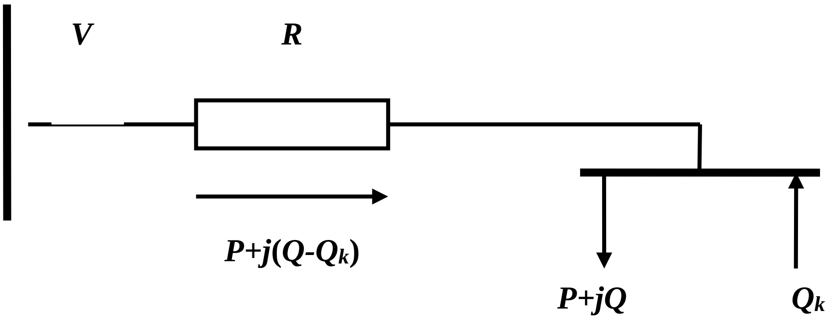


Figure 3.1 – A simple diagram of reactive power compensation

Active power loss in electricity grids is calculated as

$$P_{loss} = (P^2 + Q^2)R/V^2, \tag{3.1}$$

where V is the grid voltage, and R is the grid resistance.

With a reactive-power compensator installed in the grid, the active power loss decreases:

$$P_{loss} = (P^2 + (Q - Q_k)^2)R/V^2. \qquad (3.2)$$

Thus, reactive power compensation allows reducing active power loss in electricity grids, which, consequently, improves economic parameters of the grid.

According to (3.1) and (3.2), active power loss ΔP includes two components: loss resulted from active power P transportation through the transmission or distribution line and loss resulted from reactive power Q transportation.

Since reactive power compensation only affects the other component of power loss, $(Q - Q_k)$, we can omit the first component, active power P, to analyze active power loss as a function of reactive power transported through the transmission or distribution line.

In the distribution grids, the total compensation power may be limited for technical or economic reasons. The available compensation power must be optimally distributed or installed throughout the grid, which depends on the distribution grid layout.

3.2. Reactive power compensation in a star-type distribution grid

3.2.1 Condition of optimal reactive power compensation in a star-type grid [4]

A power source of voltage V (for example, a low-capacity power station or an MV/LV transformer substation) supplies electricity to n consumers with reactive power load Q_1, Q_2,…, Q_n connected in parallel (fig. 3.2). An electric grid with such connections of consumers is called a star-type or radial grid. Resistances of the distribution lines between the source and the consumers are R_1, R_2, … R_n.

A reactive power compensator Q_{ki} of any capacity can be installed at every (i-th) consumer. The total available compensation capacity $Q_{k\Sigma}$ should be optimally distributed among the consumers so as to minimize active power loss resulted from reactive power consumption in the grid.

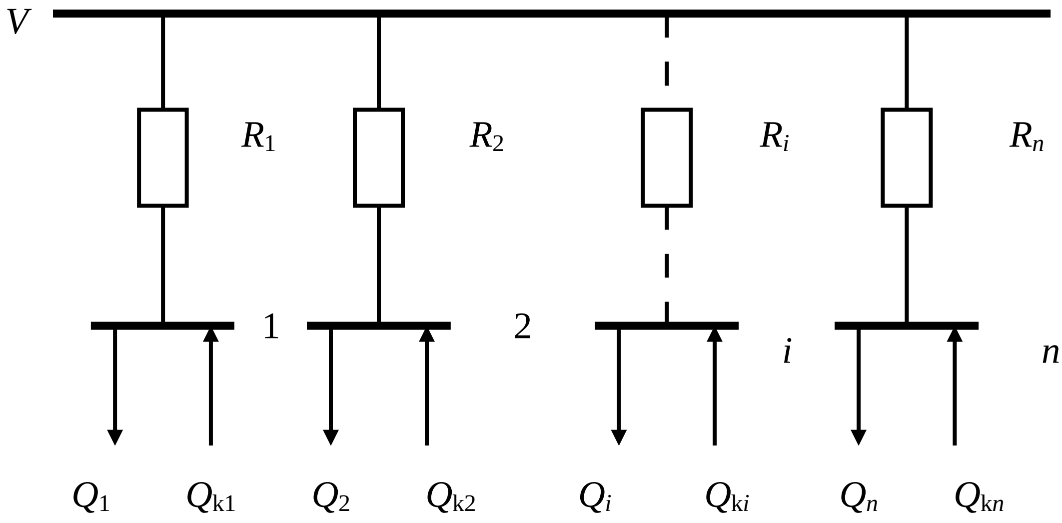


Figure 3.2 – A star-type electricity distribution grid [4]

The objective function is formulated as [4]

$$Z = \Delta P_{loss} = \sum_{i=1}^{n}((Q_i - Q_{ki})^2 R_i / V^2) \to \min \, . \tag{3.3}$$

The constraint is the limitation of the total compensation capacity

$$Q_{k1} + Q_{k2} + \ldots + Q_{kn} = \sum_{i=1}^{n} Q_{ki} = Q_{k\Sigma} \tag{3.4}$$

or

$$\sum_{i=1}^{n} Q_{ki} - Q_{k\Sigma} = 0 \, . \tag{3.5}$$

The condition of compensation capacity optimal distribution in a star-type electric grid can be found with the help of Lagrange multiplier technique.

Lagrange function L comprises objective function (3.3) and constraint (3.5) multiplied by a Lagrange multiplier:

$$L = \sum_{i=1}^{n}((Q_i - Q_{ki})^2 R_i / V^2) + \lambda \cdot (\sum_{i=1}^{n} Q_{ki} - Q_k) \to \min \tag{3.6}$$

or, in the detailed formulation,

$$\begin{aligned} L = (Q_1 - Q_{k1})^2 R_1 / V^2 + (Q_2 - Q_{k2})^2 R_2 / V^2 + \ldots + (Q_n - Q_{kn})^2 R_n / V^2 + \\ + \lambda \cdot (Q_{k1} + Q_{k2} + \ldots + Q_{kn} - Q_{k\Sigma}) \to \min \end{aligned} \quad . \quad (3.7)$$

Minimum of Lagrange function (3.7) can be found by determining its first derivatives with respect to all the variables (compensation capacities Q_{ki}, $i = 1,..n,$ and Lagrange multiplier λ) and setting them to 0.

$$\begin{aligned} &\frac{\partial L}{\partial Q_{k1}} = -2(Q_1 - Q_{k1}) R_1 / V^2 + \lambda = 0 \\ &\frac{\partial L}{\partial Q_{k2}} = -2(Q_2 - Q_{k2}) R_2 / V^2 + \lambda = 0 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad . \\ &\frac{\partial L}{\partial Q_{kn}} = -2(Q_n - Q_{kn}) R_n / V^2 + \lambda = 0 \\ &\frac{\partial L}{\partial \lambda} = Q_{k1} + Q_{k2} + \ldots + Q_{kn} - Q_{k\Sigma} = 0 \end{aligned} \qquad (3.8)$$

We obtain a system of (n+1) equations with (n+1) variables.

After transposing the other component, λ, in the first n equations of (3.8) to the left side

$$\begin{aligned} &2(Q_1 - Q_{k1}) R_1 / V^2 = \lambda \\ &2(Q_2 - Q_{k2}) R_2 / V^2 = \lambda \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &2(Q_n - Q_{kn}) R_n / V^2 = \lambda \end{aligned} \quad , \qquad (3.9)$$

we can conclude that, as the right side in equations (3.9) is the same (the same Lagrange multiplier λ), the left sides are equal:

$$2(Q_1 - Q_{k1}) R_1 / V^2 = 2(Q_2 - Q_{k2}) R_2 / V^2 = \ldots = 2(Q_n - Q_{kn}) R_n / V^2,$$

or, taking into account that the grid voltage V is constant,

$$R_1(Q_1 - Q_{k1}) = R_2(Q_2 - Q_{k2}) = \ldots = R_n(Q_n - Q_{kn}). \qquad (3.10)$$

Therefore, expression (3.10) is a **condition of optimal distribution of reactive power compensation capacity in a star-type electric grid**. This condition is applied to determine optimal compensation capacities that must be installed in the consumption nodes to decrease active power loss.

3.2.2. Optimal distribution of available compensation capacity in a star-type grid

Let us consider an example of optimal compensation capacity distribution in a star-type electric grid. It is required to determine the optimal distribution of available reactive power compensation capacity $Q_{k\Sigma}$ among three reactive power consumption nodes, Q_1, Q_2, and Q_3, in the star-type electricity supply system presented in fig. 3.3 so as to minimize the total costs of reactive power compensation installation and the active power loss coverage.

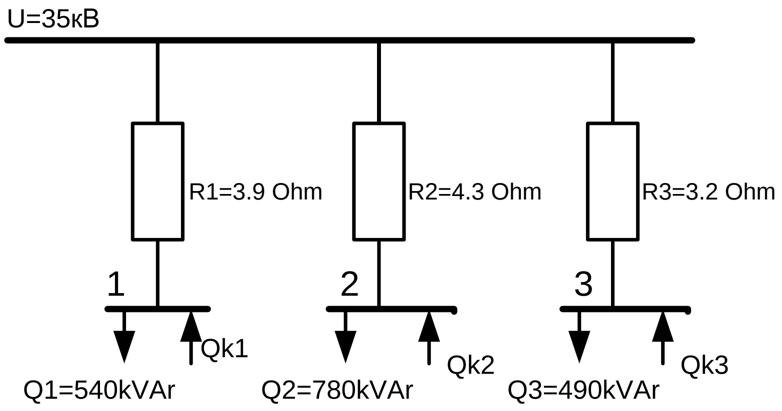


Figure 3.3 – A three-node star-type electricity supply system

The initial electrical parameters are given in the figure. Specific costs of reactive power compensation installation z_0= 21 $/kVAr; specific costs on active power loss coverage c_0= 0.15 $/kVAr.

The total available reactive power compensation capacity $Q_{k\Sigma}$ is 900 kVAr.

The mathematical model of the considered problem is the following.

The **decision variables** are compensation capacities $Q_{к1}$, $Q_{к2}$, and $Q_{к3}$ that should be installed at consumption nodes 1, 2 and 3, correspondingly.

The objective function is sum of $Q_{к1}$, $Q_{к2}$, and $Q_{к3}$ installation costs and costs resulted from coverage of active power loss cause by reactive power consumption.

The first component of the objective function is $z_0 \cdot (Q_{k1} + Q_{k2} + Q_{k3})$.

The other component is sum of active power loss at every node $(Q_i - Q_{ki})^2 R_i / V^2$ multiplied by the loss coverage cost c_0:

$$c_0 \cdot (Q_1 - Q_{k1})^2 R_1 / V^2 + c_0 \cdot (Q_2 - Q_{k2})^2 R_2 / V^2 + c_0 \cdot (Q_3 - Q_{k3})^2 R_3 / V^2 .$$

The **objective** function is

$$\begin{gathered} Z = z_0 \cdot (Q_{k1} + Q_{k2} + Q_{k3}) + c_0 \cdot (Q_1 - Q_{k1})^2 R_1 / V^2 + \\ c_0 \cdot (Q_2 - Q_{k2})^2 R_2 / V^2 + c_0 \cdot (Q_3 - Q_{k3})^2 R_3 / V^2 \to \min \end{gathered} . \quad (3.11)$$

The **constraints** are

– equality of the total compensation capacity distributed among the three VAR consumers of the star-type grid to the available capacity of VAR compensation

$$Q_{k1} + Q_{k2} + Q_{k3} = Q_{k\Sigma};$$

– the condition (6.10) of the optimal reactive power compensation capacity distribution in the star-type grid with three consumption nodes

$$R_1 (Q_1 - Q_{k1}) = R_2 (Q_2 - Q_{k2}) = R_3 (Q_3 - Q_{k3}) .$$

Substituting the initial data, we obtain the **mathematical model** of active power loss minimization problem through optimal distribution of reactive power compensation capacity in the star-type grid shown in fig. 3.3

$$Z = 21\cdot(Q_{k1}+Q_{k2}+Q_{k3}) + 0.15\cdot 3.9/35^2\cdot(540-Q_{k1})^2 +$$

$$+0.15\cdot 4.3/35^2\cdot(780-Q_{k2})^2 + 0.15\cdot 3.2/35^2\cdot(490-Q_{k3})^2 \rightarrow \min$$

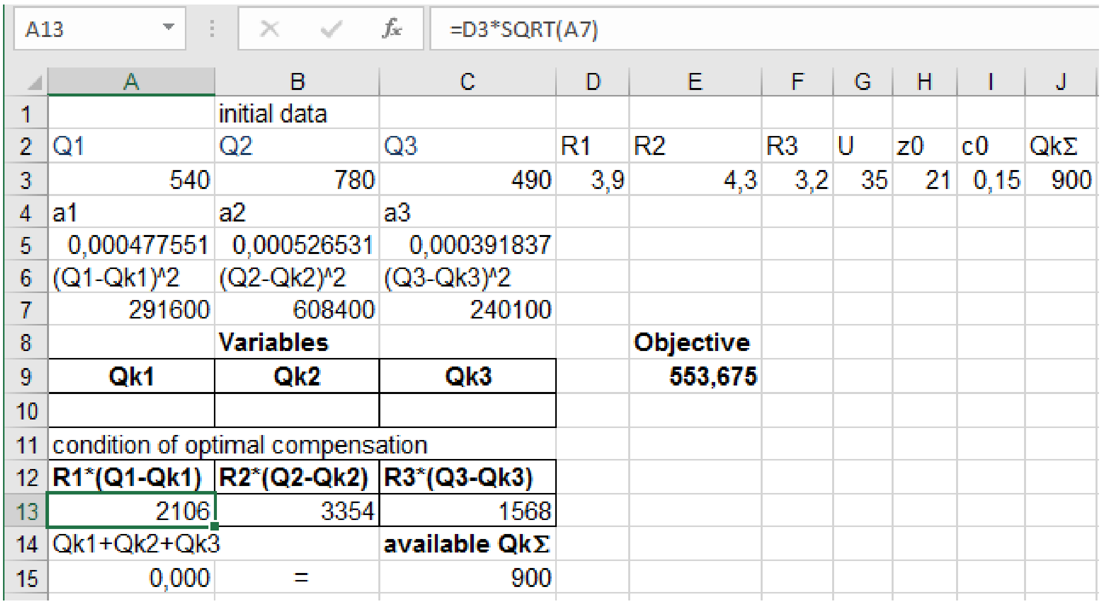
$$Q_{k1}+Q_{k2}+Q_{k3} = 900 \qquad . \tag{3.12}$$

$$3.9\cdot(540-Q_{k1}) = 4.3\cdot(780-Q_{k2}) = 3.2\cdot(490-Q_{k3})$$

$$Q_{ki}\geq 0,\quad i=1,2,3$$

This problem is solved with the help of MS Excel Solver.

