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Parameter Analysis of Vibroimpact Machines Dynamics With Variable Mass and Stiffness

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Abstract
Vibroimpact machines operate under high repeated loadings. This poses certain requirements with regards to the strength and durability of their components that have to be met by the design. In order to predict the magnitude and time distribution of the acting forces the dynamics of the vibroimpact systems need to be studied A two-body mechanical system is considered as a model of a shake-out machine designed for Azovmash company and used for extraction of steel casts from the mold The first body is the suspended frame with the shake-out grid driven by an unbalanced vibrator, while the second one is the cast. Its sand mold is damaged at every impact with the shake-out grid, which results in the gradual loss of mass. The previously developed rigid body dynamics model is extended in order to account for the variable mass factor. Two approaches for the time evolution of the cast mass has been taken. The first one suggests that mass is a predefined linear or piecewise linear function of time. Alternatively the mass is treated as an unknown variable and was determined in the course of solving the equations of motion. A constitutive law for the mass reduction based on the energy dissipated at each impact is proposed. It has been shown that this model results in more adequate description of the shake-out process compared to the fixed-law mass evolution. In addition to the variable mass the influence of stiffness characteristics has been investigated. Nonlinear double springs with variable stiffness and length difference suspending the shake-out platform are considered The survey on the combined effect of mass and stiffness parameters on the dynamics of the modelled shake-out machine allowed to determine the loads sustained by its structural elements and to make the rational design with the required strength. In particular, it has been shown how to detune the machine from resonance frequencies, in particular from the discovered dangerous subharmonic regimes due to variable mass and stiffness.

Keywords
Dynamic processes in machines, strength of machines, vibroimpact system, vibromachine, variable mass, applied theory of oscillations, subharmonic mode.

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Introduction

Nowadays vibroimpact machines find increasingly broad applications in various sectors of industry including metallurgy, metal processing, agriculture, manufacturing of building materials. Vibroimpact machines used in metal casting can serve as an example of a case when technological weight is variable. The partial destruction of sand and clay form is the part of the process of cast extraction. Sand and clay form is heterogeneous in its structure and composition. Accordingly, its physical and mechanical properties depend on a large number of factors. That's why it's difficult to provide a description of its destruction and constitute straightforwardly an accurate model for change of its weight over time and account for it in the dynamic simulations. Discrepancy in the determination of these properties of the nonlinear vibroimpact system does not allow to predict reliably various important phenomena such as subharmonic oscillation regimes of different multiplicity, change of loading frequency and others.

Vibroimpact machines has been paid substantial attention over the past decades [1-11]. The available literature on this topic covers various aspects of vibroimpact systems. Nonetheless a complete theory of vibroimpact dynamics which would account for the cumulative effect of a number of important factors (variables weight, stiffness, friction, etc.) has not yet been presented. Thus, the existing universal methods of vibroimpact systems analysis require further development. This work delivers an improvement to the previously proposed model that includes consideration of variable stiffness of elastic supports and partial destruction of technological weight.

1. Vibroimpact systems with variable parameters

1.1 The equations of motion formulation

The equations of motion for the considered mechanical system can be formulated based on Lagrange equations of the 2nd kind and Newton's law [12-14]. They can be written in the general form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial \Pi}{\partial q_i}-Q_i^{Int}(q,\dot{q})=Q_i^{Ext}(t),\ i=1,2,...n, \quad (1)$$

where $Q^{Ext}(t)$ is the external active force,

$Q^{Int}(q,\dot{q})$ is the resultant of internal forces.

The internal force is decomposed as $Q^{Int}=Q^L+Q^N$ into the linear component $Q^L(q,\dot{q})$ and the non-linear contribution $Q^N(q,\dot{q})$, both of which are functions of the phase space variables q and $\dot{q}$. For the known functions Q^L and Q^N we end-up with the problem of structural dynamics which determines the motion of the mechanical system under the action of given forces.

Unlike this standard case the proposed paper considers specific objects for which the character and the particular law of internal interaction is not defined or can not be determined robustly by an experiment. In this situation besides the problem of motion analysis there's a principle problem of force identification.

