MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL TECHNICAL UNIVERSITY
“KHARKIV POLYTECHNIC INSTITUTE”

SELF-LEARNING GUIDE

“MATHEMATICS IN QUESTIONS AND ANSWERS. Part 1”

for international students

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PREFACE

This methodic guide is designed for international students who study at the preparatory departments of Ukrainians universities. In this methodic guide students can find the basic theoretical questions and answers for the main topics to be covered during the Math educational course (1 semester).

The special attention to the basic concepts of the Mathematics theory and scientific style in Mathematics are focused in the methodic guide.

The methodic guide includes some basic problems of Arithmetic and Algebra.

Active learning style encouraged throughout, helping students really get to grip with the theory and getting them ready for successful mastery of higher Mathematics course.

This methodic guide may be useful for international students of preparatory courses at universities, secondary schools, high schools and colleges. The book is also intended to be useful for Mathematics teachers who work with international students.

Authors believe that this methodic guide should not just be a collection of mathematical facts carefully laid out, so the student can open it and cram in whatever formula they have to remember for tomorrow's exam.

A Math methodic guide, especially for international students, should be full of questions, not just exercises. Questions that require some thought to answer. After all, "it's the questions that keep the students interested, not the answers." The basic issues of the theory give the possibility to address gaps in knowledge, acquire the knowledge needed for all major sections from the school Mathematics and revise the theory independently, without any other assistance.

Topic 1. ARITHMETIC. INTRODUCTION

1. What digits do you know?

Answer. Digits are mathematical signs.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are digits.

2. What numbers do you know?

Answer. Numbers consist of digits. We know one-digit numbers, two-digit numbers, three-digit numbers, four-digit numbers, five-digit numbers, six-digit numbers, seven-digit number etc.

One sign = one digit = one-digit number.

Two signs = two digits = two-digit number.

Three signs = three digits = three-digit number.

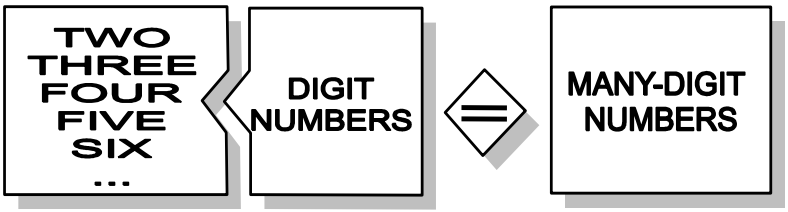
Four signs = four digits = four-digit number.

Five signs = five digits = five-digit number.

Six signs = six digits = six-digit number.

Seven signs = seven digits = seven-digit number.

REMEMBER!



3. What mathematical signs do you know?

Answer. We know: "+" plus; "–" minus; "·" multiply by or times; ":" divide by; ">" greater than; "<" less than; "≥" greater or equal to; "≤" less or equal to; "=" equals or equal to.

4. What mathematical arithmetical operations do you know?

Answer. We know: addition, subtraction, multiplication, division.

5. Write down the rule of order of operations.

Answer. Order of operations. Firstly, you should do operations in parentheses. Inside any parentheses do multiplication or division first (from left to right) and after that do addition or subtraction whichever comes first left to right.

6. Write what do you know about exact and inexact quotients.

Answer. Quotient of two numbers may be exact (without remainder ($r = 0$)) and unexact (with remainder ($r = 0$)).

$$a = bq + r,\ 0 \le r \le b,\ b \ne 0,$$

a – is a dividend; b – is a divisor; q – is a quotient; r – is a remainder.

7. What is multiple of number?

Answer. Multiple of number a is a number b, if we can divide b by a without remainder ($r = 0$).

8. What is a prime number?

Answer. Prime numbers are numbers that have only two divisors.

9. What is a composite number?

Answer. Composite numbers are numbers that have more than two divisors. 1 (one) – is neither simple number, nor composite number.

10. What is factoring of composite numbers?

Answer. Composite numbers factoring is an operation when a composite number can be written as a product of prime numbers.

11. What does the GCD mean?

Answer. Greatest Common Divisor of several numbers (GCD) is the greatest natural number, by which each given number is divisible without remainder.

12. What does the LCM mean?

Answer. Least Common Multiple (LCM) of several numbers is the smallest natural number, which is divisible by each of given numbers without remainder.

