UDC 658.51.012

The stabilization problem the flow parameters of the production line

doctor of technical sciences Pihnastyi O.M., 0000-0002-5424-9843
National Technical University "Kharkiv Polytechnic Institute", Ukraine
pihnastyi@gmail.com
doctor of phys.-math. sciences Khodusov V.D., 0000-0003-1129-3462
V.N. Karazin Kharkiv National University, ", Ukraine
vdkhodusov@karazin.ua

Annotation. The problem of designing a system for optimal control of random deviations of flow parameters of a production line from the planned value is considered. The PDE-model of the production line was used as a foundation for the development of an algorithm for optimal control of the parameters production line. The method of Lyapunov functions was used to construct a system of operational control of flow parameters of production lines. The problem of stabilization of the value flow parameters of the production line (the task of operational control of the flow production) is formulated. The equations for the production line parameters in small disturbances are obtained. To assess the technological resources of the production system, which spent on the formation of control actions, the integral of the production line quality was introduced, the minimum value of which corresponds to the rapid damping of the flow parameters disturbance. Taking into account the specified quality criterion, the Lyapunov function of the production line is determined. Control actions are found that ensure the asymptotic stability of a given planned state of the production line flow parameters for a steady and transient mode of operation.
Keywords: production line, production control system, PDE-model, flow production.

The problem formulation

The main task of operational planning and control of production lines flow parameters is to ensure the rhythm of production. The changing the work tempo of a separate technological section in the production line requires a change in the work schedule of the production line, procurement shops and other departments [1, p.113]. Ensuring the continuous nature of production is reduced to the issue of synchronization of generalized technological operations sequentially located along the technological route.

It is known that increasing the performance of production line control is achieved by reducing costs and reducing the volume of work in progress [2]. On the other hand, with an increase in the production volume, an increase in the value of interoperation backlogs is required to smooth out differences in the productivity of individual technological equipment. No matter how stable the production programs and operational tasks for the product segment are, in the production process, the deviations arise that require adjustment [3]. There is a need to stabilize the production line flow parameters to ensure the rhythm of the production process [1, p.181]. An important issue is to determine the frequency of use of control actions to stabilize the production line flow parameters. The dispatch service, which constantly monitors the production progress, collects the necessary information about the actual execution of shift tasks, the presence of interaction delays and the speed of movement of parts along the technological route of the production line. To obtain operational information about the production progress, an alert and alarm system is used, which is a variety of sensors and recording devices that automatically transmit actual readings from workplaces [4]. Information about the state of the production line parameters during a regulated period of time is accumulated, processed, compared with planned targets and transmitted to production units in the form of managerial actions: recommendations and orders. The purpose of these influences is to eliminate deviations from the planned result. [5].

The research purpose

The presence and relevance of the above-considered problems associated with ensuring the continuous nature of production and synchronizing the productivity of individual technological operations of the production line determined the purpose of the research: setting and solving the problem of stabilizing the production line flow parameters. To construct optimal controls that ensure the stabilization of the production line flow parameters, the class of PDE-models of the production line was used [6].

The research methods

To describe the state of the production flow line, most researchers have used two flow parameters with a sufficient degree of accuracy: the value of interoperation backlogs before a technological operation; the tempo of processing of parts in a technological operation [1,2,6].

In the PDE-model, the interoperation backlogs before a technological operation and the tempo of processing parts in a technological operation are determined through the functions $[\chi]_0(t,S)$ and $[\chi]_1(t,S)$ [5,7], where the S coordinate determines the technological position of the part in the technological route at time t [5,7]. The flow parameters $[\chi]_0(t,S)$, $[\chi]_1(t,S)$ of a production line without additional control cannot be in a stable stationary state for a long time [8]. Therefore, the main task of the production line flow parameters operational control is to form control actions that provide a stable state of the flow parameters for a sufficiently long time. At the same time, the consumption of technological resources for control the flow parameters $[\chi]_n(t,S)$ deviations from their unperturbed planned values $[\chi]_n^*(t,S)$ should not exceed the specified values. An equally important task is to ensure that the transition of the production system from one state with one production plan to another state with another production plan was carried out in the shortest possible time with a minimum deviation of the flow parameters from the required standard values [7]. To construct a system of operational control of production lines flow parameters, the method of Lyapunov functions is used. The disturbing factors are meant the impact on the flow parameters of the production line $[\chi]_n(t,S)$, which were not taken into account when modelling the production process due to their smallness in comparison with the main factors that affect production.