In particular, in the considered vibroimpact systems the exact law of interaction between the machine and the processed object is unknown. This generally non-linear interaction process can be attributed to the component Q^N in the analysis. According to the approach proposed by A. Grabovsky et.al. the unknown impact force is sought for in a form of Taylor series expansion. The particular expression given in the original work [1] is also suitable for the considered here two-body vibroimpact system depicted in Fig. 1. The impact force is a function of relative speed $\dot{\zeta}=(\dot{w}_1-\dot{w}_2)$ and the approach $\zeta=(w_1-w_2)$ of the two bodies with the displacements w_1 and w_2:

$$F^{\wedge}(\varsigma,\dot{\varsigma})=F(0,0)+\frac{\partial F}{\partial \varsigma}\varsigma+\frac{\partial F}{\partial \dot{\varsigma}}\dot{\varsigma}+\frac{1}{2}\frac{\partial^2 F}{\partial \varsigma^2}\varsigma^2+\frac{\partial^2 F}{\partial \varsigma \partial \dot{\varsigma}}\varsigma\dot{\varsigma}+\frac{1}{2}\frac{\partial^2 F}{\partial \dot{\varsigma}^2}\dot{\varsigma}^2+...= \\ =\alpha_{00}+\alpha_{10}\varsigma+\alpha_{01}\dot{\varsigma}+\alpha_{20}\varsigma^2+.... \quad (2)$$

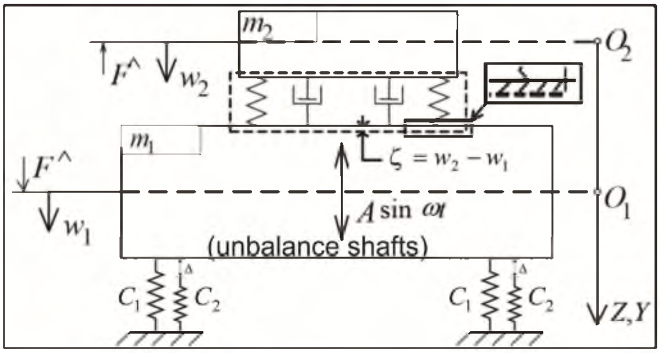

Figure 1. The modeled two-body vibroimpact system.

With this expression for the impact forces and for the given excitation force $A\sin\omega t$ exerted by the unbalanced shaft drive as well as the elastic reaction $-Cw_1$ of the resilient supports the equations of motion for the considered vibroimpact system take the following form:

$$\begin{cases} -m_1\ddot{w}_1 + m_1 g + F^{\wedge}(\varsigma,\dot{\varsigma}) - Cw_1 = A\sin\omega t, \\ m_2\ddot{w}_2 - m_2 g + F^{\wedge}(\varsigma,\dot{\varsigma}) = 0. \end{cases} \tag{3}$$

The undefined coefficients of the force series expansion (2) are identified in accordance with the mismatch minimization criterion: $\alpha_{pk} = \arg\min \left\| w^E - w^N, \dot{w}^E - \dot{w}^N \right\|$, where w^E, $\dot{w}^E$ are the experimentally measured displacements and velocities while w^N, $\dot{w}^N$ are their time distributions obtained numerically by integrating equations of motion (3).

1.2 Variable mass of system elements

Traditionally, the nature of mass change over time is defined by artificial dependence obtained experimentally or derived empirically [2]. The disadvantage of this approach is that it is often difficult to get information about qualitative and quantitative nature of this dependence for real processes. It only allows the simulation of variable mass in the first approximation. As an alternative the mass of the second body $m_2 = m_2(t)$ can be sought for as an unknown function according to a particular constitutive law. Experiments conducted on real vibroimpact machines show that during the shakeout the technological weight can lose up to 50% of its weight and more (depending on the type of product). It is proposed to simulate this process by certain reduction of the body weight.

The time evolution of weight is described by linear approximation $m_2(t) = m_2^0(1 - t/\tau) + m_2^k$ or piecewise linear function:

$$m_2(t) = m_2^0 - \frac{m_2^0 - m_2(t^*)}{t^*} \cdot t, \quad t \le t^*; \tag{4}$$

$$m_2(t) = m_2(t^*) - \frac{m_2(t^*) - m_2^k}{\tau - t^*} \cdot (t - t^*), \; t \ge t^*, \tag{5}$$

where m_2^0 - initial weight of technological weight;

m_2^k - final mass of technological weight;

t^* - moment of transition from a more intensive mode of mass change to a less intense;

τ - the duration of impact;

t - current time from the beginning of impact.