Topic 2. COMMON FRACTIONS AND DECIMALS

13. What is a common fraction?

Answer. $\frac{a}{b}$ is a common fraction, "a" is a numerator of fraction, "b" is a denominator of fraction.

14. What is a proper fraction?

Answer. If numerator of fraction is less than denominator, therefore this fraction is called a proper fraction.

15. What is an improper fraction?

Answer. If numerator of fraction is greater than denominator or equal to denominator, then this fraction is called an improper fraction.

16. What is a mixed number?

Answer. Mixed number is a sum of whole and fractional parts of given number. For example, $8\frac{2}{7} = \underset{\text{whole part}}{8} + \underset{\text{fractional part}}{\frac{2}{7}}$.

17. What is the main property of fraction?

Answer. The main property of fraction**.** The fraction's value does not change, if we multiply its numerator and denominator by the same number that is not equal to zero.

$$\frac{a}{b} = \frac{a:m}{b:m} = \frac{a \cdot n}{b \cdot n}, \quad \begin{pmatrix} m \neq 0 \\ n \neq 0 \end{pmatrix}$$

18. What is a decimal fraction?

Answer. A common fraction that has a denominator equal to 10, 100, 1000 ... is called a decimal fraction. A decimal fraction can be written: $\frac{37}{100} = 0.37$.

19. What fractions do you know?

Answer. We know: common fractions (proper and improper), decimals (finite and infinite), periodical (pure and mixed), unperiodical.

Topic 3. RATIO. PROPORTION. PERCENT

20. What is ratio?

Answer. Quotient of numbers a and b is a ratio of these numbers, if $b \neq 0$. The ratio can be written: $\frac{a}{b}$ or $a:b$, where: a and b are terms of the ratio.

21. What does ratio show if $\frac{a}{b} > 1$?

Answer. If $\frac{a}{b} > 1$, then ratio shows how many times a is greater than b. For example, $\frac{15}{3} = 5$ (ratio of fifteen to three is equal to five). This ratio shows that 15 is 5 times greater than 3.

22. What does ratio show if $\frac{a}{b} < 1$?

Answer. If $\frac{a}{b} < 1$, then ratio shows what part a consists of b. For example, $\frac{3}{12} = \frac{1}{4}$ (ratio of three to twelve is equal to fraction "one fourth"). This ratio shows that number 3 consists $\frac{1}{4}$ part of number 12.

23. What does ratio show if $\frac{a}{b} = 1$?

Answer. If $\frac{a}{b} = 1$, then numbers a and b are equal. For example, $\frac{7}{7} = 1$ (ratio of seven to seven is equal to one). This ratio shows that numbers are equal.

24. Write the main property of ratio.

Answer. Property of ratio. Ratio does not change, if we multiply or divide the terms of the ratio by the same number (nonequal to zero).

$$\frac{a}{b}=\frac{a\cdot m}{b\cdot m}=\frac{a:n}{b:n},\ \text{where: } m\neq 0,\ n\neq 0.$$

25. What is proportion?

Answer. Proportion is an equality of two numbers.

$$\frac{a}{b}=\frac{c}{d}\ \text{ or } a:b=c:d;\ \ b\neq 0;\ d\neq 0.$$

Here a and d are extremes terms of the proportion; b and c are mean terms of the proportion; $a:b$ is a left side of proportion; $c:d$ is a right side of proportion.

26. Write the basic property of proportion.

Answer. The basic property of proportion. Product of extreme terms of proportions is equal to the product of mean terms of proportion: if $\frac{a}{b}=\frac{c}{d}$, then; $a:b=c:d;\qquad b\neq 0;\ d\neq 0.$

27. What is percent?

Answer. Percent is one hundred part of a number one: $1\%=\frac{1}{100}=0.01$.

28. How can you write down percents as a fraction?

Answer. It is possible to write down percents as a fraction:

$$40\%=\frac{40}{100}=0.4;\ 100\%=\frac{100}{100}=1;\ 252\%=\frac{252}{100}=2.52.$$

29. How can you write down a fraction as percents?

Answer. It is possible to write down a fraction as percents:

$0.25\cdot 100\%=25\%;$ 0.25 is 25%;

$1.75\cdot 100\%=175\%;$ 1.75 is 175%.