The Excel presentation of the initial data is shown in fig. 3.4. The initial data are introduced in cells **A3:J3**. The decision variables Q_{ki} are introduced in cells **A10:C10**.



A13 =D3*SQRT(A7)

	A	B	C	D	E	F	G	H	I	J
1		initial data								
2	Q1	Q2	Q3	R1	R2	R3	U	z0	c0	QkΣ
3	540	780	490	3,9	4,3	3,2	35	21	0,15	900
4	a1	a2	a3							
5	0,000477551	0,000526531	0,000391837							
6	(Q1-Qk1)^2	(Q2-Qk2)^2	(Q3-Qk3)^2							
7	291600	608400	240100							
8		**Variables**			**Objective**					
9	**Qk1**	**Qk2**	**Qk3**		**553,675**					
10										
11	condition of optimal compensation									
12	**R1*(Q1-Qk1)**	**R2*(Q2-Qk2)**	**R3*(Q3-Qk3)**							
13	2106	3354	1568							
14	Qk1+Qk2+Qk3		**available QkΣ**							
15	0,000	=	900							

Figure 3.4 – Excel presentation of the initial data for example problem 3.2.2 (the pointer is in cell **A13** to show Excel formula for calculation of optimality condition (3.10) at the first node)

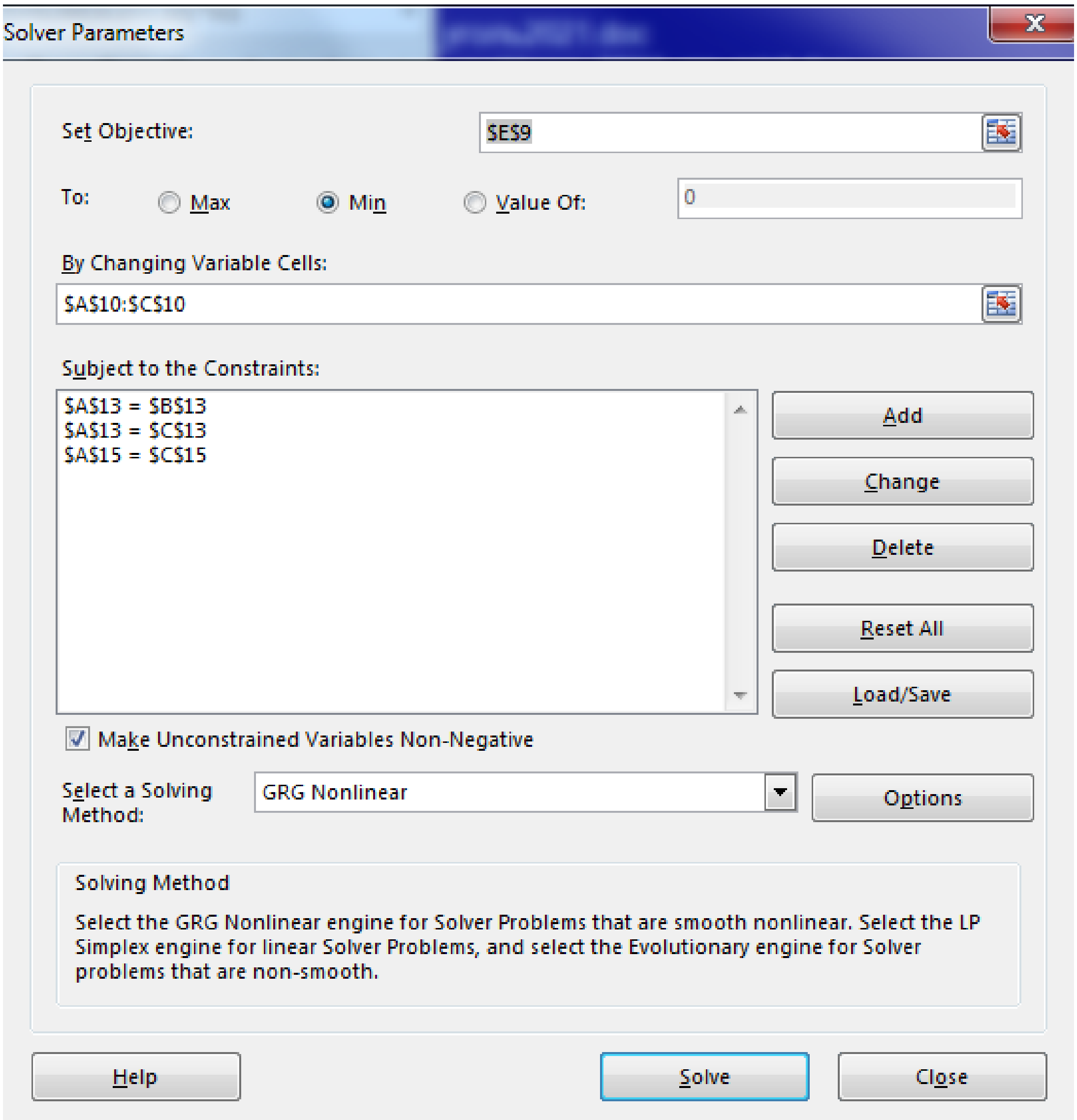
Some intermediate expressions should be calculated to facilitate introduction of mathematical model (3.12). Excel expressions for the intermediate calculations taking into account the Excel presentation of the initial data (fig. 3.4) are given in table 3.1.

Table 3.1 – Excel expressions for intermediate calculations of (3.12)

Mathematical item	Excel cell	Excel expression
Coefficients a_i		
$a_1 = c_0 \times R_1 / V^2$	**A5**	**=$I$3*D3/($G$3^2)**
$a_2 = c_0 \times R_2 / V^2$	**B5**	**=$I$3*E3/($G$3^2)**
$a_3 = c_0 \times R_3 / V^2$	**C5**	**=$I$3*F3/($G$3^2)**
Brackets $(Q_i – Q_{ki})^2$		
$(Q_1\text{-}Q_{k1})^2$	**A7**	**=(A3-A10)^2**
$(Q_2\text{-}Q_{k2})^2$	**B7**	**=(B3-B10)^2**
$(Q_3\text{-}Q_{k3})^2$	**C7**	**=(C3-C10)^2**
$R_i \cdot (Q_i – Q_{ki})$ – condition of optimal reactive power compensation distribution		
$R_1 \cdot (Q_1\text{-}Q_{k1})$	**A13**	**=D3*SQRT(A7)**
$R_2 \cdot (Q_2\text{-}Q_{k2})$	**B13**	**=E3*SQRT(B7)**
$R_3 \cdot (Q_3\text{-}Q_{k3})$	**C13**	**=F3*SQRT(C7)**
Sum of the reactive power compensation capacities installed at the nodes		
left side $Q_{k1} + Q_{k2} + Q_{k3}$	**A15**	**=SUM(A10:C10)**
right side $Q_{k\Sigma}=900$	**C15**	**=J3**
Objective		
	E9	**=H3*SUM(A10:C10)+SUMPRODUCT(A5:C5;A7:C7)**

The **Solver** parameters for solving problem (3.12) are shown in fig. 3.5.

The solution to the considered problem of optimal reactive power compensation capacity distribution in the three-node star-type grid shown in fig. 3.3 is given in fig. 3.6. The graphic presentation of the optimal reactive power compensation in this grid is given in fig. 3.7.



Set Objective: $E$9

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $A$10:$C$10

Subject to the Constraints:

$A$13 = $B$13
$A$13 = $C$13
$A$15 = $C$15

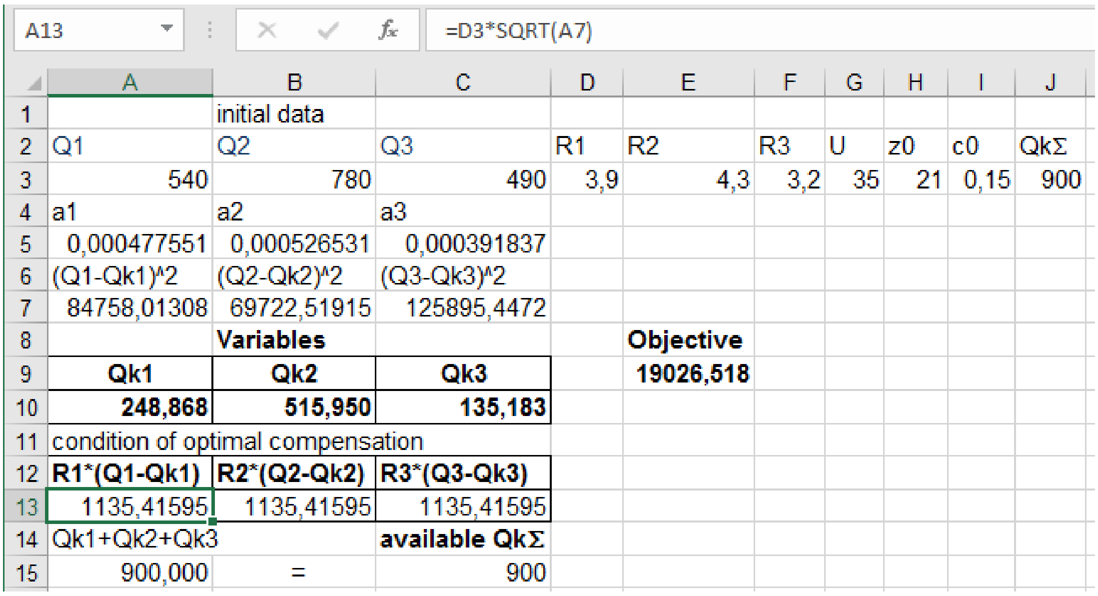
☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 3.5 – Settings of **Solver parameters** window for problem 3.2.2



A13 =D3*SQRT(A7)

	A	B	C	D	E	F	G	H	I	J
1		initial data								
2	Q1	Q2	Q3	R1	R2	R3	U	z0	c0	QkΣ
3	540	780	490	3,9	4,3	3,2	35	21	0,15	900
4	a1	a2	a3							
5	0,000477551	0,000526531	0,000391837							
6	(Q1-Qk1)^2	(Q2-Qk2)^2	(Q3-Qk3)^2							
7	84758,01308	69722,51915	125895,4472							
8		**Variables**			**Objective**					
9	**Qk1**	**Qk2**	**Qk3**		**19026,518**					
10	**248,868**	**515,950**	**135,183**							
11	condition of optimal compensation									
12	**R1*(Q1-Qk1)**	**R2*(Q2-Qk2)**	**R3*(Q3-Qk3)**							
13	1135,41595	1135,41595	1135,41595							
14	Qk1+Qk2+Qk3		**available QkΣ**							
15	900,000	=	900							

Figure 3.6 – The solution to problem 3.2.2 of optimal reactive power compensation

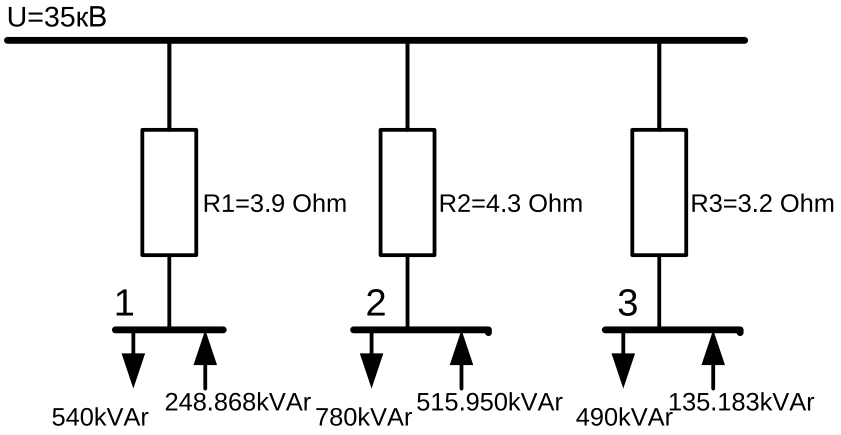


Figure 3.7 – Graphical presentation of the optimal distribution of reactive power compensation capacity in the three-node grid

Analysis of the obtained solution. According to the solution obtained in compliance with condition (3.10) of optimal distribution of reactive power compensation capacity among nodes in a star-type grid, the biggest capacity of reactive power compensators is installed at node 2 that is characterized by the highest reactive power consumption and the biggest distribution line resistance. The second high reactive power consumption is at node 1 so the second big reactive power compensation takes place there. The smallest volume of reactive power is consumed at node 3 that is also characterized by the smallest distribution line resistance, the two factors resulting in the smallest capacity of reactive power compensation at this node.

