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MATHEMATICAL MODEL OF SERVER REQUESTS INTENSITY DESCRIPTION

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ABSTRACT *The paper is devoted to the mathematical model development of non-stationary flow of requests from clients to the database in order to modulate the quality of service. The mathematical model of the queries number fluctuations to the database has the form of a regression equation and allows more accurate modeling of the connections pool size in the servlet. Connection pool is a pattern that helps to reduce responding time for queries to databases. To another hand the extra used memory wasting server resources. The task of calculating the optimal connection pool size could be solved by verity of mathematical apparatuses that demand information about intensity of stationary incoming queries flow. It is known, that real incoming queries flow is non-stationary. In the paper was suggested mathematical model of flow intensity fluctuations with daily and hourly harmonic vibrations. Statistics analyses of model adequacy was made, homogeneity of variances is checked, the significance of the coefficients of the regression equation was estimated. The obtained mathematical model describes fluctuations in the intensity of clients' requests to the servlet during the week. The mathematical model can be used to predict the load on the server or to build a simulation model of the query service system. The adequacy of the model is checked, the homogeneity of variances is checked, the significance of the coefficients of the regression equation is estimated, the adequacy of the regression equation is checked, the analysis of the autocorrelation of the residues is carried out. The results obtained in the article give further development for modeling process technologies in the field of information systems and can be used to calculate the load on the server with a non-stationary flow of requests from clients to the database.*
Keywords: *connection pool simulation; query intensity fluctuation; regression equation; statistic analyses*

МАТЕМАТИЧНА МОДЕЛЬ ОПИСУ ІНТЕНСИВНОСТІ ЗАПИТІВ ДО СЕРВЕРА

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АНОТАЦІЯ *Робота присвячена побудові математичної моделі нестаціонарного потоку запитів клієнтів до бази даних із метою подальшого моделювання показників якості обслуговування. Математична модель коливання кількості запитів до бази даних має вигляд рівняння регресії та дозволяє більш точно проводити моделювання розміру пулу з'єднань у сервлеті. Пул з'єднань - це схема, яка допомагає скоротити час відповіді на запити до баз даних. З іншого боку, додатково використовуються серверні ресурси, що витрачають пам'ять. Завдання обчислення оптимального розміру пулу підключень могло бути вирішене достовірно за допомогою математичних апаратів, які вимагають інформації про інтенсивність потоку стаціонарних вхідних запитів. Відомо, що потік реальних вхідних запитів є нестаціонарним. У статті запропонована математична модель коливань інтенсивності потоку з добовими та погодинними гармонічними коливаннями. Проведено статистичний аналіз адекватності моделі, перевірено однорідність дисперсій, оцінено значення коефіцієнтів рівняння регресії. Отримана математична модель описує коливання інтенсивності запитів клієнтів до сервлету протягом тижня. Математичну модель можна використовувати для прогнозування навантаження на сервер або для побудови імітаційної моделі системи обслуговування запитів. Перевірено адекватність моделі, перевірено однорідність дисперсій, оцінено значущість коефіцієнтів рівняння регресії, перевірено адекватність рівняння регресії, проведено аналіз автокореляції залишків. Отримані у статті результати дають подальший розвиток технологіям моделювання процесів у інформаційних системах та можуть бути використані при обчисленні навантаження на сервер при нестаціонарному потоку запитів від клієнтів до бази даних.*
Ключові слова: *моделювання пулу підключень; коливання інтенсивності запитів; рівняння регресії; статистичний аналіз*

Introduction

Connection pool in servlet is a cache of database connections maintained in memory so that the connections can be reused when future requests to the database are required. Separate database connection is fairly expensive operation; here are the sequences of steps involved into a typical database connection life cycle:

1. Opening a connection to the database using the database driver.
2. Opening a TCP socket for data manipulating (reading/writing).
3. Data manipulating (reading/writing) over the socket.
4. Closing the connection.
5. Closing the socket.

Instead of performing this list of operations every time when client sends a request, it is better to implement a database connection pool, which allows to reuse a number of existing connections, the cost of performing a huge number of expensive database trips can be effectively saved, boosting the overall performance of implemented database-driven application.

Connection pools can be tuned to maximize performance, while keeping resource utilization below the point where application will start to fail rather than just run slower. It is very good concept for those web application where number of users operate at the same time, where many connections are required at the same time, this process of creating new connection every time makes server too be busy so that performance decreases, although connection pooling method manages all the connection required by users [1].

The aim of the study

Construction of a mathematical model of non-stationary flow of user requests to the database based on the obtained statistical information about the intensity of queries and time of their execution.

Statement of the main material

Literature review. Database connection as a key resource to interact with the database is particularly important in the applications especially, in web application of multi-user. Database connection pool technology is proposed to solve the problem of frequently creating and releasing database connection that reduces the system performance in software system. An important problem in database connection pool technology is how to configure the parameters of the connection pool in order to make the software system at the optimal performance [2].

The purpose of connection pools is to enhance the performance of executing queries on database, but connection pool requires extra memory size for every extra connection kept long time in memory. In this way arises the task to find out the optimal number of opened connections in connection pool. This task can be solved, for example, using queuing theory [3] under the assumption that the intensity of the incoming and outgoing query flows [4] are known. The intensity of queries can be estimated using statistics from server but their intensities very in day time and in days of the week [5-6]. So, the task of the paper is to develop the mathematical model of query flows intensity depending on a time.