30. What does percentage show?

Answer. Percentage shows how many percents does one number make from the other one. For example, $P = \frac{120}{320} \cdot 100\% = 37.5\%$.

Topic 4. SETS

31. What sets do you know?

Answer. There are finite $\{a_1, a_2, a_3, a_4\}$ and infinite $\{a_1, a_2, a_3, a_4, \ldots\}$ sets, empty sets $\varnothing$, numerical sets $\{1, 2, 3, 4, \ldots\}$ etc.

32. What numerical sets do you know?

Answer. We can designate following numerical sets:

$N = \{1, 2, 3, 4...\}$ is a set of natural numbers.

$Z = \{\ldots -2, -1, 0, 1, 2 \ldots\}$ is a set of integers (or whole numbers).

$Q = \left\{ \frac{m}{n} \middle/ m \in Z, n \in N \right\}$ is a set of rational numbers.

Set I is a set of irrational numbers. Infinite non-periodic fractions are irrational numbers.

Set R is a set of real numbers. All rational and irrational numbers are real numbers.

33. What intervals do you know?

Answer. We know:

$[a, b]$ or $a \le x \le b$ is a closed interval or a segment;

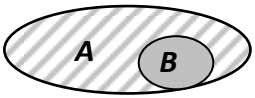
(a, b) or $a < x < b$ is an open interval;

$[a, b)$ or $a \le x < b$, $(a, b]$ or $a < x \le b$ are semi-intervals (the half-open or half-closed intervals);

numerical rays $[a, +\infty)$; $(-\infty, b]$; $(-\infty, +\infty)$.

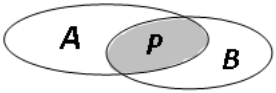
34. What does subset mean?

Answer. The set B is called a subset of set A, if each element of the set B is also an element of the set A. It can be written: $B \subset A$.



35. What does intersection of sets mean?

Answer. Intersection of sets A and B is the set P that consists of common elements of given sets: $P = A \bigcap B$.

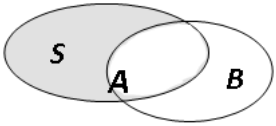


36. What does union of sets mean?

Answer. Union of sets A and B is the set C that consists of all elements of given sets. It can be written: $C = A \bigcup B$.

37. What does subtraction of sets mean?

Answer. Subtraction of the sets A and B is the set S which consists of all elements of the set A. These elements do not belong to set B: $S = A \setminus B$.



Topic 5. POWER. OPERATIONS WITH POWER

38. What is power?

Answer. Power (a^n) is product of n equal factors.

$a^n = \underbrace{a \cdot a \cdot ... \cdot a}_{n \text{ times}}$ is a power, a is a base of power, n is an index of power.

39. What operations with powers do you know?

Answer. We know: multiplication of powers with same bases, division of powers with same bases, raising power to exponent etc.

40. Write the rule of multiplication of powers with same bases.

Answer. To multiply powers with same bases, write down the base without changing and add exponents.

41. Write the rule of divison of powers with same bases.

Answer. To divide powers with same bases, write down the base without changing and subtract exponents.

42. Write the rule of raising power to exponent.

Answer. To raise power to an exponent write down the base without changing and multiply exponents.

43. What is power of number with zero exponent?

Answer. Power of number with zero exponent is equal to one if $a \neq 0$.

44. What is a number a to the power with negative exponents?

Answer. Any number a to the power with negative exponents is a fraction if $a \neq 0$.

Topic 6. ALGEBRAIC EXPRESSIONS

45. What is algebraic expression?

Answer. Algebraic expression is an expression that consists of numbers, variables and mathematical signs. This expression may contain parentheses, rational power of variable (with whole or fractional exponent) and absolute values. For example, $\sqrt{3a-4b}$, $m+n-\frac{p}{4}$, $\frac{4a^4+2a+1}{a-1}$, $\left(\frac{1}{a}+\frac{1}{b}-\frac{c}{3}\right)^3$, $\left(\sqrt[3]{8}-y\right)^4$, $x^{\frac{2}{3}}-y^{\frac{2}{3}}$ are algebraic operations.

46. What is a domain of definition (*D*) of algebraic expression?

Answer. Domain of definition (D) of algebraic expression is the set of real numbers for which an algebraic expression is defined.

Rational expression is a ratio of polynomials.