3.2.3. Optimal location of VAR compensators in a star-type grid

Let us consider an optimization problem with Boolean variables. The general objective is to determine the nodes in a star-type electric grid where reactive power compensating device of a given capacity must be installed to maximally decrease active power loss resulted from reactive power consumption.

For the three-node star-type electric grid presented in fig. 3.3, it is required to find the optimal location of two 450 kVAr compensators so as to minimize the total costs of reactive power compensation. The initial electrical parameters are given in fig. 3.4. The installation cost of a 450 kVAr compensator $z_0 = 9450$ $; specific costs on active power loss coverage $c_0 = 0.15$ $/kVAr.

The 450 kVAr compensators can be installed at any two of the three node of the considered grid.

Therefore, the variables in this problem are possibilities λ_i of the compensator installation at the nodes:

λ_1 is a possibility of 450 kVAr compensator installation at node 1;

λ_2 is a possibility of 450 kVAr compensator installation at node 2;

λ_3 is a possibility of 450 kVAr compensator installation at node 3.

λ_i are Boolean variables,

$$\lambda_i = \begin{cases} 1 \ \ \textit{if the compensator is installed at node i} \\ 0 \ \ \textit{if the compensator is not installed at node i} \end{cases}.$$

The compensation capacity at any of the three nodes will be

$$Q_{ki} = \lambda_i \cdot Q_k = \lambda_i \cdot 450 = \begin{cases} 450 \ \ \textit{if the compensator is installed at node i} \\ 0 \ \ \textit{if the compensator is not installed at node i} \end{cases}.$$

The mathematical model of the Boolean problem considered is the following.

The objective function is, like in problem 3.2.2., sum of two components specifying the totals costs on reactive power compensation.

The first component of the objective function determines the total costs on installation of the two reactive-power compensators. $Q_{к1}$, $Q_{к2}$, and $Q_{к3}$ in (3.12) are now identical and equal to $Q_k = 450$ kVAr, so the first component is calculated as $z_0 \cdot (\lambda_1 \cdot Q_k + \lambda_2 \cdot Q_k + \lambda_3 \cdot Q_k)$.

The other component computes the total costs resulted from active power loss coverage. It equals to the sum of active power loss at every node $(Q_i - \lambda_i \cdot Q_{ki})^2 R_i / V^2$ multiplied by the loss coverage cost c_0:

$$c_0 \cdot (Q_1 - \lambda_1 \cdot Q_k)^2 R_1 / V^2 + c_0 \cdot (Q_2 - \lambda_2 \cdot Q_k)^2 R_2 / V^2 + c_0 \cdot (Q_3 - \lambda_3 \cdot Q_k)^2 R_3 / V^2.$$

The **objective** function is

$$\begin{gathered} Z = z_0 \cdot (\lambda_1 \cdot Q_k + \lambda_2 \cdot Q_k + \lambda_3 \cdot Q_k) + c_0 \cdot (Q_1 - \lambda_1 \cdot Q_k)^2 R_1 / V^2 + \\ c_0 \cdot (Q_2 - \lambda_2 \cdot Q_k)^2 R_2 / V^2 + c_0 \cdot (Q_3 - \lambda_3 \cdot Q_k)^2 R_3 / V^2 \to \min \end{gathered}. \quad (3.13)$$

The **constraint** is

– equality of the installed VAR compensators to the available number of VAR compensators

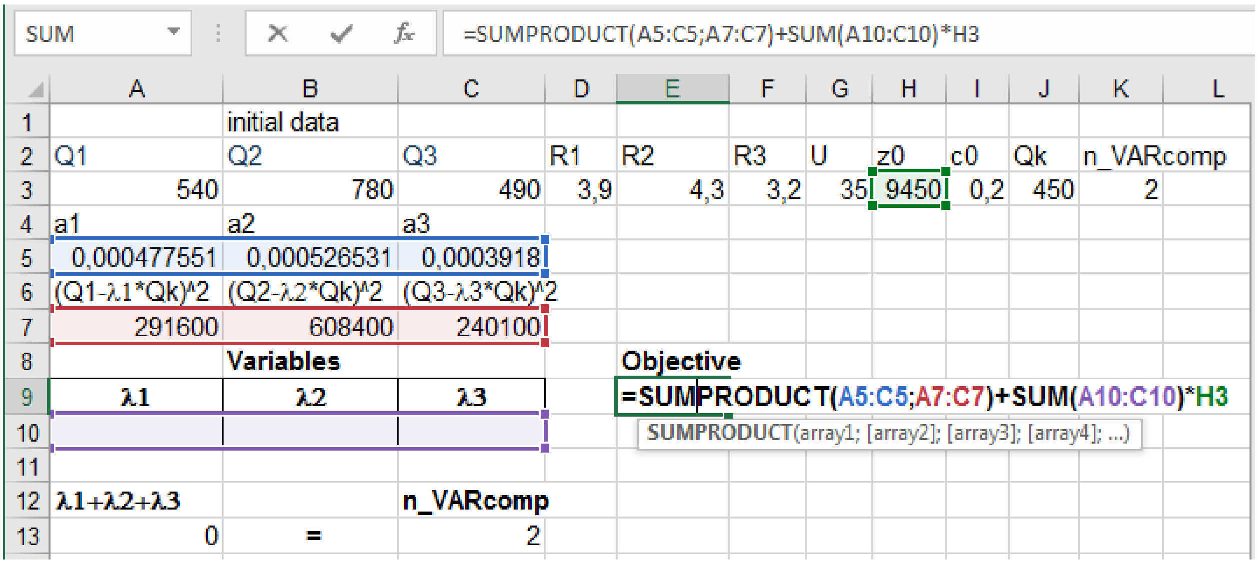
$$\lambda_1 + \lambda_2 + \lambda_3 = n_{\text{VAR\_comp}};$$

<u>**In optimization problems with Boolean variables, the condition (3.10)**</u> of the optimal reactive power compensation capacity distribution in a star-type grid cannot not satisfied and, therefore, <u>**must not be taken into account**</u>**.**

After substitution of the initial data, the **mathematical models** is

$$\begin{gathered}
Z = 9450\cdot(\lambda_1+\lambda_2+\lambda_3) + 0.15\cdot 3.9/35^2\cdot(540-\lambda_1\cdot 450)^2 + \\
+0.15\cdot 4.3/35^2\cdot(780-\lambda_2\cdot 450)^2 + 0.15\cdot 3.2/35^2\cdot(490-\lambda_3\cdot 450)^2 \to \min \\
\lambda_1+\lambda_2+\lambda_3 = 2 \\
\lambda_i = Boolean, \quad i = 1,2,3.
\end{gathered} \tag{3.14}$$

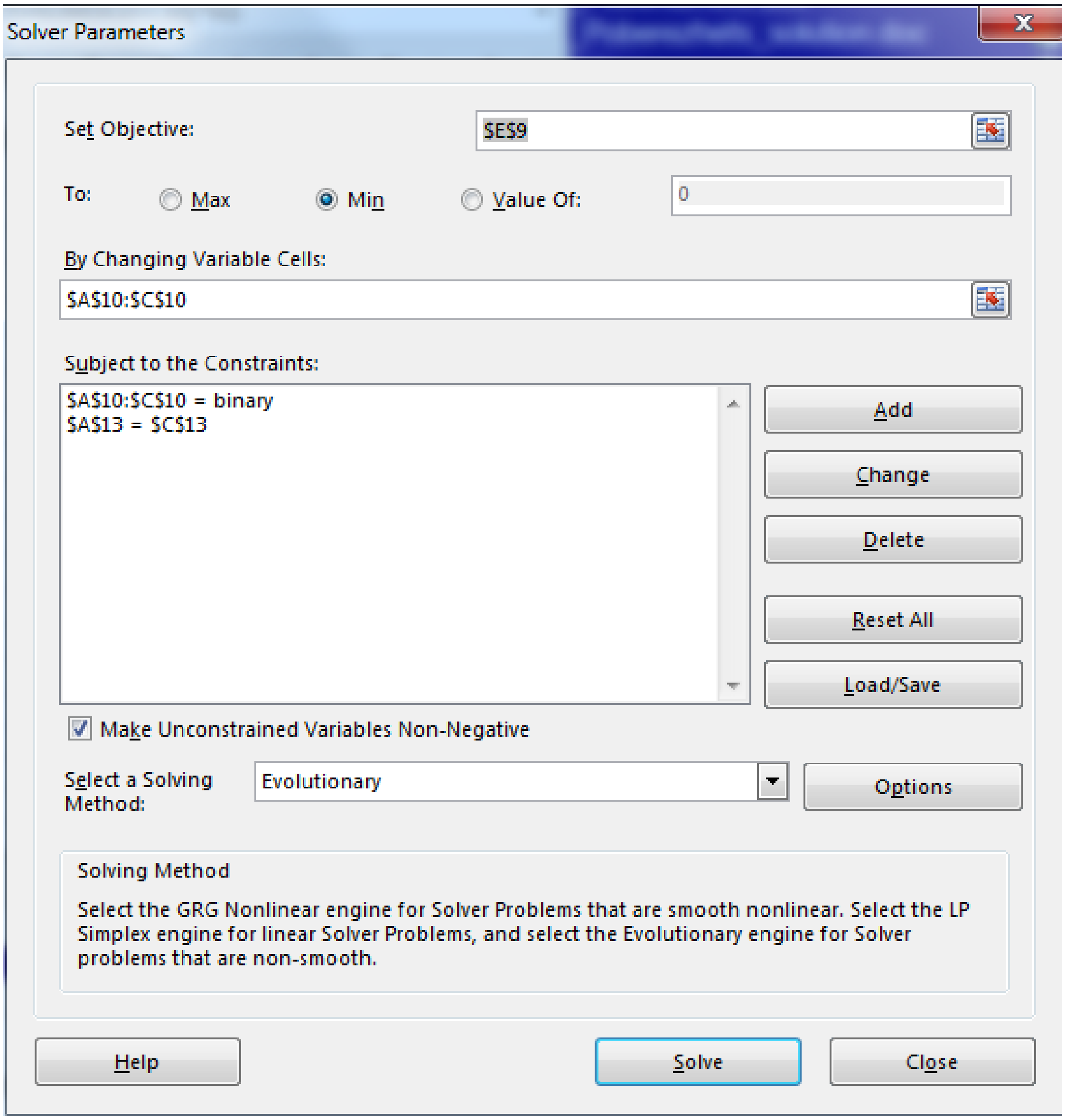
The Excel presentation of the initial data and intermediate expressions of (3.14) is shown in fig. 3.8. Figure 3.9 presents **Solver** parameters for this problem.