In [7] is given a quantitative data about the improvement of the performance of the database access based on the connection pool through performance contrast tests under three different kinds of database access modes. In [8] was proposed the optimal allocation method of database connection pool based on the log file records. In [9] was given the configuration method in detail for database connection pool parameter configuration based on XML under J2EE frame study. In [10 -14] was made a study and have obtained some significant results from database connection pool technology.

The performance analysis methods of database connection pool technology can be divided into three categories: Experimental data analysis [15], simulation study method [16] and mathematical model analysis methods [17]. In this study [18], the mathematical model analysis method is used for the database connection pool technology to establish a discrete-time multi-server queuing model. The analysis of classical discrete-time queuing model is shown in [19].

Connection pools include a set of properties that are used to control the size of the pool [20]. The properties allow the number of connections in the pool to increase and decrease as demand increases and decreases. This dynamic behavior helps conserve system resources that are otherwise lost [21] on maintaining unnecessary connections [22].

Model developing. The actual incoming request flows from the client to the servlet are non-stationary. A direct analysis of the observations of the calls reveals the presence of weekly and daily fluctuations in the intensity of calls. Let's divide the entire observation interval into a set of subintervals of length Δ (for example, $\Delta = 1$ ▯our). Let's calculate the number of calls received within each sub-interval. Let's process the aggregate random requests number which arrive at the same time-point of day and on the same day of the week (for example, all time intervals from 10:00 till 11:00 every Monday). Let's calculate the average value and variance of the number of servlet calls that arrive during each of the sub-intervals. In this case, we obtain the totality of the average values $m_1, m_2, \dots, m_n$ of the number of requests and their variances $\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2$, $n = 24 \cdot 7 = 168$.

Let's input b_0 - the average number of requests to the server during the day, b_1 - the amplitude of the daily fluctuations of the number of requests, b_2 - the amplitude of the weekly fluctuations of the number of requests. Then, the model which describes the dependence of the average intensity of incoming requests on time is represented by relation (1).

$$\begin{aligned} \lambda(t) &= \\ &= \left(b_0 + b_1 \sin\frac{2\pi}{24}t\right)\left(1 + b_2 \sin\frac{2\pi}{168}t\right) \\ &= b_0 + b_1 \sin\frac{2\pi}{24}t + b_2 b_0 \sin\frac{2\pi}{168}t \\ &+ b_1 b_2 \sin\frac{2\pi}{24}t \sin\frac{2\pi}{168}t = \\ &= a_0 + a_2 \sin\frac{2\pi}{24}t + a_2 \sin\frac{2\pi}{168}t \\ &+ a_3 \sin\frac{2\pi}{24}t \sin\frac{2\pi}{168}t \end{aligned} \tag{1}$$

Model (1) parameters a_0, a_1, a_2 are calculated using data from real observations by least squares method. Let's input the functional

$$\begin{aligned} J &= \sum_{k=1}^{n} [m_k - \lambda(t_k)]^2 \\ &= \sum_{k=1}^{n} \Big[m_k \\ &- \Big(a_0 + a_2 \sin\frac{2\pi}{24}t_k + a_2 \sin\frac{2\pi}{168}t_k \\ &+ a_3 \sin\frac{2\pi}{24}t_k \sin\frac{2\pi}{168}t_k\Big)^2 \Big] \end{aligned} \tag{2}$$

The adequacy of the model is checked according to the Fisher criterion [23], taking into account the analysis of residues autocorrelation [23].

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Model parameters calculating. Let's rewrite the functional (2) in matrix form. Let's input the matrix H and vectors A, M:

$$H=\begin{pmatrix} 1 & sin\frac{2\pi}{24}t_1 & sin\frac{2\pi}{168}t_1 & sin\frac{2\pi}{24}t_1\, sin\frac{2\pi}{168}t_1 \\ 1 & sin\frac{2\pi}{24}t_2 & sin\frac{2\pi}{168}t_2 & sin\frac{2\pi}{24}t_2\, sin\frac{2\pi}{168}t_2 \\ \dots & \dots & \dots & \dots \\ 1 & sin\frac{2\pi}{24}t_n & sin\frac{2\pi}{168}t_n & sin\frac{2\pi}{24}t_n\, sin\frac{2\pi}{168}t_n \end{pmatrix},$$

$$A=\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}, M=\begin{pmatrix} m_1 \\ m_2 \\ \dots \\ m_n \end{pmatrix}.$$

Then

$$J=(HA-M)^T(HA-M). \qquad (3)$$

Minimizing (3) over the vector A, the desired vector of regression equation estimates is obtained

$$A=(H^TH)^{-1}H^TM. \qquad (4)$$

Moreover, so as

$$H^TH=\begin{pmatrix} 1 & 1 & \dots & 1 \\ sin\frac{2\pi}{24}t_1 & sin\frac{2\pi}{24}t_2 & \dots & sin\frac{2\pi}{24}t_2 \\ sin\frac{2\pi}{168}t_1 & sin\frac{2\pi}{168}t_2 & \dots & sin\frac{2\pi}{168}t_n \\ sin\frac{2\pi}{24}t_1\, sin\frac{2\pi}{168}t_1 & sin\frac{2\pi}{24}t_2\, sin\frac{2\pi}{168}t_2 & \dots & sin\frac{2\pi}{24}t_n\, sin\frac{2\pi}{168}t_n \end{pmatrix}\times$$