The rate of change of mass and separation of fragments is low and the weight is much higher than a mass of fragments that are separated. That's why the reactive component $\dot{m}_2\dot{w}_2$ is also small and the motion of the system (Fig. 1) is described by a system of equations (3) with zero initial conditions.

The system of differential equations (3) for the 2-mass vibroimpact system has the following parameters: $m_1 = 15960$ kg, $m_2 = 5000$ kg, $C = 5280$ kN/m, $H = 127680$ Ns/m, $A = 293$ kN, $\nu = 16$ Hz. The equations of motion (3) are integrated by the Runge-Kutta method with the initial conditions: $w_1 = w_2 = 0$, $\dot{w}_1 = \dot{w}_2 = 0$. The results are shown in Fig. 2, 3.

The mass change $m_2(t)$ occurs in a limited range limited from below by the mass of the cast when it separates completely from the mold. Apparently, the its variation over time has only quantitative effect on the steady oscillations. As can be seen in Fig. 4, 5 for the given parameters of the vibroimpact system the reduction of mass does not trigger any transient processes and switching to other potentially dangerous regimes.

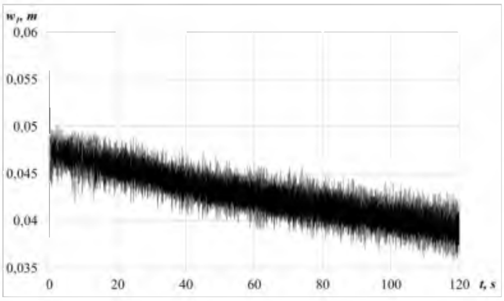

Figure 2. Vertical displacement of vibroimpact machine frame in case of linear weight loss function.

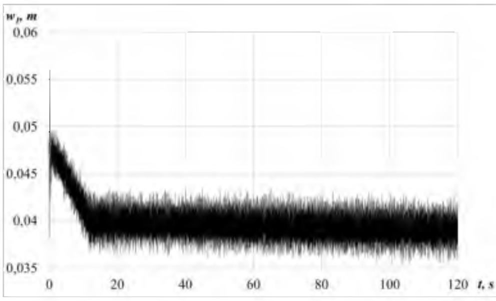

Figure 3. Vertical displacement of vibroimpact machine frame in case of piecewise linear weight loss function.

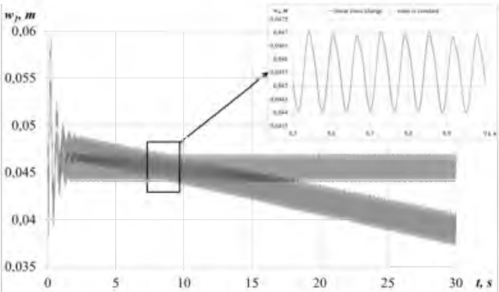

Figure 4. Distribution of vertical displacement of vibroimpact machine frame in case of constant and variable mass of technological weight.

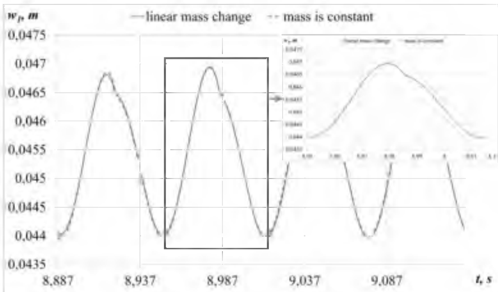

Figure 5. Distribution of vertical of vibroimpact machine frame in case of constant and variable mass of technological weight.

The exact law of mass change is not always known a priori, and therefore there is uncertainty in the nature of mass change and consequently in the description of the shakeout process. For example, in [3] the effectiveness of shakeout (destruction of sand and clay forms) is related to specific impact energy e_0. The value of e_0 is measured in units of length and is proportional to the height of the free fall of the weight required to hit the foundation with the given relative velocity. Its value is determined experimentally and depends on several factors.