For example, $\frac{6a^2}{b^2+1}$, $5b$, $\frac{4y}{y^3-1}$ are rational expressions.

Rational expressions may be whole and fractional ones.

47. What is monomial?

Answer. The monomial is a product of powers of variables with nonnegative integer exponents. For example, $-3b$; $8a^2b^3c^4$; $\frac{7}{13}mn$ are monomials, where: -3; 8; $\frac{7}{13}$ are constant coefficients (numerical factors) and b, $a^2b^3c^4$; mn are powers of variables.

48. What is a degree of monomial?

Answer. The degree of a monomial is the sum of the exponents of all its variables. For example, *the monomial* $5ax^3$ is a monomial of the fourth power $(1+3=4)$.

49. Which monomials are called like or similar ones?

Answer. Monomials are called like (or similar), if they are identical or differed only by coefficients. For example, $7a^3b^2$; $0.3a^3b^2$; $-24a^3b^2$ are like monomials.

50. What does reducing like monomials mean?

Answer. Reducing like monomials means finding their sum or difference.

51. What is polynomial?

Answer. Polynomial is an algebraic sum of monomials (their sum or difference). For example, $5a^2-3ab+c$ is a polynomial.

52. What is a degree of polynomial?

Answer. The degree of polynomial is the most of degrees of monomials, forming this polynomial. For example, a polynomial $8x^4-3x^3-x-5$ is a polynomial of forth power; a polynomial $2x^3y^2+xy^3-7xy+6$ is a polynomial of fifth power.

53. What Special Factoring Formulas do you know?

Answer. We know following Special Factoring Formulas:

1) Square of sum: $(a+b)^2=a^2+2ab+b^2$,

2) Square of difference: $(a-b)^2=a^2-2ab+b^2$,

3) Difference of squares: $a^2-b^2=(a+b)(a-b)$.

4) Cube of sum: $(a+b)^3=a^3+3a^2b+3ab^2+b^3=a^3+b^3+3ab(a+b)$.

5) Cube of difference: $(a-b)^3=a^3-3a^2b+3ab^2-b^3=a^3-b^3-3ab(a-b)$.

6) Sum of cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$.

7) Difference of cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$.

54. What does factoring polynomials mean?

Answer. To factor a polynomial means to write polynomial as a product of polynomials and monomials.

55. Which methods of factoring polynomials do you know?

Answer. It is known several methods of factoring polynomials.

1. Factoring out the Greatest Common Factor

For example, $12a^5b^6-9a^4bc^3+3a^3b^2c=3a^3b\cdot 4a^2b^5-3a^3b\cdot 3ac^3+$ $+3a^3b\cdot bc=3a^3b\cdot(4a^2b^5-3ac^3+bc)$.

2. Factoring by Grouping

For example, $x^2y-z^2x+y^2x-yz^2=(x^2y+y^2x)-(z^2x+yz^2)=$ $=xy\cdot(x+y)-z^2\cdot(x+y)=(x+y)\cdot(xy-z^2)$;

3. Factoring with Special Factoring Formulas

For example, $27a^3+125b^3=(3a)^3+(5b)^3=(3a+5b)\cdot(9a^2-15ab+25b^2)$.

4. Factoring Quadratic Polynomials

For example, $2x^2-6x-8=2(x+1)(x-4)$, because $2x^2-6x-8=0$ $\Rightarrow x_1=-1,\ x_2=4$ are roots of polynomial.

56. Write The Polynomial Remainder Theorem or Little Bézout's Theorem.

Answer. If we divide polynomial $P(x)=a_0x^n+a_1x^{n-1}+...+a_n$ $(a_0\neq 0;\ b\in R)$ by $(x-b)$, we obtain: $P(x)=(x-b)\cdot\varphi(x)+r$, where: $\varphi(x)$ is a quotient of division $P(x)$ by $(x-b)$; r is the remainder (number r does not depend on x).

57. What consequences of the polynomial remainder theorem do you know?

Answer. We know following.

1. If the polynomial $P(x)$ divides by $(x-a)$ without remainder, then a is a root of the polynomial.

2. If a is a root of the polynomial $P(x)$, then the polynomial divides by $(x-a)$ without remainder.

Topic 7. ROOT OF *n* POWER

58. What is the *n*th root of the number a?

Answer. The *n*-th root of the number a is a number x, where n is a positive integer, is a number x whose *n*-th power is a: $x^n=a$ $(n\in N, n\geq 2)$.