$$\times\begin{pmatrix} 1 & sin\frac{2\pi}{24}t_1 & sin\frac{2\pi}{168}t_1 & sin\frac{2\pi}{24}t_1\, sin\frac{2\pi}{168}t_1 \\ 1 & sin\frac{2\pi}{24}t_2 & sin\frac{2\pi}{168}t_2 & sin\frac{2\pi}{24}t_2\, sin\frac{2\pi}{168}t_2 \\ \dots & \dots & \dots & \dots \\ 1 & sin\frac{2\pi}{24}t_n & sin\frac{2\pi}{168}t_n & sin\frac{2\pi}{24}t_n\, sin\frac{2\pi}{168}t_n \end{pmatrix}=$$

$$=\begin{pmatrix} n & \sum_{k=1}^{n} sin\frac{2\pi}{24}t_k & \sum_{k=1}^{n} sin\frac{2\pi}{168}t_k & \sum_{k=1}^{n} sin\frac{2\pi}{24}t_k\, sin\frac{2\pi}{168}t_k \\ \sum_{k=1}^{n} sin\frac{2\pi}{24}t_k & \sum_{k=1}^{n} sin^2\frac{2\pi}{24}t_k & \sum_{k=1}^{n} sin\frac{2\pi}{24}t_k\, sin\frac{2\pi}{168}t_k & \sum_{k=1}^{n} sin^2\frac{2\pi}{24}t_k\, sin\frac{2\pi}{168}t_k \\ \sum_{k=1}^{n} sin\frac{2\pi}{168}t_k & \sum_{k=1}^{n} sin\frac{2\pi}{24}t_k\, sin\frac{2\pi}{168}t_k & \sum_{k=1}^{n} sin^2\frac{2\pi}{168}t_k & \sum_{k=1}^{n} sin\frac{2\pi}{24}t_k\, sin^2\frac{2\pi}{168}t_k \\ \sum_{k=1}^{n} sin\frac{2\pi}{24}t_k\, sin\frac{2\pi}{168}t_k & \sum_{k=1}^{n} sin^2\frac{2\pi}{24}t_k\, sin\frac{2\pi}{168}t_k & \sum_{k=1}^{n} sin\frac{2\pi}{24}t_k\, sin^2\frac{2\pi}{168}t_k & \sum_{k=1}^{n} sin^2\frac{2\pi}{24}t_k\, sin^2\frac{2\pi}{168}t_k \end{pmatrix}$$

$$=\begin{pmatrix} n & & & 0 \\ & \sum_{k=1}^{n} sin^2\frac{2\pi}{24}t_k & & \\ & & \sum_{k=1}^{n} sin^2\frac{2\pi}{168}t_k & \\ 0 & & & \sum_{k=1}^{n} sin^2\frac{2\pi}{24}t_k\, sin^2\frac{2\pi}{168}t_k \end{pmatrix},$$

then

$$(H^TH)^{-1}==\begin{pmatrix} n^{-1} & & & 0 \\ & \left(\sum_{k=1}^{n} sin^2\frac{2\pi}{24}t_k\right)^{-1} & & \\ & & \left(\sum_{k=1}^{n} sin^2\frac{2\pi}{168}t_k\right)^{-1} & \\ 0 & & & \left(\sum_{k=1}^{n} sin^2\frac{2\pi}{24}t_k\, sin^2\frac{2\pi}{168}t_k\right)^{-1} \end{pmatrix}$$

$$=\begin{pmatrix} \eta_{11} & & & 0 \\ & \eta_{22} & & \\ & & \eta_{33} & \\ 0 & & & \eta_{44} \end{pmatrix}.$$

Then

$$A=\begin{pmatrix}\eta_{11} & & & 0\\ & \eta_{22} & & \\ & & \eta_{33} & \\ 0 & & & \eta_{44}\end{pmatrix}\times$$

$$\times\begin{pmatrix}1 & 1 & \dots & 1\\ \sin\frac{2\pi}{24}t_1 & \sin\frac{2\pi}{24}t_2 & \dots & \sin\frac{2\pi}{24}t_n\\ \sin\frac{2\pi}{168}t_1 & \sin\frac{2\pi}{168}t_2 & \dots & \sin\frac{2\pi}{168}t_n\\ \sin\frac{2\pi}{24}t_1\sin\frac{2\pi}{168}t_1 & \sin\frac{2\pi}{24}t_2\sin\frac{2\pi}{168}t_2 & \dots & \sin\frac{2\pi}{24}t_n\sin\frac{2\pi}{168}t_n\end{pmatrix}\begin{pmatrix}m_1\\ m_2\\ \dots\\ m_4\end{pmatrix}=$$

$$=\begin{pmatrix}\eta_{11} & & & 0\\ & \eta_{22} & & \\ & & \eta_{33} & \\ 0 & & & \eta_{44}\end{pmatrix}\begin{pmatrix}\sum_{k=1}^{n}m_k\\ \sum_{k=1}^{n}m_k\sin\frac{2\pi}{24}t_n\\ \sum_{k=1}^{n}m_k\sin\frac{2\pi}{168}t_n\\ \sum_{k=1}^{n}m_k\sin\frac{2\pi}{24}t_n\sin\frac{2\pi}{168}t_n\end{pmatrix}= \quad (5)$$

$$=\begin{pmatrix}\frac{1}{n}\sum_{k=1}^{n}m_k\\ \frac{1}{\sum_{k=1}^{n}\sin^2\frac{2\pi}{24}t_n}\sum_{k=1}^{n}m_k\sin\frac{2\pi}{24}t_n\\ \frac{1}{\sum_{k=1}^{n}\sin^2\frac{2\pi}{168}t_n}\sum_{k=1}^{n}m_k\sin\frac{2\pi}{168}t_n\\ \frac{1}{\sum_{k=1}^{n}\sin^2\frac{2\pi}{24}t_n\sin^2\frac{2\pi}{168}t_n}\sum_{k=1}^{n}m_k\sin\frac{2\pi}{24}t_n\sin\frac{2\pi}{168}t_n\end{pmatrix}=\begin{pmatrix}a_0\\ a_1\\ a_2\\ a_3\end{pmatrix}$$

As a result of calculations according to formula (5), taking into account real values of m_k, $k=1,2,\dots,n$, we obtain

$$A=\begin{pmatrix}3.16\cdot10^5\\ 1.28\cdot10^5\\ 0.46\cdot10^5\\ 0.21\cdot10^5\end{pmatrix}.$$

Let us make a statistical analysis of the observations processing results. This procedure contains several steps.