They can act as instantly, which is reduced to a small change in the current state of the production line flow parameters, as continuously. This means that the equations for determining the state of the flow parameters of the technological process differ from the true ones by some small correction terms. Let us supplement the balance equations [5]

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=0\,, \tag{1}$$

$$\frac{\partial[\chi]_n(t,S)}{\partial t}+\frac{\partial[\chi]_{n+1}(t,S)}{\partial S}=nf(t,S)[\chi]_{n-1}(t,S)\,,\quad n=1,2.3....\,, \tag{2}$$

that determine the state of the production line flow parameters with control functions $\Upsilon_n(t,S)$

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=\Upsilon_0(t,S)\,, \tag{3}$$

$$\frac{\partial[\chi]_n(t,S)}{\partial t}+\frac{\partial[\chi]_{n+1}(t,S)}{\partial S}-nf(t,S)[\chi]_{n-1}(t,S)=\Upsilon_n(t,S)\,,\qquad n=1,2.3....\,, \tag{4}$$

Let the system of equations (3), (4) correspond to an unperturbed solution [8]:

$$[\chi]_n=[\chi]_n{}^*(t,S)\,,\qquad \Upsilon_n(t,S)=\Upsilon_n^*(t,S)\,, \tag{5}$$

and the flow parameters of the production line will receive at the current moment in time previously unknown random small disturbances: $[y]_n$:

$$[y]_n=[\chi]_n-[\chi]_n{}^*\,, \tag{6}$$

for the elimination of which control actions are required $u_m=u_m\left(t,[y]_n\right)$. Let us linearize the system of equations (3), (4) for the production line flow parameters taking in account to small perturbations (6) in the vicinity of the unperturbed state of the flow parameters (5):

$$\frac{\partial[y]_0}{\partial t}+\frac{\partial[y]_1}{\partial S}=\sum_{m=0}^{N_k}q_{0m}u_m\,,\qquad n=0,1,...N_n\,, \tag{7}$$

$$\frac{\partial[y]_n}{\partial t}+\frac{\partial[y]_{n+1}}{\partial S}=nf(t,S)\big|_0[y]_{n-1}+$$

$$+\sum_{m=0}^{N_n}\left(n[\chi]_{n-1}(t,S)\frac{\partial f(t,S)}{\partial[\chi]_m(t,S)}\right)\Bigg|_0[y]_m+\sum_{m=0}^{N_n}q_{nm}u_m\,, \tag{8}$$

$$\sum_{m=0}^{N_n}q_{nm}u_m=Y_n(t,S)-Y_n^*(t,S)\,,\qquad \sum_{m=0}^{N_n}q_{nm}u_m \ll Y_n^*(t,S)\,.$$

The symbol $\big|_0$ denotes that the Taylor series expansion was carried out in the vicinity of the unperturbed state of flow parameters $[\chi]_n{}^*(t,S)$ (5).

Let us formulate the problem of operational control (the problem of stabilizing the production line flow parameters) by the flow parameters of a production line. It is required to find such control actions

$u_m = u_m(t,[y]_n)$ on the deviations of the flow parameters (6) arising as a result of the production process, which will ensure the asymptotic stability of the steady (unperturbed) planned state of the flow parameters (5) of the production line. It is assumed that the dispatching service measures the current values of the flow parameters $[\chi]_n$, the data on which is received at the dispatching point by means of production signalling. On the basis of this measurement, the control device generates a control $u_m = u_m(t,[y]_n)$ that affects the pace of processing parts and the control the stocks.