In this paper the mass reduction is determined for every impact event accordingly to the amount of energy dissipated during the impact [4]. Thus, the nature of technological mass change is computed explicitly during the simulation of the vibroimpact dynamics. The proposed model law for mass decrement is constituted by the following relations:

$$dm/dt = -K_e \cdot N; \quad m(0) = m_0 \tag{6}$$

$$N = \frac{dE(w_1, w_2, \dot{w}_1, \dot{w}_2)}{dt} \tag{7}$$

where K_e - an experimentally determined coefficient that describes the mechanical properties of the mold; N - the dissipation power with regard to the impact interaction .

A comparison between the proposed law of mass change $m(E)$ related to the dissipated energy against the previously considered linear function $m(t)$ of mass over time is demonstrated in Fig. 6, 7.

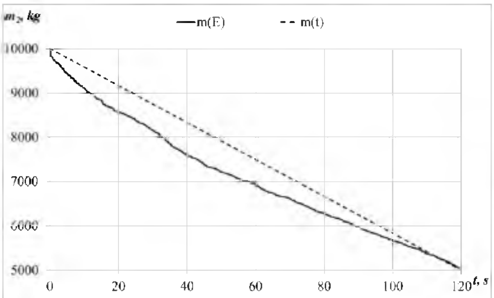

Figure 6. The mass change for 120s

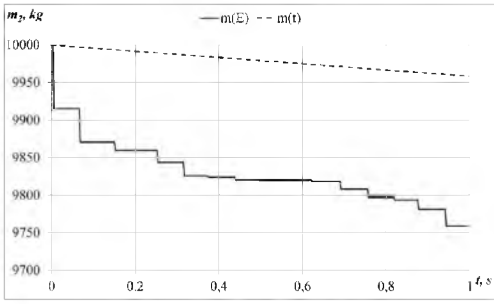

Figure 7. The mass change for 1s

The general nature of the process for both cases is similar, although there are minor differences, particularly at the beginning of shakeout corresponding to the transient processes.

The results of the dynamic simulation of vibroimpact machines confirm that description based on the use of the energy dissipation is more correct with regard to the observed non-stationary vibroimpact processes.

1.3 Variable stiffness of supports

Besides the impact interaction there is another basic factor that significantly affects the dynamics of shake-out machines, namely the mechanical properties of the elastic supports. The design options include single, double springs and other constructions which may display essentially non-linear characteristics [5, 10-11].

Dual springs support has a progressive load-displacement relation that is distinguished by the additional stiffness C_2 produced by the second coil (Fig. 1). The resulting reaction force is a piece-wise linear function of the displacement w_1 with a tangent C_1 for $w_1 < \Delta$ which is increased by the stiffness C_2 of the second coil for $w_1 > \Delta$. Here $\Delta = l_1 - l_2$ is the offset between the two coils of length l_1 and l_2. The presented in Fig. 8 oscillation diagram demonstrates that nonlinear elastic characteristics of the supports can result in existence of subharmonic modes.

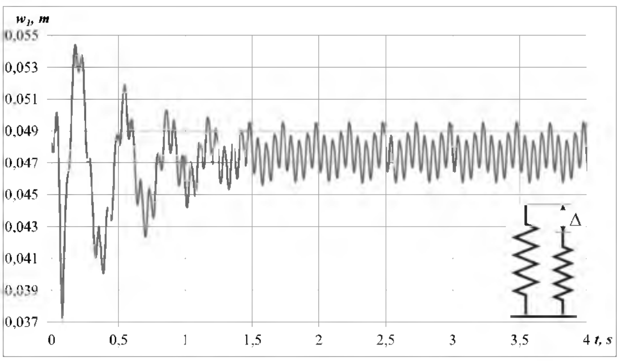

Figure 8. Transient motion and steady subharmonic regime in case of non-linear reactions provided by dual springs support

The effect of the variation of spring stiffness and coil offset on the nature of oscillations was investigated for the dual springs support. Typical phase portraits and time distributions of generalized coordinate w_1 are given in Fig. 9. It was discovered that subharmonic regimes are possible for the certain parameter values. The offset of the additional spring turned out to have more profound effect compared to its stiffness with regard to the existence of multiplicity oscillation modes.