The discussion of the results

Dispersions homogeneity checking. As at each point in the factor space of the variable t_k, $k=1,2,\dots,m$ possible values a series of observations was carried out containing 6 repeated experiments, the experiments results have the form:

$$\mathrm{X}=\begin{pmatrix}x_{11} & x_{12} & \cdots & x_{1m}\\ x_{21} & x_{22} & \cdots & x_{2m}\\ \cdots & \cdots & \cdots & \cdots\\ x_{61} & x_{62} & \cdots & x_{6m}\end{pmatrix}.$$

Here we have:

x_{ik}- random value of the observed variable at the k -th point of the experiment, obtained at the i -th week, $i=1,2,\dots,6$, $k=1,2,\dots,m$.

For each column of the matrix X, lets average the results of repeated experiments for the observed variable and calculate the variances of these results

$$m_k=\frac{1}{q}\sum_{i=1}^{6}x_{ik},\quad \sigma_k^2=\frac{1}{q-1}\sum_{i=1}^{6}(x_{ik}-M_k)^2,$$
$$k=1,2,\dots,m,\ q=6.$$

The dispersions homogeneity is checked by the Fisher criterion. For this let's find out

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$$\sigma_{max}^2 = \max_k\{\sigma_k^2\}, \sigma_{min}^2 = \min_k\{\sigma_k^2\}$$

and calculate

$$F_p = \frac{\sigma_{max}^2}{\sigma_{min}^2}.$$

The obtained value of the F_p criterion let's compare with the critical in the Fisher distribution table for a given significance level $\alpha = 0.05$ and the number of degrees of freedom ν_1 and ν_2 of the numerator and denominator, which is equal to the number of repeated experiments minus the number of estimated parameters (in this case we have $\nu_1 = \nu_1 = q - 1$).

Moreover, if $F_p < F_{kp}$, then the hypothesis of homogeneity is accepted, otherwise it should be rejected and steps for homogeneity improvement should be done (for example, to make additional experiments at a point with maximum dispersion) [24].

In the considering problem eliminating of the dispersions heterogeneity by providing additional experiments is impossible. Moreover, to evaluate the parameters of the regression equations in the case of dispersion heterogeneity, the usege of least square method in the form (4) is incorrect. In order to take into account, the differences in the experimental variances, lets input the dispersion matrix

$$D = \begin{pmatrix} \sigma_1^2 & & 0 \\ & \sigma_2^2 & \ddots & \\ 0 & & \sigma_n^2 \end{pmatrix}.$$

Using it the functional on least squares (3) transforms to

$$J = (HA - M)^T D^{-1} (HA - M).$$

This functional minimization on model parameters vector A leads to equation

$$\hat{A} = (H^T D^{-1} H)^{-1} H^T D^{-1} M. \qquad (6)$$

If the homogeneity hypothesis is accepted, then the variance of the experiments is averaged by the formula

$$s_0^2 = \frac{1}{n}\sum_{k=1}^{n} \sigma_k^2 \qquad (7)$$

and thus, a dispersion of reproducibility of the experiment is obtained. Moreover, here $f_0 = n(q-1)$ - is the number of degrees of freedom.

So, if during the calculation of the variance reproducibility of the experiment, the homogeneity of the variances at different points of the factor space was established, then using the relations (4), the coefficients of the regression equation (1) are calculated. If the heterogeneity of dispersions is revealed, then for evaluating the components of the vector A is necessary to use formula (6).

In current task we have $\sigma_{max}^{2^5}$ $\sigma_{min}^{2^5}$ Wherein $F_\text{p} = 1{,}516$. According to the table of critical points of Student distribution for the significance level $\alpha = 0{,}05$ and the number of degrees of freedom $q - 1 = 5$, we obtain $F_\text{кр} = 2{,}57$. So, as $F_p < F_\text{кр}$ the experiment is homogeneous. After averaging the variances, we obtain the reproducibility dispersion $s_0^2 = 3.74 \cdot 10^5$.

After calculating the coefficients of the regression equation, lets proceed directly to the statistical analysis of the results, which is carried out in two stages:

1. Assessment of the coefficients significance of the regression equation.

2. Assessment of the model adequacy.

Let's run into their consideration.