These influences should ensure the asymptotic stability of the given planned state of the flow parameters (5). The functions $u_m = u_m(t,[y]_n)$ are assumed to satisfy the equalities

$$u_m(t,0) = 0, \tag{9}$$

are defined and continuous in the considered domain, and are also not constrained by any additional inequalities.

Main results

Applied problems of stabilization of flow parameters of a production line along with the requirements of asymptotic stability of the unperturbed state of flow parameters (5) contain wishes about the best quality of the transient process during the transition to the unperturbed state at $t \to \infty$. At the same time, wishes were expressed about the lowest possible cost of technological resources (energy, raw materials, materials, labour resources, etc.) spent on the formation of control actions. $u_m = u_m(t,[y]_n)$. We express these wishes in the form of a condition for the minimality of some integral:

$$I = \int_{t_0}^{\infty} \omega(t,[y]_n,u_m)dt. \tag{10}$$

The function $\omega(t,[y]_n,u_m)$ is a non-negative function, when choosing which one should take into account: a) the condition of the minimum of the quality integral (10) should ensure a sufficiently fast decay of the arisen disturbance of the flow parameter $[y]_n$;

b) the value of the quality integral (10) should satisfactorily assess the technological resources of the production system spent on the formation of control actions $u_m = u_m(t,[y]_n)$; c) the function $\omega(t,[y]_n,u_m)$ should provide a simple solution to the problem and, if possible, in a closed-form.. In many cases, interesting from a practical point of view, these requirements are met by a function $\omega(t,[y]_n,u_m)$ chosen in the form of a definitely positive quadratic form

$$\omega = \sum_{n,m=0}^{N_n} \alpha_{n,m}[y]_n[y]_m + \sum_{n,m=0}^{N_n} \beta_{n,m}u_n u_m. \tag{11}$$

where $\alpha_{n,m}$, $\beta_{n,m}$ the coefficients characterizing the costs associated with the presence of the flow parameters deviations and the costs associated with the control actions necessary to eliminate these deviations.

The problem of stabilizing the production line flow parameters under the condition of the minimum quality criterion (10) is the problem of optimal operational control of the production line flow parameters. The problem is formulated as follows: let the quality criterion be chosen for the mode of operation of the production line in the form of an integral (10). It is required to find such control actions $u_m = u_m(t,[y]_n)$ that will ensure the asymptotic stability of the given state of the production line flow parameters $[\chi]_n(t,S)$ due to the balance equations in small perturbations (8). In this case, whatever other control actions $u_m = u_m(t,[y]_n)$ that solve the problem, the inequality must be satisfied

$$I = \int_{t_0}^{\infty} \omega(t,[y]_n^0,u_m^0)dt \le \int_{t_0}^{\infty} \omega(t,[y]_n^*,u_m^*)dt \qquad (12)$$

for all initial conditions in the domain of existence of solutions to the system of equations (3), (4).

The functions $u_m = u_m(t,[y]_n)$ that solve the problem (8) are the optimal operational control of the deviations $[y]_n(t,S)$ of the flow parameters $[\chi]_n(t,S)$ of the production line from the given undisturbed state $[\chi]_n^*(t,S)$ (5). The solution to the problem (3), (4), as a rule, has a unique solution [9]. The set of functions $u_m = u_m(t,[y]_n)$ that solve the problem contains a lot of arbitrariness.

For the problem of stabilization of the production line flow parameters, as well as for the general problem of stability, a research theory can be developed in the first approximation [10].

There are cases when the solution to the problem is determined by the first approximation, as well as critical cases when the possibility of solving the problem and the desired actions $u_m = u_m(t,[y]_n)$ themselves are determined by terms of higher-order of smallness in the system of equations (3), (4).