=SUMPRODUCT(A5:C5;A7:C7)+SUM(A10:C10)*H3

	A	B	C	D	E	F	G	H	I	J	K	L
1		initial data										
2	Q1	Q2	Q3	R1	R2	R3	U	z0	c0	Qk	n_VARcomp	
3	540	780	490	3,9	4,3	3,2	35	9450	0,2	450	2	
4	a1	a2	a3									
5	0,000477551	0,000526531	0,0003918									
6	(Q1-λ1*Qk)^2	(Q2-λ2*Qk)^2	(Q3-λ3*Qk)^2									
7	291600	608400	240100									
8		**Variables**			**Objective**							
9	**λ1**	**λ2**	**λ3**		**=SUMPRODUCT(A5:C5;A7:C7)+SUM(A10:C10)*H3**							
10					SUMPRODUCT(array1; [array2]; [array3]; [array4]; ...)							
11												
12	**λ1+λ2+λ3**		**n_VARcomp**									
13	0	**=**	2									

Figure 3.8 –Excel presentation of considered problem 3.2.3 with initial data and formulae of the mathematical model (the pointer is in cell **E9** to show the Excel expression that introduces objective function (3.13))



Set Objective: $E$9

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells:
$A$10:$C$10

Subject to the Constraints:
$A$10:$C$10 = binary
$A$13 = $C$13

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: Evolutionary

Solving Method

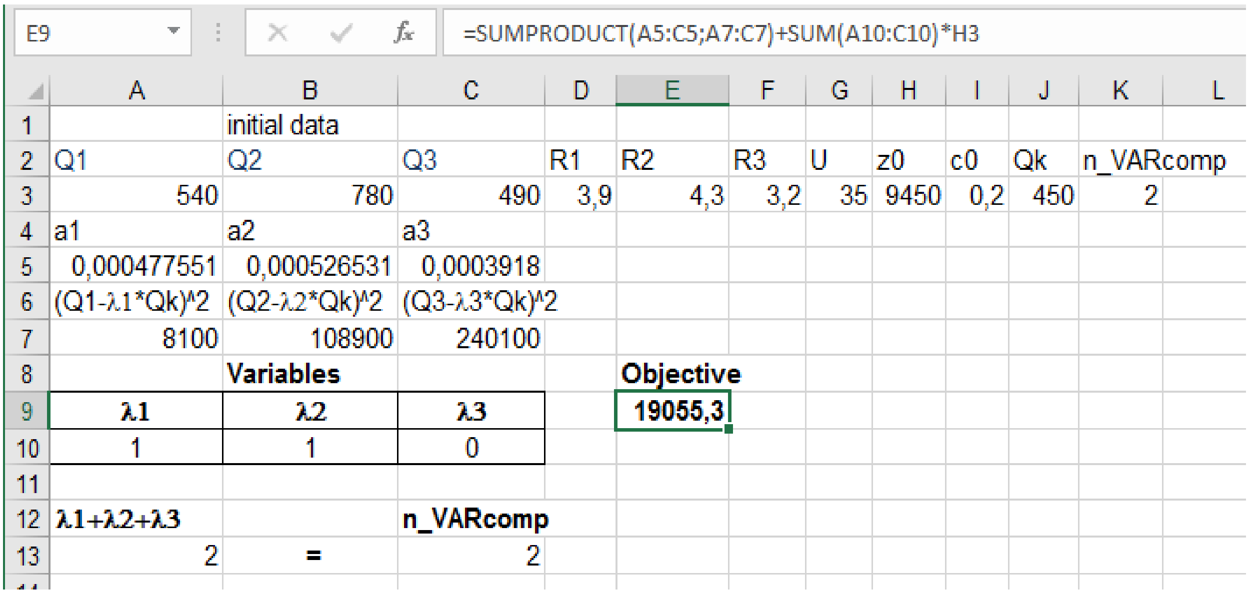
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 3.9 – Settings of **Solver parameters** window for problem 3.2.3

The solution to considered problem 3.2.3 of reactive power compensation capacity distribution in a three-node star-type grid is shown in fig. 3.10. The optimal configuration of reactive power compensation is given in fig. 3.11.

Analysis of the obtained solution. The optimal location of two 450 kVAr compensators are node 1 and node 2, which results from their high reactive power consumption and higher line resistance.

Comparing the total costs of reactive power compensation for the same grid but different conditions of VAR compensation, we can conclude that it is distribution of available compensating capacity among all the nodes depending on their reactive power consumption that allows more decrease in total active power loss in the grid.



E9

=SUMPRODUCT(A5:C5;A7:C7)+SUM(A10:C10)*H3

	A	B	C	D	E	F	G	H	I	J	K	L
1		initial data										
2	Q1	Q2	Q3	R1	R2	R3	U	z0	c0	Qk	n_VARcomp	
3	540	780	490	3,9	4,3	3,2	35	9450	0,2	450	2	
4	a1	a2	a3									
5	0,000477551	0,000526531	0,0003918									
6	(Q1-λ1*Qk)^2	(Q2-λ2*Qk)^2	(Q3-λ3*Qk)^2									
7	8100	108900	240100									
8		**Variables**			**Objective**							
9	**λ1**	**λ2**	**λ3**		**19055,3**							
10	1	1	0									
11												
12	**λ1+λ2+λ3**		**n_VARcomp**									
13	2	**=**	2									

Figure 3.10 – The solution to problem 3.2.3 of optimal location of two VAR compensators in the three-node star-type electric grid shown in fig. 3.3

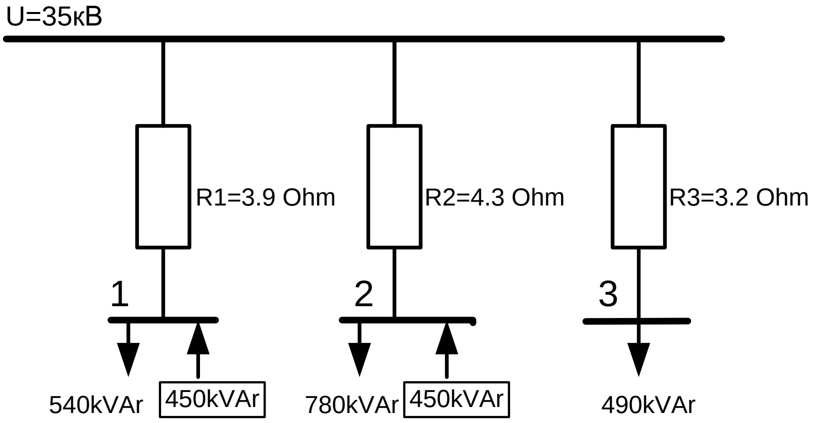


Figure 3.11 – Graphical presentation of the optimal location of two 450 kVAr compensator in the three-node grid

3.3. Reactive power compensation in a series distribution grid

3.3.1. Condition of optimal reactive power compensation in series grid [4]

Let us consider an electric grid (fig. 3.12) with a power source of voltage V that supplies electricity to n series-connected consumers with reactive power load Q_1, Q_2, … Q_n. Like in problem 3.2, it is required to optimally distribute the available compensation capacity $Q_{k\Sigma}$ among the consumers so as to obtain maximum possible decrease in active power loss resulted from reactive power consumption in the grid.

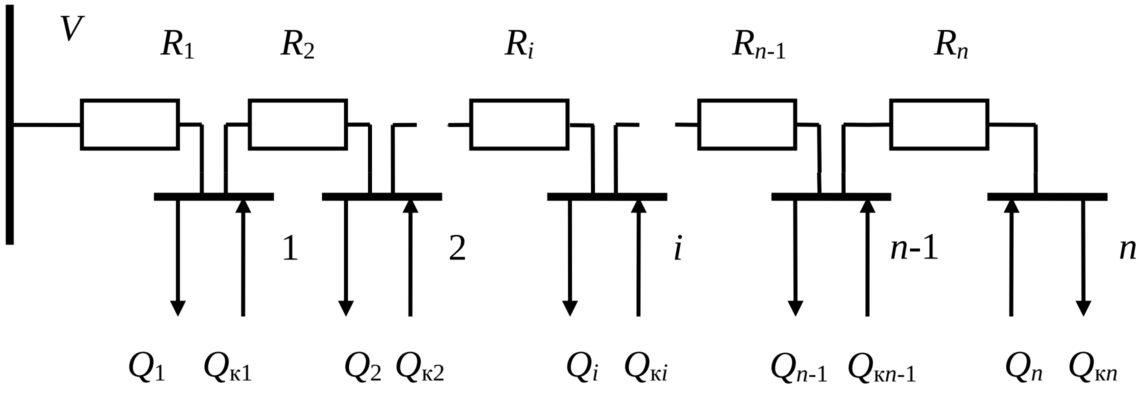


Figure 3.12 – An electricity distribution grid with series-connected nodes [4]

In a distribution grid with series-connected nodes, active power loss at the *i*-th node is affected by power consumption at this and farther located load nodes up to the last one.

For example, active power loss at the 1st node is affected by power consumption at the 1st, 2nd, and farther up to *n*-th, nodes. Active power loss at the 2nd load is affected by power consumption at the 2nd, 3rd, …, *n*-th nodes. And so on. Active power loss at the last, *n*-th, node is only affected by the power consumption at this node. The total power loss in the distribution grid is sum of power loss at all the series-connected nodes.

Therefore, the **objective** function to minimize is

$$\begin{aligned}\Delta P_{loss} = R_1/V^2\Big(\sum_1^n Q_i - \sum_1^n Q_{ki}\Big)^2 + R_2/V^2\Big(\sum_2^n Q_i - \sum_2^n Q_{ki}\Big)^2 + \\ + R_3/V^2\Big(\sum_3^n Q_i - \sum_3^n Q_{ki}\Big)^2 + \ldots + R_n/V^2(Q_n - Q_{kn})^2 \to \min\end{aligned} . \qquad (3.15)$$

The **constraint** is

$$\sum_{i=1}^{n} Q_{ki} = Q_{k\Sigma}$$

or

$$\sum_{i=1}^{n} Q_{ki} - Q_{k\Sigma} = 0 . \qquad (3.16)$$

To determine the condition of optical reactive power compensation capacity distribution in a bulk power supply system, we compose Lagrange function:

$$\begin{aligned} L = {} & R_1/V^2(\sum_1^n Q_i - \sum_1^n Q_{ki})^2 + R_2/V^2(\sum_2^n Q_i - \sum_2^n Q_{ki})^2 + \\ & + R_3/V^2(\sum_3^n Q_i - \sum_3^n Q_{ki})^2 + \ldots + R_n/V^2(Q_n - Q_{kn})^2 + \lambda \cdot (\sum_{i=1}^n Q_{ki} - Q_{k\Sigma}) \to \min \end{aligned} \quad . \tag{3.17}$$

Minimum of Lagrange function (3.17) can be found by determining its derivatives over all the variables (compensation capacities Q_{ki}, $i = 1,..n$, and Lagrange multiplier λ) and setting them to 0.

$$\begin{aligned} & \frac{\partial L}{\partial Q_{k1}} = -2R_1/V^2(\sum_1^n Q_i - \sum_1^n Q_{ki}) + \lambda = 0 \\ & \frac{\partial L}{\partial Q_{k2}} = -2R_1/V^2(\sum_1^n Q_i - \sum_1^n Q_{ki}) - 2R_2/V^2(\sum_2^n Q_i - \sum_2^n Q_{ki}) + \lambda = 0 \\ & \frac{\partial L}{\partial Q_{k3}} = -2R_1/V^2(\sum_1^n Q_i - \sum_1^n Q_{ki}) - 2R_2/V^2(\sum_2^n Q_i - \sum_2^n Q_{ki}) - \\ & \qquad\qquad\qquad - 2R_3/V^2(\sum_3^n Q_i - \sum_3^n Q_{ki}) + \lambda = 0 \end{aligned}$$

……

$$\begin{aligned} & \frac{\partial L}{\partial Q_{ki}} = -2R_1/V^2(\sum_1^n Q_i - \sum_1^n Q_{ki}) - 2R_2/V^2(\sum_2^n Q_i - \sum_2^n Q_{ki}) - \ldots \\ & \qquad\qquad\qquad \ldots - 2R_i/V^2(\sum_i^n Q_i - \sum_i^n Q_{ki}) + \lambda = 0 \end{aligned}$$

……

$$\begin{aligned} & \frac{\partial L}{\partial Q_{k\,n-1}} = -2R_1/V^2(\sum_1^n Q_i - \sum_1^n Q_{ki}) - 2R_2/V^2(\sum_2^n Q_i - \sum_2^n Q_{ki}) - \ldots \\ & \quad - 2R_{n-1}/V^2(Q_{n-1} + Q_n - Q_{k\,n-1} - Q_{k\,n}) + \lambda = 0 \\ & \frac{\partial L}{\partial Q_{k\,n}} = -2R_1/V^2(\sum_1^n Q_i - \sum_1^n Q_{ki}) - 2R_2/V^2(\sum_2^n Q_i - \sum_2^n Q_{ki}) - \ldots \\ & \quad - 2R_n/V^2(Q_n - Q_{k\,n}) + \lambda = 0 \\ & \frac{\partial L}{\partial \lambda} = Q_{k1} + Q_{k2} + \ldots + Q_{kn} - Q_{k\Sigma} = 0 \end{aligned} \quad . \tag{3.18}$$

From the 1st equation of (3.18) we obtain

$$2R_1/V^2(\sum_{i=1}^{n}Q_i - \sum_{i=1}^{n}Q_{ki}) = \lambda. \tag{3.19}$$

Substituting (3.19) into the 2nd equation of (3.18), we obtain

$$-\lambda - 2R_2/V^2(\sum_{i=2}^{n}Q_i - \sum_{i=2}^{n}Q_{ki}) + \lambda = 0$$

or

$$(\sum_{i=2}^{n}Q_i - \sum_{i=2}^{n}Q_{ki}) = 0. \tag{3.20}$$

Substituting (3.19) and (3.20) into the 3rd equation of (3.18), we obtain

$$-\lambda - 2R_2/V^2 \cdot 0 - 2R_3/V^2(\sum_{i=3}^{n}Q_i - \sum_{i=3}^{n}Q_{ki}) + \lambda = 0$$

or

$$(\sum_{i=3}^{n}Q_i - \sum_{i=3}^{n}Q_{ki}) = 0. \tag{3.21}$$

Going on this way, in the 3rd equation from the end of (3.18) we obtain

$$Q_{n-1} + Q_n - Q_{k\,n-1} - Q_{k\,n} = 0, \tag{3.22}$$

and substituting (3.22) to the 2nd equation from the end of (3.18), we obtain

$$Q_n - Q_{k\,n} = 0$$

or

$$Q_{k\,n} = Q_n. \tag{3.23}$$

Equation (3.23) describes the requirement for complete compensation of reactive power consumption at the end, *n*-th, node in the series grid.