Assessment of the coefficients significance of the regression equation. Firstly, lets calculate the accuracy variance of the regression coefficient estimates. For this let's calculate the covariance matrix of calculating errors of the regression equation parameters by the formula:

$$\Psi = s_0^2 (H^T H)^{-1} = s_0^2 \begin{pmatrix} \sigma_{a_0}^2 & cov(a_0, a_1) & \dots & cov(a_0, a_m) \\ cov(a_1, a_0) & \sigma_{a_1}^2 & \dots & cov(a_1, a_m) \\ \dots & \dots & \dots & \dots \\ cov(a_m, a_0) & cov(a_m, a_1) & \dots & \sigma_{a_m}^2 \end{pmatrix}.$$

In current task

$$(H^T H)^{-1} = \begin{pmatrix} \eta_{11} & & & 0 \\ & \eta_{22} & & \\ & & \eta_{33} & \\ 0 & & & \eta_{44} \end{pmatrix}.$$

So

$$\Psi = \begin{pmatrix} s_0^2\eta_{11} & & & 0 \\ & s_0^2\eta_{22} & & \\ & & s_0^2\eta_{33} & \\ 0 & & & s_0^2\eta_{44} \end{pmatrix}.$$

Lets

a_i- is the real value of the i -th coefficient of the regression equation,

$\hat{a}_i$- assessment of this value according to the experiment results.

Then the random variable

$$T_i = \frac{|\hat{a}_i - a_i|}{\sigma_{a_i}}. \qquad (8)$$

is distributed according to Student's law with $\sum_{j=1}^{n}(q_j - 1)$ degrees of freedom. Let's create the confidence interval for a_i. Let ε_{a_i} - will be a half of this interval. Lets take ε_{a_i} so that $P\left(|\hat{a}_i - a_i| < \varepsilon_{a_i}\right) = \gamma$, where γ is the confidence probability. Let's consider the left part of this equation to a random variable T_i:

$$P\left(\frac{|\hat{a}_i - a_i|}{\sigma_{a_i}} < \frac{\varepsilon_{a_i}}{\sigma_{a_i}}\right) = \gamma$$

or

$$P\left(T_i < \frac{\varepsilon_{a_i}}{\sigma_{a_i}}\right) = \gamma.$$

Using tables of T-distribution, let's find t_{a_i}so, that

$$P\left(T_i < t_{a_i}\right) = \gamma.$$

Then

$$\frac{\varepsilon_{a_i}}{\sigma_{a_i}} = t_{a_i}.$$

Wherein $\varepsilon_{a_i} = \sigma_{a_i} t_{a_i}$ and $|\hat{a}_i - a_i| < \sigma_{a_i} t_{a_i}$ or

$$\hat{a}_i - \sigma_{a_i} t_{a_i} < a_i < \hat{a}_i + \sigma_{a_i} t_{a_i}. \quad (9)$$

Thus, a confidence interval is found $\left[\hat{a}_i - \sigma_{a_i} t_{a_i}, \hat{a}_i + \sigma_{a_i} t_{a_i}\right]$, which cover the real value of coefficient a_i with probability not less then γ.

Further, it is believed that a_i coefficient is significant with reliability γ if the corresponding confidence interval does not cover zero. Otherwise, this coefficient should be considered as insignificant and equal it to zero. The corresponding factor should be excluded from the regression equation.

In current task we have

$$(H^T H)^{-1} = \begin{pmatrix} 6{,}9 \cdot 10^{-3} & & & 0 \\ & 2{,}1 \cdot 10^{-3} & & \\ & & 1{,}4 \cdot 10^{-3} & \\ 0 & & & 0{,}72 \cdot 10^{-3} \end{pmatrix}$$

Then

$$\Psi = \begin{pmatrix} 2{,}59 \cdot 10^{3} & & & 0 \\ & 7{,}88 \cdot 10^{2} & & \\ & & 5{,}25 \cdot 10^{2} & \\ 0 & & & 2{,}62 \cdot 10^{2} \end{pmatrix}$$

So, as critical value $t_a = 1.98$ (with confidence probability $\gamma = 0.95$), then

$$\varepsilon_{a_0} = \sigma_{a_0} \cdot t_a = 100.8,$$
$$\varepsilon_{a_1} = \sigma_{a_1} \cdot t_a = 55.59,$$
$$\varepsilon_{a_2} = \sigma_{a_2} \cdot t_a = 45.37,$$
$$\varepsilon_{a_3} = \sigma_{a_3} \cdot t_a = 32.05.$$

Moreover, in all cases, the confidence interval does not cover zero and, therefore, all the coefficients of the regression equation (1) are significant.

Checking the adequacy of the regression equation. To check the adequacy of the regression equation obtained after discarding insignificant factors, it is necessary, in addition to reproducibility variance, to calculate the adequacy variance which characterizes the degree of deviation of the regression line from the value of the response function at the points of the factor space corresponding to the performed experiments. The adequacy variance is calculated by the formula

$$s_{\text{ад}}^2 = \frac{1}{n-\ell} \sum_{j=1}^{n} \left(\overline{y}_j - \sum_{i=0}^{\ell-1} \hat{a}_i x_{ji}\right)^2 = \frac{1}{n-\ell} \sum_{j=1}^{n} (m_k - \lambda(t_k))^2. \quad (10)$$

For the $s_{\text{ад}}^2$ calculation, lets use the response function values averaged over the results of repeated experiments $\bar{y}_j, \quad j = 1, \ldots, n$. Therefore, the variance of the adequacy does not depend on the variance of the measurement errors. In relation (10) ℓ - is the number of terms in the regression equation remaining after the significance assessment. Thus, the adequacy variance characterizes the residual sum of squares of the results deflection of real experiments from the created regression equation per one free experiment (i.e., one degree of freedom). Now, the adequacy of the obtained regression equation is checked according to the Fisher test, comparing the equation

$$F_P = \frac{s_{\text{ад}}^2}{s_0^2}$$

with critical value

$$F_P < F_{\kappa p}. \quad (11)$$

For the number of degrees of freedom $f_{\text{ад}} = n - \ell$ и $f_0 = \sum_{j=1}^{n} (q_j - 1)$ set value of significance α.