$\sqrt[n]{a}=x$ is a n-th root of the number a, where: a is called the radicand; n is called the index; x is called the value of the root.

59. What is extraction of the root?

Answer. The operation of finding the root of given number (or expression) is called extraction of the root.

60. What is an arithmetical root?

Answer. Arithmetical root is a nonnegative *n*-th root of the nonnegative number. For example, $\sqrt{9}=3$ is an arithmetical square root of nine: $3^2=9$ (numbers 3 and 9 are nonnegative ones).

61. What is the basic property of root?

Answer. The basic property of root: $\sqrt[nk]{a^{mk}}=\sqrt[n]{a^m}$, $a\geq 0$, $n\in N$, $m\in N$, $k\in N$.

The value of root does not change, if the root indexes and radicands multiply or divide by the same natural number.

62. What operations with roots do you know?

Answer. We know multiplication of roots, division of roots, extraction a root of a root, raising a root to the power (*n*-th Root of a-to-the *m*-th Power),

63. Write the rule about multiplication of roots with identical and different indexes.

Answer. $\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{ab}$. In order to multiply roots with identical indexes, it is necessary to write a root index without change, and to multiply radicands. For example, $\sqrt{2}\cdot\sqrt{6}=\sqrt{12}=2\sqrt{3}$.

$\sqrt[n]{a}\cdot\sqrt[m]{b}=\sqrt[n\cdot m]{a^m\cdot b^n}$. In order to multiply roots with different indexes, it is necessary to reduce roots to the common denominator (use the basic property of fraction) and then make their multiplication. For example, $\sqrt[5]{3}\cdot\sqrt{2}=\sqrt[10]{3^2}\cdot\sqrt[10]{2^5}=\sqrt[10]{3^2\cdot 2^5}=\sqrt[10]{9\cdot 32}=\sqrt[10]{288}$.

64. Write the rule about division of roots with identical and different indexes.

Answer. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}},\ b\neq 0.$ In order to divide roots with identical indexes, it is necessary to write down a root index without change and to divide radicands. For example, $\sqrt[3]{6a^4}:\sqrt[3]{2a}=\sqrt[3]{\frac{6a^4}{2a}}=\sqrt[3]{3a^3}=a\sqrt[3]{3}$.

$\frac{\sqrt[n]{a}}{\sqrt[m]{b}}=\frac{\sqrt[n\cdot m]{a^m}}{\sqrt[n\cdot m]{b^n}}=\sqrt[n\cdot m]{a^m:b^n}$. In order to divide roots with different indexes, it is necessary to reduce roots to the common denominator (use the basic property of fraction) and then make their division.

For example, $\frac{\sqrt[5]{3}}{\sqrt{2}}=\frac{\sqrt[10]{3^2}}{\sqrt[10]{2^5}}=\sqrt[10]{\frac{3^2}{2^5}}=\sqrt[10]{\frac{9}{32}}$.

65. Write the rule about extraction a root of a root.

Answer. $\sqrt[m]{\sqrt[n]{a}}=\sqrt[m\cdot n]{a}$. In order to extract a root of a root, it is necessary to multiply the indexes of the root and write down the radicands without change. For example, $\sqrt[3]{\sqrt{7}}=\sqrt[6]{7}$.

66. Write the rule about raising a root to the power (n-th Root of a-to-the m-th-Power).

Answer. $\left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m}$. Raising a root to the power (n-th Root of a-to-the m-th-Power)**.** For example, $\left(\sqrt[3]{-2}\right)^5=\sqrt[3]{(-2)^5}=\sqrt[3]{-32}=-\sqrt[3]{32}$.

67. Write the rule about carrying out a multiplier from the root.

Answer. Carrying out a multiplier from the root.

$$\sqrt[n]{a^{n+m}}=\sqrt[n]{a^n\cdot a^m}=\sqrt[n]{a^n}\cdot\sqrt[n]{a^m}=a\sqrt[n]{a^m},\ a>0.$$

For example, $\sqrt[5]{2^7}=\sqrt[5]{2^5\cdot 2^2}=\sqrt[5]{2^5}\cdot\sqrt[5]{2^2}=2\sqrt[5]{2^2}=2\sqrt[5]{4}$.