Substituting (3.23) to (3.22), we obtain the same requirement of complete compensation of VAR consumption at the last but one, (*n*-1)-th, node

$$Q_{n-1} = Q_{k\,n-1}. \tag{3.24}$$

And so on, until the available compensation capacity is installed.

Therefore, **the condition of optimal reactive power compensation** in a series distribution grid is the following.

VAR compensation capacity must be distributed from the end consumer with complete compensation of its VAR consumption towards the beginning of the grid, to the first consumer (from the *n*-th node to the 1st node), until all the available VAR compensation capacity is installed (the constraint $\sum_{i=1}^{n} Q_{ki} = Q_{k\Sigma}$ is fulfilled).

If the available VAR compensation capacity finishes at the *i*-th node, the reactive power consumption remains uncompensated at the 1st, 2nd, …, (*i*-1)st nodes.

3.3.2. Optimal distribution of available compensation capacity in a series electric grid

Let us consider an example of optimal VAR compensation capacity distribution in a series electric grid.

The grid (fig. 3.13) connects three nodes with the same reactive power consumption, Q_1, Q_2, and Q_3, as the VAR consumers in the star-type grid shown in fig. 3.3.

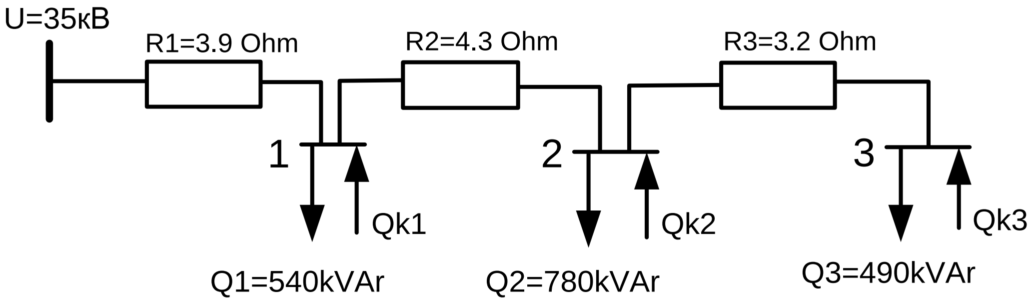


Figure 3.13 – An electricity supply system with three series-connected nodes

The initial electrical parameters of the series grid (given in fig. 3.13) are identical to those of the star-type grid. Specific costs of reactive power compensation installation and specific costs on active power loss coverage are also the same, $z_0 = 21$ \$/kVAr and $c_0 = 0.15$ \$/kVAr, respectively.

It is required to optimally distribute the available reactive power compensation capacity of 900 kVAr among the VAR consumers connected in series so as to decrease active power loss and minimize the total costs of VAR compensation in the grid.

The mathematical model of the problem is the following.

The **decision variables** are VAR compensation capacities $Q_{к1}$, $Q_{к2}$, and $Q_{к3}$ that must be installed in nodes 1, 2 and 3, correspondingly.

The objective function is sum of $Q_{к1}$, $Q_{к2}$, and $Q_{к3}$ installation costs and costs resulted from coverage of active power loss cause by reactive power consumption.

The first component of the objective function is $z_0 \cdot (Q_{k1} + Q_{k2} + Q_{k3})$.

Power loss at node 1 is function of VAR consumption at node 1 and farther located nodes 2 and 3. Power loss at node 2 depends on reactive power consumption at node 2 and farther located nodes 3. Power loss at node 3 is function of reactive power consumption at node 3 only. Therefore, the other component is sum of active power loss at every node $(\sum_i^3 Q_i - \sum_i^3 Q_{ki})^2 R_i / V^2$ multiplied by the loss coverage cost c_0:

$$c_0 \cdot (Q_1 + Q_2 + Q_3 - Q_{k1} - Q_{k2} - Q_{k3})^2 R_1 / V^2 + c_0 \cdot (Q_2 + Q_3 - Q_{k2} - Q_{k3})^2 R_2 / V^2 +$$
$$+ c_0 \cdot (Q_3 - Q_{k3})^2 R_3 / V^2.$$

The **objective** function is

$$\begin{gathered} Z = z_0 \cdot (Q_{k1} + Q_{k2} + Q_{k3}) + c_0 \cdot (Q_1 + Q_2 + Q_3 - Q_{k1} - Q_{k2} - Q_{k3})^2 R_1 / V^2 + \\ + c_0 \cdot (Q_2 + Q_3 - Q_{k2} - Q_{k3})^2 R_2 / V^2 + c_0 \cdot (Q_3 - Q_{k3})^2 R_3 / V^2 \to \min \end{gathered} . \quad (3.25)$$

The **constraint** is equality of the total VAR compensation capacity distributed among the VAR consumers in the grid to the available VAR compensation capacity:

$$Q_{k1} + Q_{k2} + Q_{k3} = Q_{k\Sigma}. \quad (3.26)$$

The condition of optimal reactive power compensation distribution among series-connected consumers, including (3.23) and (3.24), are not included into the mathematical models as constraints. It is taken into account when we analyze the obtained solution.

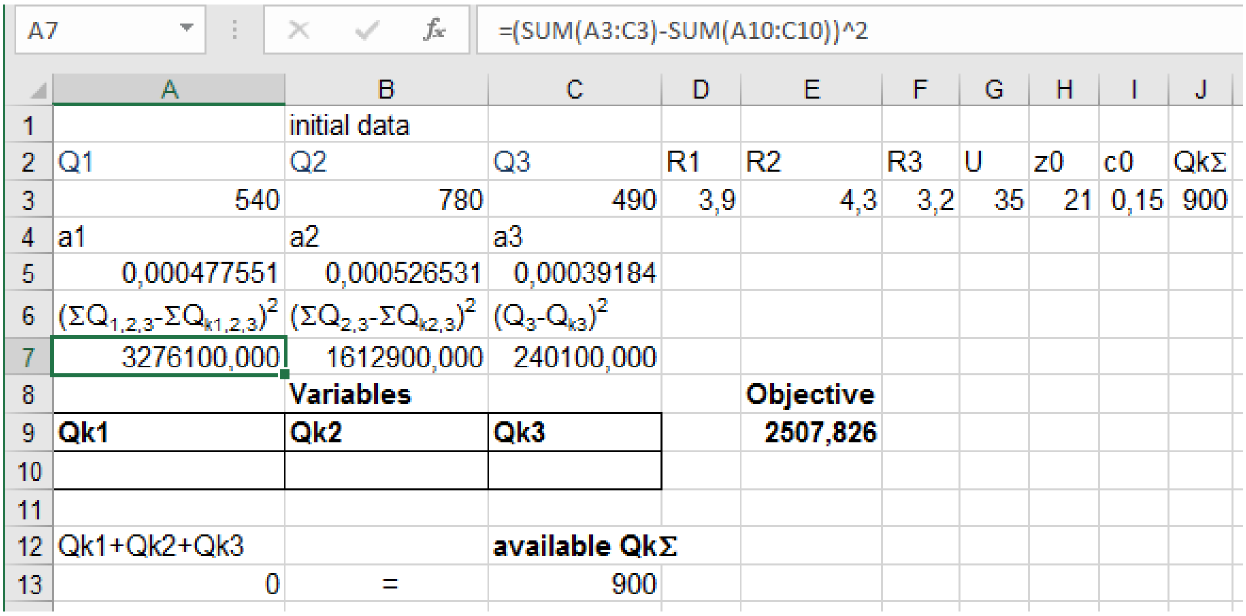
After substitution of the initial data, the mathematical model is as follows.

$$\begin{aligned} Z = {} & 21\cdot(Q_{k1}+Q_{k2}+Q_{k3})+ \\ & +0.15\cdot 3.9/35^2\cdot(540+780+490-Q_{k1}-Q_{k2}-Q_{k3})^2+ \\ & +0.15\cdot 4.3/35^2\cdot(780+490-Q_{k2}-Q_{k3})^2+ \\ & +0.15\cdot 3.2/35^2\cdot(490-Q_{k3})^2\rightarrow \min . \end{aligned} \qquad (3.27)$$

$$Q_{k1}+Q_{k2}+Q_{k3}=900$$
$$Q_{ki}\geq 0,\qquad i=1,2,3$$

The problem is solved with the help of MS Excel Solver.

The Excel presentation of mathematical model (3.27) of the considered problem is shown in fig. 3.14. The initial data are introduced in cells **A3:J3**. The decision variables Q_{ki} are introduced in cells **A10:C10**. The objective function (3.25) is written in cell **E9**.



A7 =(SUM(A3:C3)-SUM(A10:C10))^2

	A	B	C	D	E	F	G	H	I	J
1		initial data								
2	Q1	Q2	Q3	R1	R2	R3	U	z0	c0	QkΣ
3	540	780	490	3,9	4,3	3,2	35	21	0,15	900
4	a1	a2	a3							
5	0,000477551	0,000526531	0,00039184							
6	$(\Sigma Q_{1,2,3}-\Sigma Q_{k1,2,3})^2$	$(\Sigma Q_{2,3}-\Sigma Q_{k2,3})^2$	$(Q_3-Q_{k3})^2$							
7	3276100,000	1612900,000	240100,000							
8		**Variables**			**Objective**					
9	**Qk1**	**Qk2**	**Qk3**		**2507,826**					
10										
11										
12	Qk1+Qk2+Qk3		**available QkΣ**							
13	0	=	900							

Figure 3.14 –Excel presentation of problem 3.3.2 with initial data and intermediate formulae (the pointer is in cell **A7** to show Excel expression for calculating intermediate expression $(540+780+490-Q_{k1}-Q_{k2}-Q_{k3})^2$ for the 1st VAR consumer in the grid)

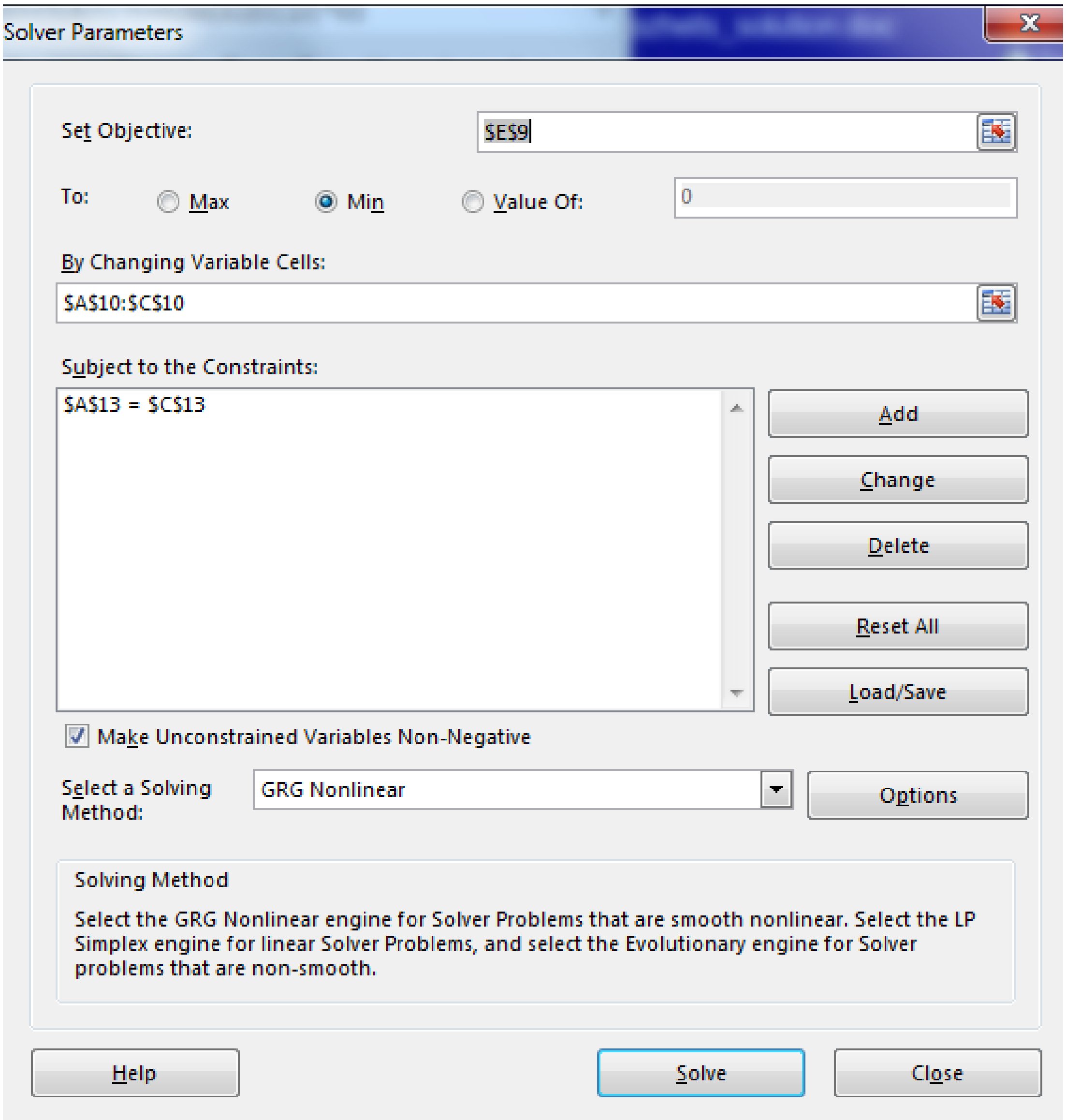
Excel formulae for calculating intermediate expressions of mathematical model (3.27) are given in table 3.2.