If inequality (11) holds, then the regression equation is considered as adequate; otherwise, it is not.

Adopting the hypothesis of adequacy is equivalent to accepting the hypothesis of equality of variances of adequacy and reproducibility. The adequacy of the regression equation shows that the scattering of experimental data relative to the regression equation is of the same order as the scattering associated with experimental accuracy. Moreover, the deviation of the experimental points relative to the constructed regression equation is explained precisely by these errors, and not by the erroneous hypothesis about the structure of the model. In the considered problem a direct calculation of the variance of adequacy by formula (10) gave the following result $\sigma_{\text{ад}}^2 = 4{,}17 \cdot 10^5$. In this case, the calculated value of the Fisher test

$$F_{\text{р}} = \frac{4.17 \cdot 10^5}{3.75 \cdot 10^5} = 1{,}083.$$

So, as a critical value of Fisher's criterion for $\alpha = 0{,}05$ equals to $F_{\text{кр}} = 2{,}38$, and $F_{\text{р}} < F_{\text{кр}}$, consequently current model is adequate.

The residuals autocorrelation analyses. The residuals autocorrelation in the trend deflections could be detected using the Darbin-Watson statistic, which can be calculated by the formula

$$d = \frac{\sum_{j=1}^{n} (\xi_{j+1} - \xi_1)^2}{\sum_{j=1}^{n} {\xi_j}^2}. \quad (12)$$

Where ξ_j - deflection from trend in j -th observation.

The values distribution of the Darbin-Watson statistic is given in the table.

Let's compare the calculated value of the criterion d with the tabular d_1, d_2. The following cases are possible:

- if $d < d_1$, the hypothesis of the autocorrelation absence is rejected (positive correlation is present);

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- if $d_2 < d < 4 - d_2$, the hypothesis of the autocorrelation absence is accepted;
- if $d_1 \le d \le d_2$ or $4 - d_2 \le d \le 4 - d_1$, then further researches are necessary (for example, on a larger number of observations);
- if $d > 4 - d_1$, then the hypothesis of the correlation absence is rejected (negative correlation is present).

Relation (12) directly follows from the relation used to detect the random process autocorrelation presence $\xi(t)$. This equation has the form

$$r(t) = \frac{M[(\xi(t+1) - m(t+1)) - (\xi(t) - m(t))]^2}{M[(\xi(t) - m(t))^2]}. \quad (13)$$

Let's rewrite (13) as a follow

$$r(t) = \frac{M[(\xi(t) - m(t))^2] + M[(\xi(t+1) - m(t+1))^2]}{M[(\xi(t) - m(t))^2]} - \frac{2M\left[(\xi(t) - m(t))(\xi(t+1) - m(t+1))\right]}{M[(\xi(t) - m(t))^2]} = \frac{D(t+1) + D(t) - 2K(t, t+1)}{D(t)}. \quad (14)$$

Here $D(t)$, $D(t+1)$ - process variation at time points t and $t+1$; $K(t, t+1)$ - covariation between random values of process at time points t and $t+1$.

If the process is stationary, then $D(t) \cong D(t+1)$ and (14) can be approximately changed as follows:

$$r(t) \cong 2 - 2k(t, t+1),$$

where $k(t, t+1)$ - correlation coefficient between random process values at time points t and $t+1$.

Concerning to this fact, it is clear that if the correlation between these random values is absent (or small), then the value of $r(t)$ is approximately equals to 2.

If the correlation coefficient increases, the value of $r(t)$ decreases, approaching to zero (for a positive correlation), or four (for a negative correlation).

It is clear that (12) is a special case of (13) when the process is observed at discrete time instants $j = 1,2,\dots,n$ and obtained residues autocorrelation after trend exclusion is analyzed.

In considered problem $d_1 = 1{,}41$, $d_2 = 1{,}72$, and the value of the Darbin-Watson statistic $d = 1{,}87$. Therefore, it should be considered that there is no autocorrelation of residues, confirming the conclusion about the adequacy of the model.

Conclusions

The obtained mathematical model describes fluctuations in the intensity of clients' requests to the servlet during the week. The mathematical model can be used to predict the load on the server or to build a simulation model of the query service system. The adequacy of the model is checked, the homogeneity of variances is checked, the significance of the coefficients of the regression equation is estimated, the adequacy of the regression equation is checked, the analysis of the autocorrelation of the residues is carried out.