68. Write the rule about *taking out a factor of the radical.*

Answer. Taking out a factor of the radical.

$$a\cdot\sqrt[n]{b}=\sqrt[n]{a^n}\cdot\sqrt[n]{b}=\sqrt[n]{a^n b},\quad a\ge 0, b\ge 0\,.$$

For example, $3\sqrt[3]{6}=\sqrt[3]{3^3\cdot 6}=\sqrt[3]{27\cdot 6}=\sqrt[3]{162}\,.$

69. Which formulas for rationalization of numerator or denominator of algebraic fractions do you know?

Answer. For rationalization of numerator or denominator of fractions the following formulas are used:

$$\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2=a-b;$$

$$\left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(\left(\sqrt[3]{a}\right)^2-\sqrt[3]{a}\cdot\sqrt[3]{b}+\left(\sqrt[3]{b}\right)^2\right)=\left(\sqrt[3]{a}\right)^3+\left(\sqrt[3]{b}\right)^3=a+b$$

$$\left(\sqrt[3]{a}-\sqrt[3]{b}\right)\left(\left(\sqrt[3]{a}\right)^2+\sqrt[3]{a}\cdot\sqrt[3]{b}+\left(\sqrt[3]{b}\right)^2\right)=\left(\sqrt[3]{a}\right)^3-\left(\sqrt[3]{b}\right)^3=a-b\,.$$

For example,

$$\frac{2}{\sqrt{5}+\sqrt{3}}=\frac{2\cdot(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}=\frac{2\cdot(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}=\frac{2\cdot(\sqrt{5}-\sqrt{3})}{5-3}=\sqrt{5}-\sqrt{3}\,.$$

Where: $\sqrt{5}-\sqrt{3}$ и $\sqrt{5}+\sqrt{3}$ are reciprocally conjugate expressions.

70. What is reducing of like (similar) roots?

Answer. ***Reducing similar roots*** means their addition and subtraction.

For example, $\sqrt[3]{108}+3\sqrt[3]{32}=\sqrt[3]{27\cdot 4}+3\sqrt[3]{8\cdot 4}=3\sqrt[3]{4}+6\sqrt[3]{4}=9\sqrt[3]{4}\,.$

71. What is an arithmetic average (arithmetic mean)?

Answer. Arithmetic average of numbers $a_1, a_2 \ldots a_n$ is a number A_n if

$$A_n=\frac{a_1+a_2+\ldots+a_n}{n}.$$

72. What do we need to determine the arithmetic average of numbers?

Answer. To determine the arithmetic average of n numbers, it is necessary: 1) to find the sum of given numbers; 2) to divide this sum by the quantity of items (n).

73. What is a geometric average (geometric mean)?

Answer. Geometric average of nonnegative numbers $a_1, a_2 \dots a_n$ is a number G_n: $G_n = \sqrt[n]{a_1 \cdot a_2 \cdot \dots \cdot a_n}$.

74. What do we need to determine the geometric average of numbers?

Answer. To determine a geometric average of n positive numbers, it is necessary: 1) to find the product of given numbers; 2) to extract the n-th root of this product.

Topic 8. EQUALITY. IDENTITY. EQUATION

75. What is an equality?

Answer. Equality is a relationship between two mathematical expressions (quantities), asserting that the quantities have the same value or that the expressions represent the same mathematical object. For example, $x = y$ is an equality where x is a left side of the equality, y is a right side of the equality.

76. What is an identity?

Answer. The identity is an equality relation *A*=*B*, such that *A* and *B* contain some variables and give the same result when the variables are substituted by any values (usually numbers).

For example, $x^2 - y^2 = (x-y)(x+y)$, $(a+b)^2 = a^2 + 2ab + b^2$, $\frac{x^2-1}{x+1} = x-1$, if $x \neq -1$; $\frac{x}{\sqrt{x}} = \sqrt{x}$, if $x > 0$ are identities.

77. What is an equation?

Answer. The equation is a formula of the form *A*=*B*, where *A* and *B* are expressions that may contain one or several variables called unknowns, and "=" denotes the equality binary relation. For example, $f(x)=\varphi(x)$ is an equation with one variable x, where $f(x)$ and $\varphi(x)$ are algebraic expressions; x is a variable or unknown one. For example, $(x+7)(x-4)=0$ is an equation with one variable x; $3x+4y=0$ is an equation with two variables x and y.