Table 3.2 – Excel expressions for intermediate calculations of (3.27)

Mathematical item	Cell	Excel expression
Coefficients a_i		
$a_1 = c_0 \times R_1 / V^2$	**A5**	**=$I$3*D3/($G$3^2)**
$a_2 = c_0 \times R_2 / V^2$	**B5**	**=$I$3*E3/($G$3^2)**
$a_3 = c_0 \times R_3 / V^2$	**C5**	**=$I$3*F3/($G$3^2)**
Brackets $(\sum_i^3 Q - \sum_i^3 Q_{ki})^2$		
$(540 + 780 + 490 - Q_{k1} - Q_{k2} - Q_{k3})^2$	**A7**	**=(SUM(A3:C3)-SUM(A10:C10))^2**
$(780 + 490 - Q_{k2} - Q_{k3})^2$	**B7**	**=(SUM(B3:C3)-SUM(B10:C10))^2**
$(490 - Q_{k3})^2$	**C7**	**=(SUM(C3)-SUM(C10))^2**
Sum of the reactive power compensation capacities installed at the nodes		
left side $Q_{k1} + Q_{k2} + Q_{k3}$	**A13**	**=SUM(A10:C10)**
right side $Q_{k\Sigma}$=900	**C13**	**=J3**
Objective		
	E9	**=H3*SUM(A10:C10)+SUMPRODUCT(A5:C5;A7:C7)**

The **Solver** parameters for solving the problem are shown in fig. 3.15.

The solution to considered problem 3.3.2 of VAR compensation capacity distribution in a three-node series grid is shown in fig. 3.16. Graphical presentation of the optimal distribution of available 900 kVAr compensation among the three series connected nodes is given in fig. 3.17.



Solver Parameters

Set Objective: $E$9

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $A$10:$C$10

Subject to the Constraints:

$A$13 = $C$13

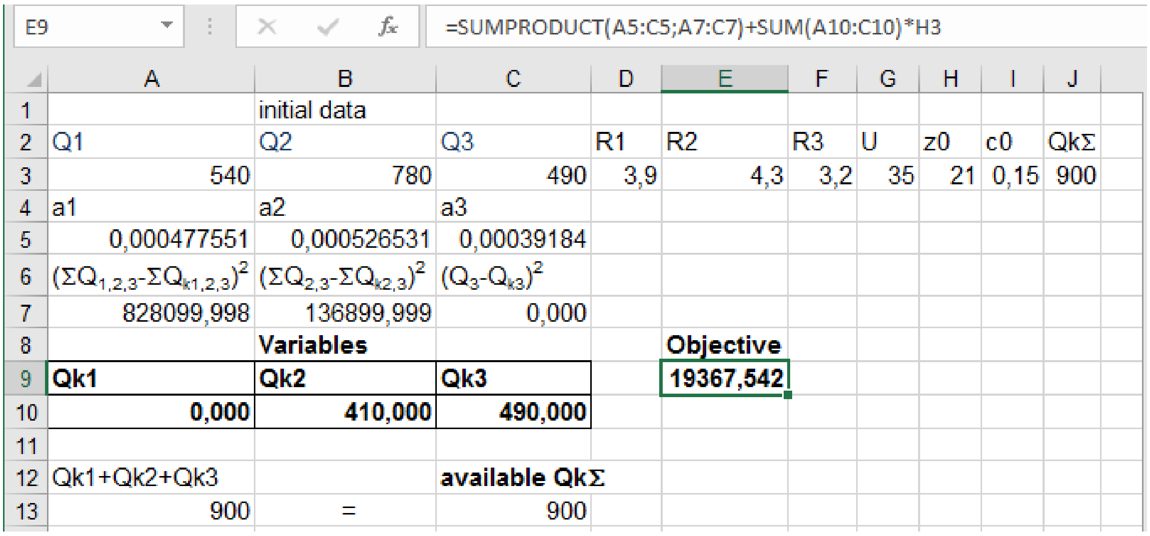
☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 3.15 – Settings of **Solver parameters** window for problem 3.3.2



E9 =SUMPRODUCT(A5:C5;A7:C7)+SUM(A10:C10)*H3

	A	B	C	D	E	F	G	H	I	J
1		initial data								
2	Q1	Q2	Q3	R1	R2	R3	U	z0	c0	QkΣ
3	540	780	490	3,9	4,3	3,2	35	21	0,15	900
4	a1	a2	a3							
5	0,000477551	0,000526531	0,00039184							
6	$(\Sigma Q_{1,2,3}-\Sigma Q_{k1,2,3})^2$	$(\Sigma Q_{2,3}-\Sigma Q_{k2,3})^2$	$(Q_3-Q_{k3})^2$							
7	828099,998	136899,999	0,000							
8		**Variables**			**Objective**					
9	**Qk1**	**Qk2**	**Qk3**		**19367,542**					
10	**0,000**	**410,000**	**490,000**							
11										
12	Qk1+Qk2+Qk3		**available QkΣ**							
13	900	=	900							

Figure 3.16 – The solution to problem 3.3.2 of optimal VAR compensation in the three-node serial grid shown in fig. 3.13

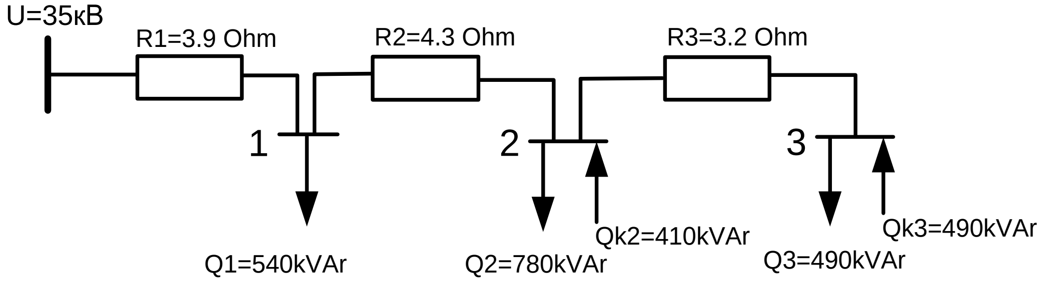


Figure 3.17 – Optimal distribution of VAR compensation capacity in problem 3.3.2

Analysis of the obtained solution. According to the obtained optimal solution, 490 kVAr out of available 900 kVAr compensation capacity must be installed at node 3 to compensate its 490 kVAr consumption. The remaining 410 kVAr of the available compensation capacity must be installed at node 2 to partly compensate its 780 kVAr consumption. At node 1, reactive power consumed is not compensated at all.

Analyzing the total costs of VAR compensation in the considered serial grid that are equal to 19367.542 $ and comparing them with costs of VAR compensation in the star-type grid with identical VAR consumers (19026.518 $, see fig. 3.6), we can conclude that reactive power compensation is more efficient in the star-type grid.

3.3.3. Optimal location of VAR compensators in a series grid

Let us find the optimal location of a 900 kVAr compensator in the three-node serial grid shown in fig. 3.13. The cost of 900 kVAr compensator installation z_0=18900 $; the specific costs on active power loss coverage c_0=0.15 $/kVAr. The objective is minimization of overall costs of VAR compensation.

It is a problem with Boolean variables. The grid connects three nodes 1, 2, and 3 in series. The 900 kVAr compensator can be installed at any of them.

Therefore, the decision variables in this problem are Boolean variables λ_i, i = 1, 2, 3, that denote possibilities of the compensator installation at the node:

λ_1 is a possibility of 900 kVAr compensator installation at node1;

λ_2 is a possibility of 900 kVAr compensator installation at node2;

λ_3 is a possibility of 900 kVAr compensator installation at node3.

The compensation capacity at any node will be

$$Q_{ki} = \lambda_i \cdot Q_k = \lambda_i \cdot 900 = \begin{cases} 900 \;\; if\ the\ compensator\ is\ installed\ at\ node\ i \\ 0 \;\; if\ the\ compensator\ is\ not\ installed\ at\ node\ i \end{cases}.$$

The mathematical model of problem 3.3.3 is the following.

The objective function is sum of the compensator installation cost and costs resulted from active power loss coverage.

The first component of the objective function is $z_0 \cdot (\lambda_1 \cdot Q_k + \lambda_2 \cdot Q_k + \lambda_3 \cdot Q_k)$.

The other component of the objective function is sum of active power loss at every node $(\sum_i^3 Q_i - \sum_i^3 (\lambda_i \cdot Q_{ki}))^2 R_i / V^2$ multiplied by the loss coverage cost c_0:

$$\begin{aligned} &c_0 \cdot (Q_1 + Q_2 + Q_3 - \lambda_1 \cdot Q_k - \lambda_2 \cdot Q_k - \lambda_3 \cdot Q_k)^2 R_1 / V^2 + \\ &+ c_0 \cdot (Q_2 + Q_3 - \lambda_2 \cdot Q_k - \lambda_3 \cdot Q_k)^2 R_2 / V^2 + c_0 \cdot (Q_3 - \lambda_3 \cdot Q_k)^2 R_3 / V^2 \end{aligned}.$$

Therefore, the **objective** function is

$$\begin{aligned} Z = {} & z_0 \cdot (\lambda_1 \cdot Q_k + \lambda_2 \cdot Q_k + \lambda_3 \cdot Q_k) + \\ & + a_1 \cdot (Q_1 + Q_2 + Q_3 - \lambda_1 \cdot Q_k - \lambda_2 \cdot Q_k - \lambda_3 \cdot Q_k)^2 + \\ & + a_2 \cdot (Q_2 + Q_3 - \lambda_2 \cdot Q_k - \lambda_3 \cdot Q_k)^2 + a_3 \cdot (Q_3 - \lambda_3 \cdot Q_k)^2 \rightarrow \min \end{aligned}, \qquad (3.28)$$

where

$a_1 = c_0 \times R_1 / V^2$; $a_2 = c_0 \times R_2 / V^2$; $a_3 = c_0 \times R_3 / V^2$.

The **constraint** is equality of the possible locations of the 900 kVAr compensator installation to the available number of them

$$\lambda_1 + \lambda_2 + \lambda_3 = 1.$$

In case the VAR compensator capacity Q_k is higher than the reactive power consumption Q_n at the nth (end) node, it is necessary to add a **constraint** that prevents overcompensation at this node:

$$Q_n - \lambda_n \cdot Q_k \geq 0 . \tag{3.29}$$

If the capacity of the VAR compensator Q_k is bigger than reactive power consumed all together at the n-th (end) and (n-1)-st (end but one) nodes, a constraint preventing overcompensation at the two last nodes must be added

$$Q_{n-1} + Q_n - \lambda_{n-1} \cdot Q_k - \lambda_n \cdot Q_k \geq 0, \tag{3.30}$$

and so on.