In vibroimpact machines impact force is the dominant source of mechanical stresses on the structure. The realization of subharmonic modes leads to its substantial growth. The distribution of impact interaction forces in time for the subharmonic modes is shown in Figure 10.

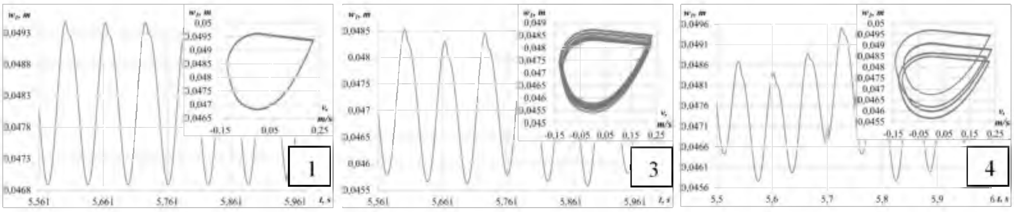

Figure 9. Typical phase portraits of different multiplicity modes (1, 3, 4) and the distribution of oscillations in time

In order to quantify the effect of subharmonic regimes on the stress level a dimensionless measure factor $\sigma = F_{\max}^{cur} / F_{\max}^{ref}$ is introduced as the ratio between the maximal impact force $F_{\max}^{cur} = F_{\max}^{k}$ for the current mode with certain multiplicity k to the value $F_{\max}^{ref} = F_{\max}^{1}$ for the reference harmonic mode with unit multiplicity computed for each particular set of the design parameters. This ratio gives an idea about the level of dynamic loading by impact forces. The exact computed values are $\sigma^1 = F_{\max}^1 / F_{\max}^1 = 1$, $\sigma^3 = F_{\max}^3 / F_{\max}^1 \approx 1.12$, $\sigma^4 = F_{\max}^4 / F_{\max}^1 \approx 1.07$. As can be seen the subharmonic oscillation mode of multiplicity 3 is the most dangerous, since it has the highest amplitude of impact force compared to harmonic oscillations.

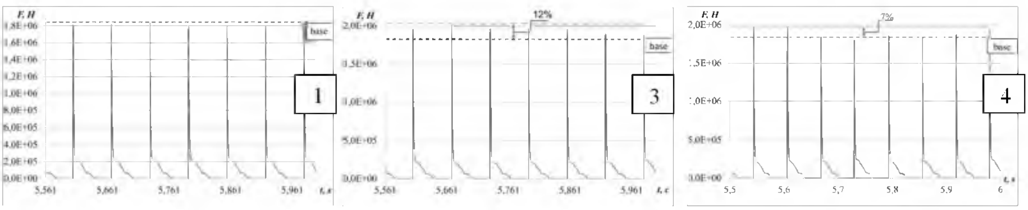

Figure 10. The distribution of impact interaction forces in time for different multiplicity modes (1, 3, 4)

Conclusions

The paper solves scientific and practical problems of the dynamic analysis of vibroimpact machines taking into account the influence of the variable mass of the technical load and nonlinear stiffness of elastic supports. Based on the obtained results the design recommendations were given for the considered vibroimpact machines. The main scientific and practical results of the work are as follows:

1. The approach to the modeling of the mass reduction of the technological load with regard to the energy dissipated during impacts is proposed and tested. It was found that the account for mass change process by the proposed relation in terms of dissipated energy is more adequate, although the results do not differ significantly from the results obtained using previously postulated law. It was shown that the character of the mass change does not trigger non-stationary transitional regimes.

2. The existence of subharmonic modes that are characterized by high impact force magnitudes compared to harmonic oscillation was predicted for the case of elastic supports with nonlinear response. The parameter study with respect for the offset length and additional stiffness performed for the dual springs provided the full specter of possible subharmonic modes. The spring offset was found to have more profound influence on the oscillations. It was found that in the considered vibroimpact machine the realization of a subharmonic mode with the multiplicity of 3 induces 12% increase of the impact force and should be avoided at the design stage.

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