Список літератури

1. Ankit Kr. Tiwari, Surendra Yadav, Manish Mathuria, Manish Sharma, Hemlata Chaudhary. Performance Optimization of Web Applications using Connection Pooling. *Proceedings of International Conference on Innovations in information Embedded and Communication Systems (ICIIECS'16).* 2016.
2. Chen S., Liu Y., Gorton I., Liu A. Performance prediction of component-based applications. *Journal of Systems and Software.* 2005. 74(1). P. 35-43. doi: 10.1016/j.jss.2003.05.005.
3. Donald Gross, John F. Shortle, James M. Thompson, Carl M. Harris. *Fundamentals of Queueing Theory 4th Edition*. New Jercy: Wiley, 2008.
4. Ancans A., Petersons E., Ipatovs A. Vehicular wireless network access controller parameter estimation. *In 2017 Progress in Electromagnetics Research Symposium-Fall (PIERS-FALL) IEEE (2017, November)*. p. 2152-2159. doi: 10.1109/PIERS-FALL.2017.8293496.
5. Pustovoitov P., Sokol G., Hroza P., Tyrtyshnikov O., Rvachova N. Mathematical Model of Single-Channel Infocommunication Node with Several Packet Flows. *Proceedings of 2018 International Scientific-Practical Conference on Problems of Infocommunications. Science and Technology.* 2018. P. 166-170. doi: 10.1109/INFOCOMMST.2018.8632128.
6. Pustovoitov P., Sokol G. The model of dual channel network node with shared memory. *Proceedings of 2nd International Conference on Advanced Information and Communication Technologies.* 2017. P. 266-269. doi: 10.1109/AIACT.2017.8020116.
7. Li B. Z., Wu F. S., Jing Z. J. and Huang H. J. Self-adaptive database connection pool technology in NVMS. *Comput. Eng.* 2008. No. 23. P. 41-43.
8. Liang Q. H., Shen Z. F., Luo J. C., Fan H., Ming D. P., Li J. L. Study of database connection pool in LBS platform. *Comput. Eng.* 2006. No. 32. P. 39-41.
9. Liu F. A method of design and optimization of database connection pool. *Proceedings of the 4th International Conference on Intelligent Human-Machine Systems and Cybernetics, August 26-27* 2012, Nanchang, Jiangxi. 2012. V. 2. P. 272-274.
10. Lv J. B., Dai G. Z. and Mu D. J. Implementation of absolute delay guarantee in database connection pool in Web application servers. *Applic. Res. Comput*. 2012. No. 29. P. 1838-1841.
11. Meng P. C., Hu S. B., Shu H. and Yan F. Y. Optimization strategy of connection pool based on ADO. *Comput. Eng. Design*. 2013. No. 34. P. 1706-1710.
12. Tian N., Zhang Z. G. *Vacation Queueing Models: Theory and Applications.* Springer, New York, 2006.

13. Wang Y. J., Wang F., Yang W. B. Performance comparison test of database access based on database connection pool. *Ind. Control Comput*. 2010. No. 6. P. 92-94.
14. Zeng G. L., Fu X. F. A new model of database connection pool. *Comput. Digital Eng*. 2011. No. 39. P. 163-166.
15. Silva F. D. A. E., Azevedo C. D. The Assistat Software Version 7.7 and its use in the analysis of experimental data. *African Journal of Agricultural Research*. 2016. No. 11(39). P. 3733-3740.
16. Latimer N. R., Abrams K. R., Lambert P. C., Crowther M. J., Wailoo A. J., Morden J. P., Campbell M. J. Adjusting for treatment switching in randomised controlled trials–a simulation study and a simplified two-stage method. *Statistical methods in medical research.* 2017. No. 26(2). P. 724-751. doi: 10.1177/0962280214557578.
17. Arregui I., Salvador B., Ševčovič D., Vázquez C. Total value adjustment for European options with two stochastic factors. Mathematical model, analysis and numerical simulation. *Computers & Mathematics with Applications.* 2018. No. 76(4). P. 725-740. doi: 10.1016/j.camwa.2018.05.012.
18. Wu J., Huo Z. Performance Analysis of Database Connection Pool Technology Based on Queueing Model with Mixed Waiting and Loss. *Journal of Software Engineering.* 2016. 10(4). P. 383-391. doi: 10.3923/jse.2016.383.391.
19. Zhu C. S. and Shane Y. F. Study on self-adaptive database connection pool. *Comput. Eng. Applic*. 2003. No. 36. P. 187-209.
20. Jain R., Ramachandra A. G. M. Merging connection pools to form a logical pool of connections during a preset period of time thereby more efficiently utilizing connections in connection pools. U.S. Patent Application No. 16/260,565, 2019.
21. Ghosh S., Varakantham P., Adulyasak Y., Jaillet P. Dynamic repositioning to reduce lost demand in bike sharing systems. *Journal of Artificial Intelligence Research.* 2017. No. 58. P. 387-430.
22. Murphy B. A. *The Brandeis-Frankfurter Connection: The Secret Political Activities of Two Supreme Court Justices*. Plunkett Lake Press, 2019.
23. Durrett R. *Probability: theory and examples*. V. 49. Cambridge university press, 2019.
24. Greenwood J. A., Hartley H. O. *Guide to tables in mathematical statistics*. Princeton University Press, 2017.

References (transliterated)