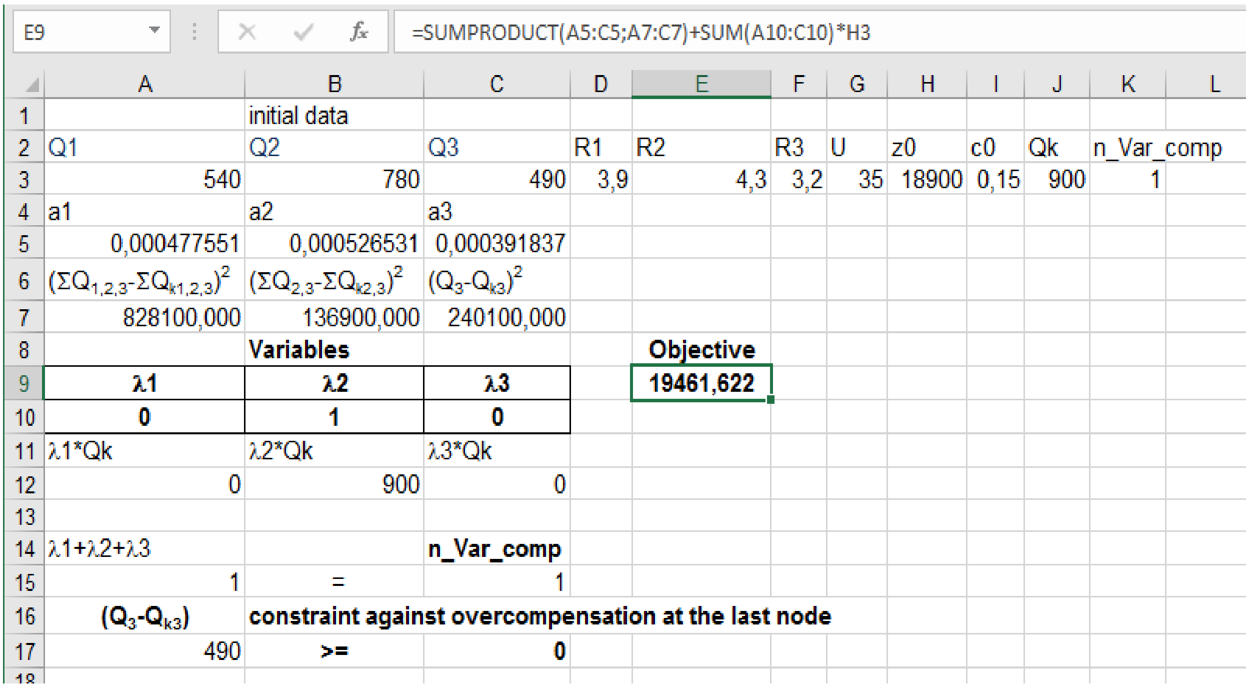
In our problem, the reactive power consumed at node 3 is 490kVAr which is lower than the VAR compensator capacity of 900 kVAr so we must add constraint (3.29).

The reactive power consumption at node 2 and node 3 is (780 + 490 = 1270) kVAr, which is bigger than 900 kVAr so constraint (3.30) may be omitted.

Substituting the initial data, we obtain the mathematical model in numerical form

$$\begin{gathered}
Z = z_0 \cdot (\lambda_1 \cdot 900 + \lambda_2 \cdot 900 + \lambda_3 \cdot 900) + \\
+ a_1 \cdot (Q_1 + Q_2 + Q_3 - \lambda_1 \cdot 900 - \lambda_2 \cdot 900 - \lambda_3 \cdot 900)^2 + \\
+ a_2 \cdot (Q_2 + Q_3 - \lambda_2 \cdot 900 - \lambda_3 \cdot 900)^2 + a_3 \cdot (Q_3 - \lambda_3 \cdot 900)^2 \to \min \\
\lambda_1 + \lambda_2 + \lambda_3 = 1 \\
Q_3 - \lambda_3 \cdot 900 \geq 0 \\
\lambda_i = Boolean, \quad i = 1,2,3
\end{gathered} . \tag{3.31}$$

The Excel solution to problem 3.3.3 of the optimal location of a 900 kVAr compensator in the three-node serial electric grid is shown in fig. 3.18. The graphical presentation of the optimal VAR compensation is given in fig. 3.19.



E9 =SUMPRODUCT(A5:C5;A7:C7)+SUM(A10:C10)*H3

	A	B	C	D	E	F	G	H	I	J	K	L
1		initial data										
2	Q1	Q2	Q3	R1	R2	R3	U	z0	c0	Qk	n_Var_comp	
3	540	780	490	3,9	4,3	3,2	35	18900	0,15	900	1	
4	a1	a2	a3									
5	0,000477551	0,000526531	0,000391837									
6	$(\Sigma Q_{1,2,3}-\Sigma Q_{k1,2,3})^2$	$(\Sigma Q_{2,3}-\Sigma Q_{k2,3})^2$	$(Q_3-Q_{k3})^2$									
7	828100,000	136900,000	240100,000									
8		**Variables**			**Objective**							
9	**λ1**	**λ2**	**λ3**		**19461,622**							
10	**0**	**1**	**0**									
11	λ1*Qk	λ2*Qk	λ3*Qk									
12	0	900	0									
13												
14	λ1+λ2+λ3		**n_Var_comp**									
15	1	=	1									
16	**(Q_3-Q_{k3})**	**constraint against overcompensation at the last node**										
17	490	**>=**	**0**									

Figure 3.18 – The solution to problem 3.3.3 of optimal location of a 900 kVAr compensator in the three-node serial grid shown in fig. 3.13

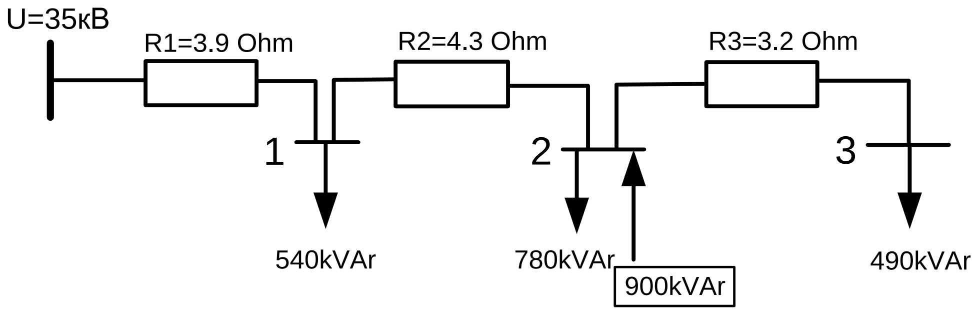


Figure 3.19 – Optimal location of 900 kVAr compensator in problem 3.3.3

Analysis of the obtained solution. The optimal location of the available 900 kVAr compensator is node 2 which is the last but one node in the serial grid. As the installation of the compensator at node 3 is forbidden, the next node to install the compensation capacity is node 2.

The costs of reactive power compensation with one powerful VAR compensator are higher than those made for installation of smaller-capacity VAR compensators of the same total capacity at several nodes (19461,62 $ vs 19367.54 $), we can conclude that distribution of available compensating capacity among separate nodes is more efficient.

3.4. Reactive power compensation in a hybrid distribution grid

Let us consider an example of optimal VAR compensation capacity distribution in a series-star electric grid [4].

The grid (fig. 3.20) connects three nodes with the same reactive power consumption, Q_1, Q_2, and Q_3, as the VAR consumers in the star-type grid and series grid shown in fig. 3.3. and fig. 3.13, respectively. Nodes 2 and 3 are connected in parallel to each other while both of them are connected in series to node 1.

The electric parameters are shown in the figure. It is required to determine the distribution of available reactive power compensation 900 kVAr among nodes 1, 2 and 3. so as to minimize the active power loss in the grid.

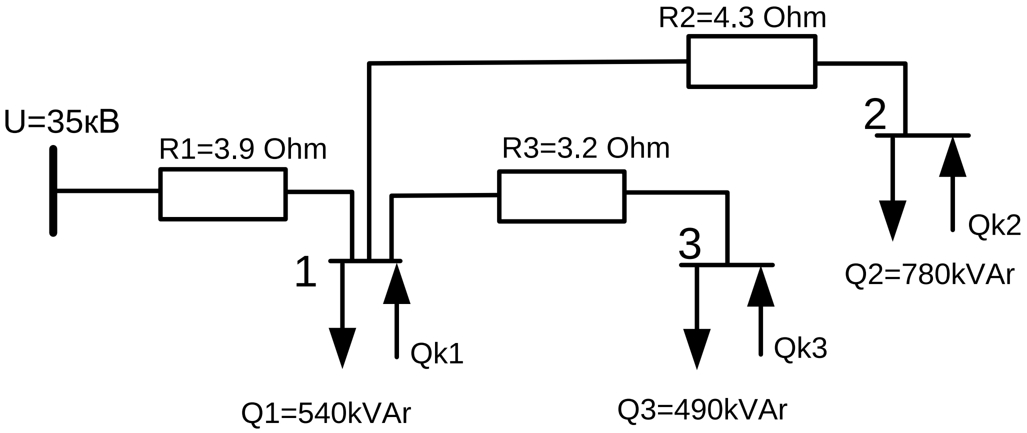


Figure 3.20 – Three-node hybrid series-star electric grid

The mathematical model of the problem is the following.

Nodes 2 and 3 are located in parallel to each other. Active power loss caused by reactive power consumption at these nodes is equal to $\Delta P_{loss2} = (Q_2 - Q_{k2})^2 \cdot R_2 / V^2$ and $\Delta P_{loss3} = (Q_3 - Q_{k3})^2 \cdot R_3 / V^2$, correspondingly.

Active power loss at node 1 is affected by reactive power consumption at node 2 and 3 which are series-connected to node 1 so $\Delta P_{loss1} = (Q_1 + Q_2 + Q_3 - Q_{k1} - Q_{k2} - Q_{k3})^2 \cdot R_1 / V^2$.

The **objective** function is minimization of the active power loss caused by reactive power consumption at all the nodes:

$$\begin{aligned} Z = \Delta P_{loss} = a_1(Q_1 + Q_2 + Q_3 - Q_{k1} - Q_{k2} - Q_{k3})^2 + \\ + a_2(Q_2 - Q_{k2})^2 + a_3(Q_3 - Q_{k3})^2 \to \min \end{aligned} \tag{3.32}$$

where $a_1 = R_1 / V^2,\ a_2 = R_2 / V^2, a_3 = R_3 / V^2$

The **constraints** are:

– equality of the total VAR compensation capacity distributed among the VAR consumers in the grid to the available VAR compensation capacity:

$$Q_{k1} + Q_{k2} + Q_{k3} = Q_{k\Sigma}; \tag{3.33}$$

- the condition (3.10) of the optimal reactive power compensation capacity distribution for parallel nodes 2 and 3:

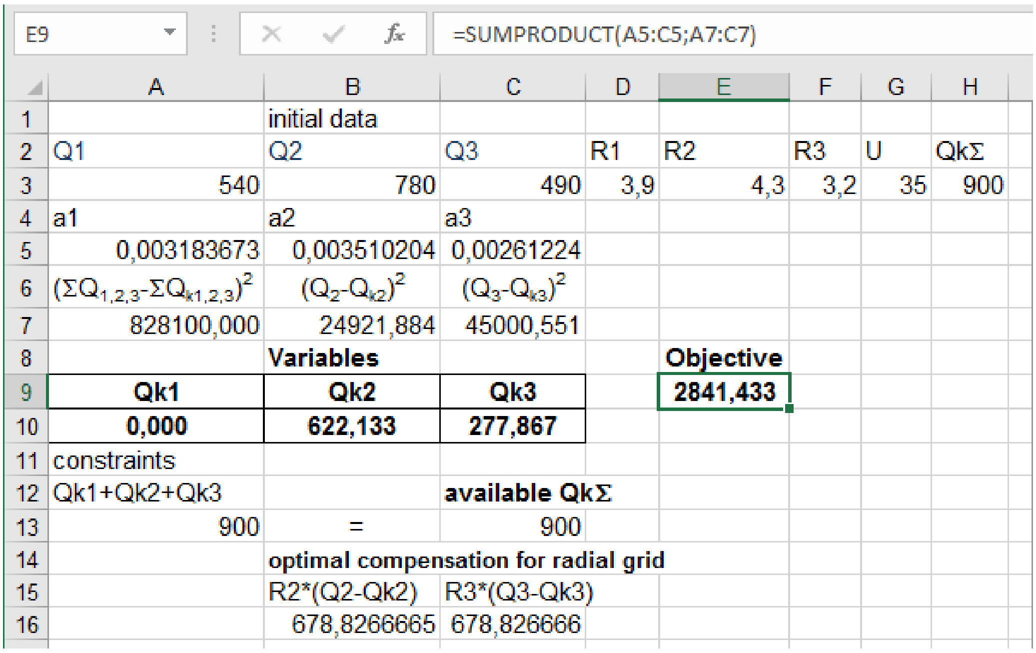
$$R_2(Q_2 - Q_{k2}) = R_3(Q_3 - Q_{k3}). \tag{3.34}$$

Substituting the initial data, we obtain

$$\begin{gathered} Z = 3.9/35^2 \cdot (540 + 780 + 490 - Q_{k1} - Q_{k2} - Q_{k3})^2 + \\ + 4.3/35^2 \cdot (780 - Q_{k2})^2 + 3.2/35^2 \cdot (490 - Q_{k3})^2 \to \min \\ Q_{k1} + Q_{k2} + Q_{k3} = 900 \\ 4.3 \cdot (780 - Q_{k2}) = 3.2 \cdot (490 - Q_{k3}) \\ Q_{ki} \geq 0, \quad i = 1,2,3. \end{gathered} \tag{3.35}$$