Let us investigate the problem of stabilizing the production line flow parameters for the linearized system of equations (3), (4). As a quality criterion, let's choose:

$$\omega(t) = \frac{1}{S_d}\int_0^{S_d}\left(\sum_{n,m=0}^{N_n}\alpha_{nm}[y]_n[y]_m + \sum_{n,m=0}^{N_n}\beta_{nm}u_m u_n\right)dS\,, \qquad (13)$$

where S_d is the coordinate of the technological position of the final product manufacturing operation. The choice of the type of quality criterion (13) is due to the wishes about the least expenditure of technological resources of the enterprise for the formation of control actions with the requirements to ensure the least deviation $[y]_n(t,S)$ of the flow parameters from $[\chi]_n(t,S)$ the undisturbed state $[\chi]_n^*(t,S)$. Let's use the expansion of small perturbations $[y]_n(t,S)$ of flow parameters $[\chi]_n(t,S)$ and control actions $u_m(t,S)$ in a Fourier series with coefficients $\{y_n\}_0$, $\{y_n\}_j$, $[y_n]_j$, $\{u_n\}_0$, $\{u_n\}_j$, $[u_n]_j$:

$$[y]_n = \{y_n\}_0 + \sum_{j=1}^{\infty}\{y_n\}_j \sin[k_j S] + \sum_{j=1}^{\infty}[y_n]_j \cos[k_j S], \qquad k_j = \frac{2\pi j}{S_d}$$

$$[u]_n = \{u_n\}_0 + \sum_{j=1}^{\infty}\{u_n\}_j \sin[k_j S] + \sum_{j=1}^{\infty}[u_n]_j \cos[k_j S], \qquad (14)$$

The integrand $\omega(t)$ (13) of the quality integral has the form

$$\omega(t) = \sum_{n,m=0}^{N_n}\alpha_{n,m}\left(\{y_n\}_0\{y_m\}_0 + \frac{1}{2}\sum_{j=1}^{\infty}\{y_n\}_j\{y_m\}_j + \frac{1}{2}\sum_{j=1}^{\infty}[y_n]_j[y_m]_j\right) +$$

$$+ \sum_{n,m=0}^{N_n}\beta_{n,m}\left(\{u_n\}_0\{u_m\}_0 + \frac{1}{2}\sum_{j=1}^{\infty}\{u_n\}_j\{u_m\}_j + \frac{1}{2}\sum_{j=1}^{\infty}[u_n]_j[u_m]_j\right),$$

$$\beta_{mn} = \beta_{nm}. \qquad (15)$$

Let us define the Lyapunov function $V^0\left(t,\{y_n\}_0,\{y_n\}_j,[y_n]_j\right)$ as a quadratic form:

$$V^0(t,[y]_n) = \frac{1}{S_d}\int_0^{S_d}\sum_{m,n=0}^{N_n} c_{nm}[y]_n[y]_m dS =$$

$$= \sum_{n,m=0}^{N_n} c_{n,m}\left(\{y_n\}_0\{y_m\}_0 + \frac{1}{2}\sum_{j=1}^{\infty}\{y_n\}_j\{y_m\}_j + \frac{1}{2}\sum_{j=1}^{\infty}[y_n]_j[y_m]_j\right).$$

(16)

Let's compose an expression:

$$B[V^0,t] = \sum_{n=0}^{N_n}\sum_{j=1}^{\infty}\frac{\partial V^0}{\partial[y_n]_j}\frac{d[y_n]_j}{dt} + \omega \qquad (17)$$

which by $u_m = u_m\left(t,[y]_n\right)$ must have a minimum and tend to zero in this case. Hence the first equation for constructing the Lyapunov function $V^0\left(t,\{y_n\}_0,\{y_n\}_j,[y_n]_j\right)$ and optimal control actions $u_m = u_m\left(t,[y]_n\right)$:

$$B[V^0,t] = 0. \qquad (18)$$

Substituting instead of small perturbations and the corresponding control actions their expansion (14), equation (18) is obtained in the following form:

$$B[V^0,t]=\sum_{n,m=0}^{N_n}\frac{dc_{n,m}}{dt}\left(\{y_n\}_0\{y_m\}_0+\frac{1}{2}\sum_{j=1}^{\infty}\{y_n\}_j\{y_m\}_j+\frac{1}{2}\sum_{j=1}^{\infty}[y_n]_j[y_m]_j\right)+$$