1. Ankit Kr. Tiwari, Surendra Yadav, Manish Mathuria, Manish Sharma, Hemlata Chaudhary. Performance Optimization of Web Applications using Connection Pooling. *Proceedings of International Conference on Innovations in information Embedded and Communication Systems (ICIIECS'16)*, 2016.
2. Chen S., Liu Y., Gorton I., Liu A. Performance prediction of component-based applications. *Journal of Systems and Software,* 2005, 74(1), p. 35-43, doi: 10.1016/j.jss.2003.05.005.
3. Donald Gross, John F. Shortle, James M. Thompson, Carl M. Harris. *Fundamentals of Queueing Theory 4th Edition*. New Jercy, Wiley, 2008.
4. Ancans A., Petersons E., Ipatovs A. Vehicular wireless network access controller parameter estimation. *In 2017 Progress in Electromagnetics Research Symposium-Fall (PIERS-FALL) IEEE (2017, November),* p. 2152-2159, doi: 10.1109/PIERS-FALL.2017.8293496.
5. Pustovoitov P., Sokol G., Hroza P., Tyrtyshnikov O., Rvachova N. Mathematical Model of Single-Channel Infocommunication Node with Several Packet Flows. *Proceedings of 2018 International Scientific-Practical Conference on Problems of Infocommunications. Science and Technology,* 2018, p. 166-170, doi: 10.1109/INFOCOMMST.2018.8632128.
6. Pustovoitov P., Sokol G. The model of dual channel network node with shared memory. *Proceedings of 2nd International Conference on Advanced Information and Communication Technologies,* 2017, p. 266-269, doi: 10.1109/AIACT.2017.8020116.
7. Li B. Z., Wu F. S., Jing Z. J. and Huang H. J. Self-adaptive database connection pool technology in NVMS. *Comput. Eng.*, 2008, No. 23, p. 41-43.
8. Liang Q. H., Shen Z. F., Luo J. C., Fan H., Ming D. P., Li J. L. Study of database connection pool in LBS platform. *Comput. Eng.,* 2006, No. 32, p. 39-41.
9. Liu F. A method of design and optimization of database connection pool. *Proceedings of the 4th International Conference on Intelligent Human-Machine Systems and Cybernetics, August 26-27* 2012, Nanchang, Jiangxi, 2012, V. 2, p. 272-274.
10. Lv J. B., Dai G. Z. and Mu D. J. Implementation of absolute delay guarantee in database connection pool in Web application servers. *Applic. Res. Comput*., 2012, No. 29, p. 1838-1841.
11. Meng P. C., Hu S. B., Shu H. and Yan F. Y. Optimization strategy of connection pool based on ADO. *Comput. Eng. Design*., 2013, No. 34, p. 1706-1710.
12. Tian N., Zhang Z. G. *Vacation Queueing Models: Theory and Applications.* Springer, New York, 2006.
13. Wang Y. J., Wang F., Yang W. B. Performance comparison test of database access based on database connection pool. *Ind. Control Comput*., 2010, No. 6, p. 92-94.
14. Zeng G. L., Fu X. F. A new model of database connection pool. *Comput. Digital Eng*., 2011, No. 39, p. 163-166.
15. Silva F. D. A. E., Azevedo C. D. The Assistat Software Version 7.7 and its use in the analysis of experimental data. *African Journal of Agricultural Research,* 2016, No. 11(39), p. 3733-3740.
16. Latimer N. R., Abrams K. R., Lambert p. C., Crowther M. J., Wailoo A. J., Morden J. P., Campbell M. J. Adjusting for treatment switching in randomised controlled trials–a simulation study and a simplified two-stage method. *Statistical methods in medical research,* 2017, No. 26(2), p. 724-751, doi: 10.1177/0962280214557578.
17. Arregui I., Salvador B., Ševčovič D., Vázquez C. Total value adjustment for European options with two stochastic factors. Mathematical model, analysis and numerical simulation. *Computers & Mathematics with Applications,* 2018, No. 76(4), p. 725-740, doi: 10.1016/j.camwa.2018.05.012.
18. Wu J., Huo Z. Performance Analysis of Database Connection Pool Technology Based on Queueing Model with Mixed Waiting and Loss. *Journal of Software Engineering,* 2016, 10(4), p. 383-391, doi: 10.3923/jse.2016.383.391.
19. Zhu C. S. and Shane Y. F. Study on self-adaptive database connection pool. *Comput. Eng. Applic*., 2003, No. 36, p. 187-209.
20. Jain R., Ramachandra A. G. M. Merging connection pools to form a logical pool of connections during a preset period of time thereby more efficiently utilizing connections in connection pools. U.S. Patent Application No. 16/260, 565, 2019.
21. Ghosh S., Varakantham P., Adulyasak Y., Jaillet p. Dynamic repositioning to reduce lost demand in bike sharing systems. *Journal of Artificial Intelligence Research,* 2017, No. 58, p. 387-430.

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22. Murphy B. A. *The Brandeis-Frankfurter Connection: The Secret Political Activities of Two Supreme Court Justices*. Plunkett Lake Press, 2019.
23. Durrett R. *Probability: theory and examples,* V. 49, Cambridge university press, 2019.
24. Greenwood J. A., Hartley H. O. *Guide to tables in mathematical statistics*. Princeton University Press, 2017.

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АННОТАЦИЯ *Работа посвящена построению математической модели нестационарного потока запросов клиентов к базе данных с целью дальнейшего моделирования показателей качества обслуживания. Математическая модель колебания количества запросов к базе данных имеет вид уравнения регрессии и позволяет более точно проводить моделирование размера пула соединений в сервлет. Пул соединений - это схема, которая помогает сократить время ответа на запросы к базам данных. С другой стороны, дополнительно используются серверные ресурсы, тратят память. Задача вычисления оптимального размера пула подключения могла быть решена достоверно с помощью математических аппаратов, которые требуют информации об интенсивности потока стационарных входящих запросов. Известно, что поток реальных входящих запросов является нестационарным. В статье предложена математическая модель колебаний интенсивности потока с суточными и почасовыми гармоничными колебаниями. Проведен статистический анализ адекватности модели, проверена однородность дисперсий, оценены значения коэффициентов уравнения регрессии. Полученная математическая модель описывает колебания интенсивности запросов клиентов к сервлету в течение недели. Математическую модель можно использовать для прогнозирования нагрузки на сервер или для построения иммитационной модели системы обслуживания запросов. Проверена адекватность модели, проверена однородность дисперсий, оценена значимость коэффициентов уравнения регрессии, проверена адекватность уравнения регрессии, проведен анализ автокорреляции остатков. Полученные в статье результаты дают дальнейшее развитие технологиям моделирования процессов в информационных системах и могут быть использованы при вычислении нагрузки на сервер при нестационарном потоке запросов от клиентов к базе данных.*

Ключевые слова: *моделирование пула подключений; колебания интенсивности запросов; уравнения регрессии; статистический анализ*

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