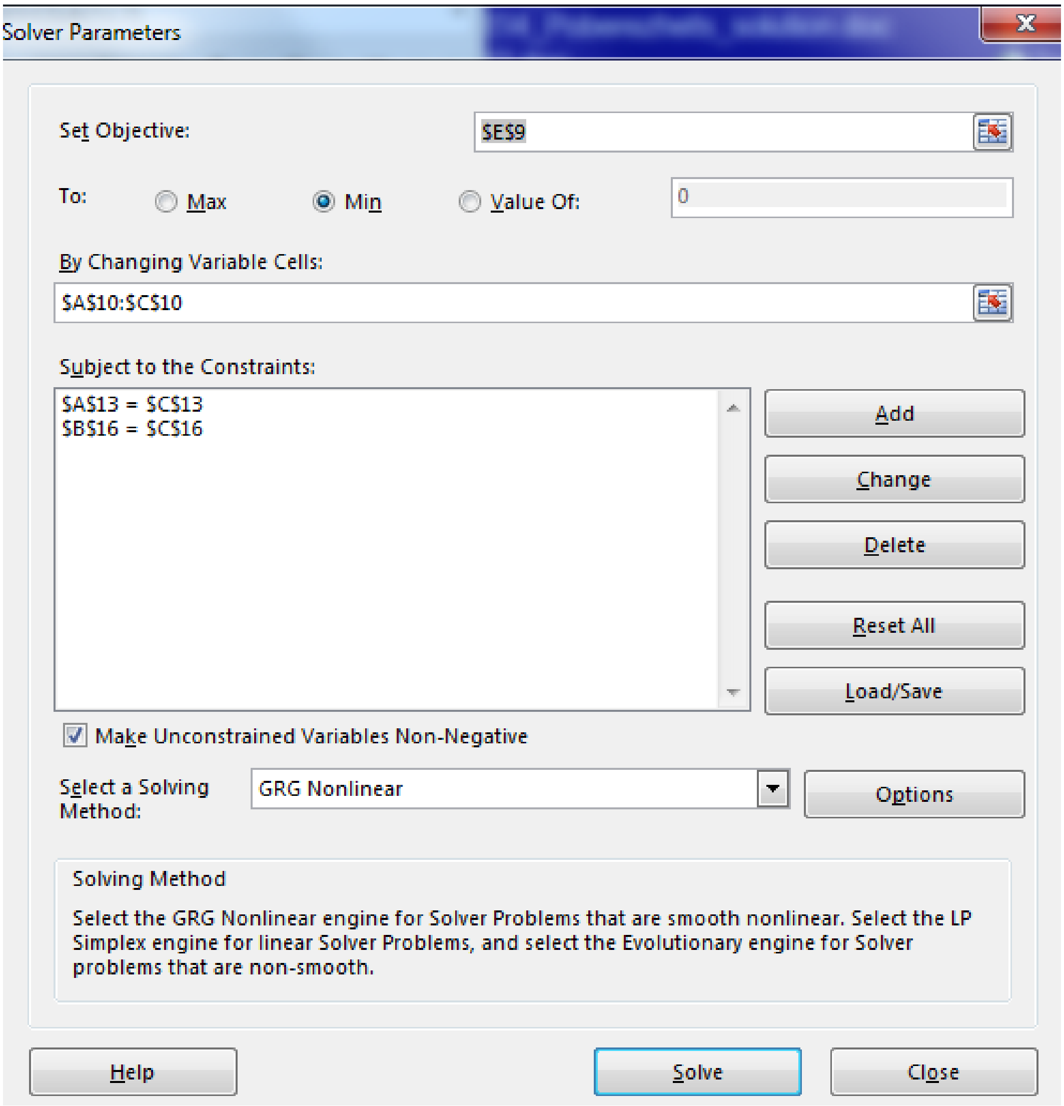
The Excel solution to this problem is given in fig. 3.21. The initial data are introduced in cells **A3:H3**. The decision variables Q_{ki} are introduced in cells **A10:C10**, the objective function Z – in cell **E9**.

The **Solver** parameters are described in fig. 3.22. Fig 3.23 shows the graphic presentation of the optimal distribution of available 900 kVAr of compensating capacity among the three nodes of the hybrid grid.


E9 =SUMPRODUCT(A5:C5;A7:C7)

	A	B	C	D	E	F	G	H
1		initial data						
2	Q1	Q2	Q3	R1	R2	R3	U	QkΣ
3	540	780	490	3,9	4,3	3,2	35	900
4	a1	a2	a3					
5	0,003183673	0,003510204	0,00261224					
6	$(\Sigma Q_{1,2,3}-\Sigma Q_{k1,2,3})^2$	$(Q_2-Q_{k2})^2$	$(Q_3-Q_{k3})^2$					
7	828100,000	24921,884	45000,551					
8		**Variables**			**Objective**			
9	**Qk1**	**Qk2**	**Qk3**		**2841,433**			
10	**0,000**	**622,133**	**277,867**					
11	constraints							
12	Qk1+Qk2+Qk3		**available QkΣ**					
13	900	=	900					
14		**optimal compensation for radial grid**						
15		R2*(Q2-Qk2)	R3*(Q3-Qk3)					
16		678,8266665	678,826666					

Figure 3.21 – The optimal solution to problem 3.4


Solver Parameters

Set Objective: $E$9

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $A$10:$C$10

Subject to the Constraints:

$A$13 = $C$13
$B$16 = $C$16

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 3.22 – Settings of **Solver parameters** window for problem 3.4

The optimal distribution of available 900 kVAr of compensation capacity in the hybrid series-star electric grid considered (see fig. 3.20) is shown in fig. 3.23.

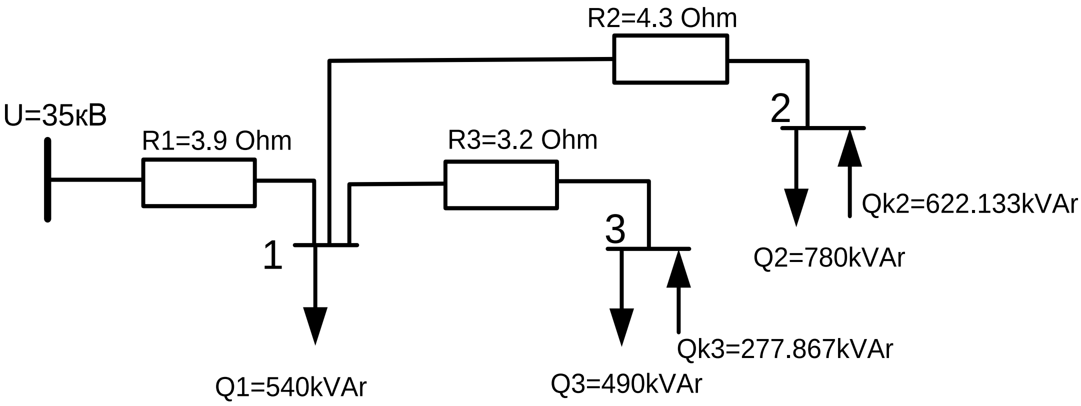


Figure 3.23 – Optimal distribution of 900 kVAr compensation in the three-node hybrid series-star electric grid

Analysis of the results. Node 2 and 3 are located at the end of the hybrid series-star grid so all the available VAR compensating capacity in installed there: 622 kVAr out of the available 900 kVAr compensation capacity must be installed at node 2 to partly compensate 780 kVAr consumption, the remaining 278 kVAr installed at node 3 to partly compensate 490 kVAr consumption. The distribution of the compensating capacity is make in compliance with the optimality condition (3.10).

Node 1 is the first node in the hybrid grid and 540 kVAr consumed there can be compensated only in case there is some of the available VAR compensating capacity left. As all the available 900 kVAr of compensation is installed farther in the grid, there is no reactive power compensation at node 1.

3.5. Assignment

3.5.1. Assignment 1

According to the variant (table 3.3), determine the optimal distribution of available VAR compensation capacity among the nodes so as to minimize the active power loss:

1) in the star-type grid; 2) in the series grid.

Table 3.3 – Initial data for the optimal VAR compensation distribution problem

№ variant	Number of nodes	V, kV	Node resistance, Ω				Reactive load of the nodes, kVAr				$Q_{\square}$
			R_1	R_2	R_3	R_4	Q_1	Q_2	Q_3	Q_4	
1	4	10	0.51	0.43	0.28	0.34	260	350	210	270	880
2	4	35	1.3	1.6	1.2	1.1	490	420	475	405	1100
3	4	110	3.5	2.9	3.2	3.0	680	570	605	590	1500
4	4	10	0.38	0.27	0.21	0.29	310	280	295	240	750
5	4	110	2.9	2.2	1.7	1.9	720	640	680	585	1800
6	4	35	1.02	0.98	0.81	0.94	455	395	430	408	1200

3.5.2. Assignment 2

According to the variant in assignment 3.5.1, determine the optimal location of available VAR compensators (table 3.4) so as to minimize the active power loss:

1) in the star-type grid; 2) in the series grid.

The electric parameters and grid configurations should be taken from table 3.3.

Table 3.4 – Initial data for the optimal VAR compensator location problem

№ variant	V, kV	Reactive load of the nodes, kVAr				VAR compensators
		Q_1	Q_2	Q_3	Q_4	
1	10	260	350	210	270	2×250 kVAr
2	35	390	320	275	350	2×350 kVAr
3	110	580	470	650	510	1×600 kVAr
4	10	310	280	295	240	2×300 kVAr
5	110	510	760	630	920	2×750 kVAr
6	35	750	680	410	650	1×700 kVAr

REFERENCES

1. Allen J. Wood. Power Generation, Operation, and Control. 3rd Edition // Allen J. Wood, Bruce F. Wollenberg, Gerald B. Sheble. – Wiley Interscience, 2013. – 632 p. (Access code: https://www.academia.edu/39673423/Allen_J_Wood_Bruce_F_Wollenberg_Gerald_B_Shebl%C3%A9_Power_Generation_Operation_and_Control_Wiley_Interscience_2013_)
2. Excel Esay. Access code: https://www.excel-easy.com/data-analysis/solver.html
3. Operations Research Simplified. Access code: http://www. universalteacherpublications.com/univ/ebooks/or/index1.htm
4. Kostin, V. N. Optimization problems of power engineering. Manual for students // Saint-Petersburb: SZTU, 2003. – 120 p. (in Russian) (Костин В.Н. Оптимизационные задачи электроэнергетики: Учебное пособие. – СПб.: СЗТУ, 2003. – 120 с.) [Access code: http://window.edu.ru/resource/987/24987]
5. Jie Zhong Zhu. Optimization of Power System Operation // IEEE Press Series on Power Engineering: WILEY. 2015 – 664 p. (Access code: https://www.perlego.com/book/996588/optimization-of-power-system-operation-pdf)
6. V. A. Venikov. Optimization of Power Plant and Power System Operation. Manual for students of higher –educational institutes // Venikov V. A., Zhuravlyov V. G., Filippova G. A. – Energoizdat Publishing House, 1981. – 464 p. (in Russian) (Веников В. А. Оптимизация режимов электростанций и энергосистем: Учебник для вузов / В. А. Веников, В. Г. Журавлев; Г. А. Филиппова. – М.: Энергоиздат, 1981. – 464 с.)

1. M. E. El-Hawary. Optimal economic operation of electric power systems // M. E. El-Hawary, G. S. Christensen – New York : Academic Press, 1979. – 278 p. (Access code: https://www.engineeringbookspdf.com/optimal-economic-operation-of-electric-power-systems-by-m-e-el-hawary-and-g-s-christensen_11301/)
2. Ken Bluttman. Microsoft Excel Formulas and Functions for Dummies. 5th Edition. – WILEY, 2019. – 383 p. (access code: http://booksupport.wiley.com)

CONTENT

Навчальне видання

ЛИСЕНКО Людмила Іванівна
МАХОТІЛО Костянтин Володимирович
ЧЕРКАШИНА Галина Ігорівна

ОПТИМІЗАЦІЙНІ ЗАДАЧІ ДИСПЕТЧЕРСЬКОГО КЕРУВАННЯ РЕЖИМАМИ ЕНЕРГОСИСТЕМ

Навчально-методичний посібник
для студентів спеціальності 141
«Електроенергетика, електротехніка
та електромеханіка»
денної та заочної форми навчання

Відповідальний за випуск проф. Лазуренко О. П.
Роботу до видання рекомендував проф. Любарський Б. Г.

В авторській редакції

Видання англійською мовою

План 2021 р., поз. 5

Підп. до друку 07.07.2021. Формат 60 х 84 / 16.
Гарнітура Times New Roman. Ум. друк. арк. 3,3.
Друк цифровий. Наклад 30 прим. Зам. № 8/07/21.

Видавець та виготовлювач: ФОП Панов А.М.
Свідоцтво серії ДК № 4847 від 06.02.2015 р.
м. Харків, вул. Жон Мироносиць, 10, оф. 6,
тел.: +38(057)714-06-74, +38(050)976-32-87
copy@vlavke.com