78. What is a root of equation?

Answer. The root (solution) of the equation is the value of a variable where the equation will be true numerical equality.

79. What is solving the equation?

Answer. Solving the equation is finding all its roots or proving that the equation does not have solution.

80. What is the domain of definition of the equation (D)**?**

Answer. The domain of definition of the equation (D) is a set of values of a variable x, where the left and right sides of the equation make sense (are defined).

81. What equations are called equivalent ones?

Answer. Two equations $f(x)=0$ and $\varphi(x)=0$ are called equivalent ones, if the sets of their roots are the same: $f(x)=0 \Leftrightarrow \varphi(x)=0$ ($\Leftrightarrow$ is an equivalent sign). For example, equations $x+4=3x$ and $x-2=0$ are equivalent ones, because they have the same root: $x=2$.

82. What types of equations do you know?

Answer. There are different types of equations:

- ✓ linear: $ax+b=0$;
- ✓ quadratic: $ax^2+bx+c=0$;

- ✓ rational (the higher degrees):
  $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0; \quad (n \in N, \quad a_0 \neq 0);$
- ✓ irrational: $\sqrt{ax+b} - \sqrt[3]{cx-d} = 0;$
- ✓ with modulus (absolute value): $|x+a| - |x-b| = 0;$
- ✓ logarithmic: $\log_a x = m;$
- ✓ exponential: $a^{\sqrt{x}} = m;$
- ✓ trigonometric: $\sin x = a$ and others.

83. What equations is called a linear equation?

Answer. The $ax + b = 0, \quad a \neq 0$ equation is called the linear equation with one variable, where a is a coefficient at the variable x, b is a constant (free term). The root (solution) of linear equation is: b.

84. What equations is called a quadratic equation?

Answer. The $ax^2 + bx + c = 0$ $(a \neq 0)$ equation is called quadratic equation with one variable, where a is a coefficient at x^2 (the first coefficient), b is a coefficient at x (the second coefficient), c is a constant (free term).

85. What equations is called a full quadratic equation?

Answer. If $b \neq 0, c \neq 0$, then quadratic equation is called a full quadratic equation.

86. What equations is called an incomplete quadratic equation?

Answer. If $b = 0$ and $c = 0$, then quadratic equation is called ***incomplete*** quadratic equation. For example, $ax^2 = 0; \quad ax^2 + bx = 0;$ $ax^2 + c = 0$ are incomplete quadratic equations.

87. What equations is called a reduced quadratic equation?

Answer. If $a = 1$, then quadratic equation is called reduced quadratic equation. The resulted quadratic equation is written as: $x^2 + px + q = 0$.

88. What is the formula of finding roots of quadratic equation?

Answer. The roots of quadratic equation $ax^2 + bx + c = 0$ can be found with following formula: $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$, where $D = b^2 - 4ac$ is a discriminator of quadratic equation.

89. What is different cases of finding roots of quadratic equation?

Answer.

1. If $D < 0$, then $ax^2 + bx + c = 0$ equation does not have real roots.
2. If $D = 0$, then $ax^2 + bx + c = 0$ equation has equal roots: $x_1 = x_2 = \frac{-b}{2a}$.
3. If $D > 0$, then $ax^2 + bx + c = 0$ equation has two equal roots.

90. Write the Viet Theorem.

Answer. The Viet theorem. If x_1 and x_2 are roots of quadratic equation $ax^2 + bx + c = 0$, then their sum is: $x_1 + x_2 = -\frac{b}{a}$, and product is: $x_1 \cdot x_2 = \frac{c}{a}$.

91. Write the Viet Theorem for resulted quadratic equation.

Answer. The Viet theorem for resulted quadratic equation $x^2 + px + q = 0$. If the resulted quadratic equation $x^2 + px + q = 0$ has two real roots x_1 and x_2, then their sum is: $x_1 + x_2 = -p$, and the product is: $x_1 \cdot x_2 = q$.

92. Write Inverse to the Viet Theorem for resulted quadratic equation.

Answer. ***Inverse to the Viet theorem.*** If the sum of two numbers x_1 and x_2 equals $(-p)$, and their product equals q, then numbers x_1 and x_2 are the roots of the equation $x^2 + px + q = 0$.