$$+\sum_{n,m=0}^{N_n}c_{n,m}\left(2\{y_n\}_0\frac{d\{y_m\}_0}{dt}+\sum_{j=1}^{\infty}\{y_n\}_j\frac{d\{y_m\}_j}{dt}+\sum_{j=1}^{\infty}[y_n]_j\frac{d[y_m]_j}{dt}\right)+$$

$$+\sum_{n,m=0}^{N_n}\alpha_{n,m}\left(\{y_n\}_0\{y_m\}_0+\frac{1}{2}\sum_{j=1}^{\infty}\{y_n\}_j\{y_m\}_j+\frac{1}{2}\sum_{j=1}^{\infty}[y_n]_j[y_m]_j\right)+$$

$$+\sum_{n,m=0}^{N_n}\beta_{n,m}\left(\{y_n\}_0\{y_m\}_0+\frac{1}{2}\sum_{j=1}^{\infty}\{y_n\}_j\{y_m\}_j+\frac{1}{2}\sum_{j=1}^{\infty}[y_n]_j[y_m]_j\right)=0$$

(19)

Differentiating $B[V^0,t]$ on $\{u_m\}_0$, $\{u_m\}_j$, $[u_m]_j$ and equating the results to zero, we obtain the missing equations for determining the Lyapunov function $V^0\left(t,\{y_n\}_0,\{y_n\}_j,[y_n]_j\right)$ and control actions $u_m=u_m\left(t,[y]_n\right)$:

$$\frac{B[V^0,t]}{\partial\{u_n\}_0}=\sum_{m=0}^{N_n}\frac{\partial V^0}{\partial\{y_n\}_0}\cdot\frac{\partial\left[\frac{d\{y_m\}_0}{dt}\right]}{\partial\{u_n\}_0}+2\sum_{m=0}^{N_n}\beta_{mn}\{u_m\}_0=0\,;$$

$$\frac{B[V^0,t]}{\partial\{u_n\}_j}=\sum_{m=0}^{N_n}\frac{\partial V^0}{\partial\{y_n\}_j}\frac{\partial\left[\frac{d\{y_m\}_j}{dt}\right]}{\partial\{u_n\}_0}+\sum_{m=0}^{N_n}\beta_{mn}\{u_m\}_j=0\,;$$

$$\frac{B[V^0,t]}{\partial[u_n]_j}=\sum_{m=0}^{N_n}\frac{\partial V^0}{\partial[y_m]_j}\frac{\partial\left[\frac{d[y_m]_j}{dt}\right]}{\partial[u_n]_0}+\sum_{m=0}^{N_n}\beta_{mn}[u_m]_j=0\,. \quad (20)$$

The resulting equations can be to solve for $\{u_m\}_0$, $\{u_m\}_j$, $[u_m]_j$:

$$\{u_m\}_0=-\frac{1}{2}\sum_{k=0}^{N_n}\frac{\Delta_{km}}{\Delta}\sum_{n=0}^{N_n}\frac{\partial V^0}{\{y_n\}_0}\frac{\partial\left[\frac{d\{y_n\}_0}{dt}\right]}{\partial\{u_k\}_0}\,,$$

$$\{u_m\}_j=-\frac{1}{2}\sum_{k=0}^{N_n}\frac{\Delta_{km}}{\Delta}\sum_{n=0}^{N_n}\frac{\partial V^0}{\{y_n\}_j}\frac{\partial\left[\frac{d\{y_n\}_j}{dt}\right]}{\partial\{u_k\}_j}\,,$$

$$[u_m]_j = -\frac{1}{2}\sum_{k=0}^{N_n}\frac{\Delta_{km}}{\Delta}\sum_{n=0}^{N_n}\frac{\partial V^0}{[y_n]_j}\frac{\partial\left[\frac{d[y_n]_j}{dt}\right]}{\partial[u_k]_j}, \qquad (21)$$

where Δ_{km} is the algebraic complement of the element of the k-th row and m-th column, and Δ is the determinant of the system of equations (20).

Substituting the obtained values $\{u_m\}_0$, $\{u_m\}_j$, $[u_m]_j$ into equations (8) taking into account the expansion (14), we obtain an equation for determining the Lyapunov function $V^0\left(t,\{y_n\}_0,\{y_n\}_j,[y_n]_j\right)$. Equating the coefficients of the products $\{y_n\}_0\{y_m\}_0$, $\{y_n\}_j\{y_m\}_j$, $[y_n]_j[y_m]_j$ linearly independent quantities $\{y_n\}_0$, $\{y_n\}_j$, $[y_n]_j$ to zero, we obtain equations for determining the coefficients $c_{nm}(t)$.
In the case of a steady production process, the Lyapunov function $V^0\left(\{y_n\}_0,\{y_n\}_j,[y_n]_j\right)$ может быть представлена квадратичной can be represented by a quadratic form with constant coefficients $c_{n,m} = const$:

$$V^0 = \sum_{n,m=0}^{N_n} c_{n,m}\left(\{y_n\}_0\{y_m\}_0 + \frac{1}{2}\sum_{j=1}^{\infty}\{y_n\}_j\{y_m\}_j + \frac{1}{2}\sum_{j=1}^{\infty}[y_n]_j[y_m]_j\right) \qquad (22)$$

Sufficient conditions for the solvability of the problem on the definitely positiveness of form (22) are determined by the rank of the matrix $W = \{Q, QB_j\}$, where Q is the matrix $\{q_{nm}\}$, B_j the matrix for the coefficients at $\{y_n\}_0$, $\{y_n\}_j$, $[y_n]_j$ in the system of equations (8). In the case when the matrix $W = \{Q, QB_j\}$ has a rank equal to the order of system (8), then the problem of stabilizing the flow parameters of the production line has a unique solution.

Conclusions and research prospects

In this paper, a method for designing optimal control systems for the parameters of production flow lines is proposed, which ensures the asymptotic stability of their state. The prospect for further research is the development of algorithms for optimal control of the productivity of technological equipment of the production line for transient and stationary modes of operation.

References

1. Shkurba V.V. Discrete production planning in an automated control system. / V.V.Shkurba et al. - Kiev: Technics, 1975. - P. 296.(In Russian)

2. Kempf K. Optimization models for production planning. Planning Production and Inventories in the Extended Enterprise: A State of the Art Handbook. / K.Kempf, P.Keskinocak, R.Uzsoy –Springer-Verlag, New York, 2010. – P. 587.
3. Hopp W.J. Factory Physics: Foundations of Manufacturing Management. / W. J. Hopp, M.L. Spearman. – Irwin/McGraw-Hill, Boston, 2001. – P. 698. https://doi.org/10.1080/15320370108500206
4. Sinitsa L.M. Organization of production. / L.M. Tit. - Minsk: UP IVC Minfin, 2003. –C. 512. (In Russian)
5. Korobetsky Yu.P. Simulation models in flexible systems. / Yu.P. Korobetsky, S.K. Ramazanov. - Lugansk: Ed. VNU, 2003. - P. 280 (In Russian)
6. Pihnastyi O.M. Statistical theory of control systems of the flow production / O.M. Pihnastyi. –LAP LAMBERT Academic Publishing. – 2018. - 436 p. –ISBN: 978-613-9-95512-1.
7. Berg R. Partial differential equations in modelling and control of manufacturing systems/ R.Berg. –Netherlands, Eindhoven Univ. Technol., 2004. –P. 157
8. Pihnastyi O.M. Questions of stability of macroscopic parameters of technological processes of mass production / V.P. Demutskiy, O.M.Pihnastyi // Reports of the National Academy of Sciences of Ukraine. –Kiev: Publishing House "Academperiodika". - 2006. –№ 3. –C. 63-67. (In Russian)
9. Malkin I.G. The theory of stability of motion / I.G. Malkin. - Moscow: Nauka, 1966 .-- P. 531. (In Russian)
10. Lyapunov A.M. General problem of motion stability / A.M. Lyapunov. - M .: Gostekhizdat, 1950 .-- P. 395. (In Russian)