93. What is a trinomial equation?

Answer. The equation $ax^{2n}+bx^{n}+c=0$ is called trinomial equation, if $n\geq 2$, $n\in N$, $a\neq 0$, $b\neq 0$, $c\neq 0$.

94. What is a biquadratic equation?

Answer. If $n=2,$ then equation $ax^{4}+bx^{2}+c=0$ is called biquadratic equation. Trinomial equation $ax^{2n}+bx^{n}+c=0$ may be replaced to the quadratic equation $at^{2}+bt+c=0$ with $x^{n}=t$ transformation.

95. What is a definition of an absolute value?

Answer. Absolute value (modulus) of the number ***x*** is: $|x|$.

The definition of absolute value (modulus):

$$|x|=\begin{cases} x, & x\geq 0,\\ -x, & x<0,\end{cases} \qquad |f(x)|=\begin{cases} f(x), & f(x)\geq 0,\\ -f(x), & f(x)<0.\end{cases}$$

96. What do you know about irrational equations?

Answer. Irrational equations are the equations, that contains variable under the radical or variable is a base of power with fractional exponent. For example, $\sqrt{1-x}=3,\ \sqrt{x^{2}}+\sqrt{x^{5}}=5,\ \sqrt[3]{x}+\sqrt[3]{x-1}=-1,$ $x^{\frac{1}{3}}+4=0$ is an irrational equation.

97. What methods used to solve irrational equations do you know?

Answer. There are two methods that are used to solve irrational equations:

1) method of raising both sides of equation to a same power;

2) method of introducing new variable.

Topic 9. SYSTEMS OF EQUATIONS

98. What is a system of algebraic equations?

Answer. A system of equations is a collection of two or more equations with the same set of unknowns.

99. What does it mean to solve system of the equations?

Answer. To solve system of the equations means to find values for each of the unknowns that will satisfy equation in the system or to prove that there are no solutions.

100. Which systems of equations do you know?

Answer. The system is called compatible one if it has at least one solution and incompatible one if it has no one solution.

101. What is a system of linear equations?

Answer. The system of linear equations is a collection of linear equations involving the same set of variables.

102. How many solutions does the system of n linear equations contains n unknown have?

Answer. If the system of the n linear equations contains n unknown terms then three cases are possible:

1) the system has no solutions;
2) the system has only one solution;
3) the system has infinite set of solutions.

103. What does investigating the system of equations mean?

Answer. To investigate system means to define values where the system has the unique solution; or no solutions; or infinite set of solutions.

104. Write the system of linear equations with two unknowns in standard view?

Answer. The system of linear equations with two unknowns: $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$, where x and y are unknown terms; a_1, a_2, b_1, b_2 are coefficients; c_1, c_2 are constants.

105. Write the determinant of the second order?

Answer. The determinant of the second order (main determinant of the system) consists of coefficients at variables:

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1.$$

Numbers a_1, a_2, b_1, b_2 are called determinant elements.

Elements a_1, a_2 or b_1, b_2 located on vertical, form determinant columns.

Elements a_1, b_1 or a_2, b_2, located on rows, form determinant rows.

Elements a_1, b_2 form the main diagonal of determinant.

Elements a_2, b_1 form an auxiliary diagonal of determinant.

106. Write an auxiliary determinant?

Answer. The auxiliary determinant consists of coefficients at variable and constants: $\Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1b_2 - c_2b_1$, $\Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1c_2 - a_2c_1$.

107. Write the Cramer's rule?

Answer. Cramer's rule. If the main determinant of the system is not equal to zero then this system has the unique solution:

$x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$. In this case: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, $\Delta \neq 0$.

108. What is a solution if the main determinant is equal to zero?

Answer. If the main determinant is equal to zero, and at least one auxiliary determinant is not equal to zero, then the system has no solutions (incompatible). The incompatibility condition can be written down as: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, $\Delta \neq 0$, $\Delta_x \neq 0$, $\Delta_y \neq 0$.

109. What is a solution if the both main determinant and auxiliary determinants are equal to zero?

Answer. If the both main determinant and auxiliary determinants are equal to zero, then the system has infinite set of solutions. It can be written down as: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, $\Delta = \Delta_x = \Delta_y = 0$.

110. How could we solve nonlinear equations?

Answer. To solve nonlinear systems of algebraic equations are used the same methods as for solving the systems of linear equations:

1) method of substitution;
2) method of algebraic addition.

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