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*IU. KHOROSHAYLO, S. YEFYMENKO, I. SEZONOVA***MATHEMATICAL MODEL OF COLOR OPTOELECTRONIC MEASUREMENT**

The work is devoted to the analysis of methods of mathematical modeling, one of the most important in human life – the organ of vision, which accounts for more than 90% of all incoming information about the external world. The human eye is a unique (natural) natural apparatus. Therefore, the principles of his work have long attracted the attention of scientists. At the heart of the tasks posed in this paper were questions closely related to the theory of human perception of color.

Still I. Newton the ground for creation of linear model of color vision was prepared. For the first time one of such models was suggested by T. Young. But T. Young was interested not so much in the psychophysical side of the phenomenon as in the physiological aspect, therefore, from the point of view of psychophysics, his results look like an episode. The most consistent continuation of Newton's ideas was Maxwell, who attempted to experimentally determine a specific kind of weight functions that characterize the sensitivity of the eye to radiation with different wavelengths, called spectral sensitivity functions of the eye. The first attempt to create an axiomatic theory of the eye was undertaken by Grassmann. Relying on Newton's results and his idea of a "null-organ", he formulated the laws of color vision – the law of additivity, the law of three-dimensionality, the law of continuity. Starting from these laws, Schrödinger attempted to derive Newton's transformations from them in a purely formal way. However, the imperfection of the mathematical algorithm used by him and the formulation of the laws of vision did not allow him to do this correctly enough. But on the whole, Schroedinger's research should be seen as an essential step in the development of the theory of vision.

Based the analysis of existing mathematical methods of vision research, the present work accepts the interpretation of input signals in the form of Hilbert spaces. The main reason for this is as follows. As studies of sensory systems show, the description of the work of sense organs leads us to linear functionals whose general form is known in Hilbert spaces or in spaces of type L_p .

The authors on of the paper propose a method and an apparatus for colorimetric measurements. The main element of the color tone recognition device is a sensor (color sensor) in which to increase the sensitivity and reliability of measurements, as well as simplify the tuning process, not three, but more (number) photodetectors are used.

Keywords: colorimetry, perception of color, sensory systems, color coordinates, a photodetector, light sensor.

*Ю. Є. ХОРОШАЙЛО, С. А. ЕФИМЕНКО, І. К. СЕЗОНОВА***МАТЕМАТИЧНА МОДЕЛЬ КОЛОРИМЕТРІЇ**

Робота присвячена аналізу методів математичного моделювання однією з найбільш важливих в життєдіяльності людини - органу зору, на частку якого припадає понад 90% всієї що надходить інформації про зовнішній світ. Людське око є унікальний (природний) природний апарат. Тому принципи його роботи здавна привертала увагу вчених. В основі поставлених в даній роботі завдань лежали питання, тісно пов'язані з теорією сприйняття кольору людиною.

Грунтуючись на аналізі існуючих математичних методів дослідження зору в даній роботі прийнята інтерпретація вхідних сигналів у вигляді Гільбертових просторів. Основна причина подібного в наступному. Як показують дослідження сенсорних систем, опис роботи органів почуттів призводить нас до лінійним функціоналом, загальний вигляд яких відомий в Гільбертових просторах або в просторах типу L_p .

Авторами статті пропонується метод і пристрій для колориметричних вимірювань. Основним елементом пристрою розпізнавання кольорних відтінків є датчик (сенсор) кольору, в якому для збільшення чутливості і достовірності вимірювань, а також спрощення процесу налаштування, використовується не три, а більшу кількість (число) фотоприймачів.

Ключові слова: колориметрія, сприйняття кольору, сенсорні системи, кольорні координати, фотоприймач, датчик світла.

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Работа посвящена анализу методов математического моделирования одной из наиболее важных в жизнедеятельности человека – органу зрения, на долю которого приходится более 90% всей поступающей информации о внешнем мире. Человеческий глаз представляет собой уникальный (природный) естественный аппарат. Поэтому принципы его работы издавна привлекали внимание ученых. В основе поставленных в данной работе задач лежали вопросы, тесно связанные с теорией восприятия цвета человеком.

Основываясь на анализе существующих математических методов исследования зрения в настоящей работе принята интерпретация входных сигналов в виде гильбертовых пространств. Основная причина подобного в следующем. Как показывают исследования сенсорных систем, описание работы органов чувств приводит нас к линейным функционалам, общий вид которых известен в гильбертовых пространствах или в пространствах типа L_p .

Авторами статьи предлагается метод и устройство для колориметрических измерений. Основным элементом устройства распознавания цветовых оттенков является датчик (сенсор) цвета, в котором для увеличения чувствительности и достоверности измерений, а также упрощения процесса настройки, используется не три, а большее количество (число) фотоприемников.

Ключевые слова: колориметрия, восприятие цвета, сенсорные системы, цветовые координаты, фотоприемник, датчик света.

The problems in criminalistics, astronomy, nondestructive test, psychophysics are linked to research of sensor systems [1, 2, 3]. One of the most important studied now system in human life is the organ of vision, which accounts for more than 90% of all incoming information about the external world. The human eye is a unique natural apparatus. Therefore, the principles of his work have long attracted the attention of scientists. At the heart of the tasks posed in this paper were questions closely related to the theory of human perception of color.

Still I. Newton the ground for creation of linear model of color vision was prepared. For the first time one of such models was suggested by T. Young. He has noted coordinates in the form of following integrals:

$$\begin{aligned}
 U_1 &= \int_{\lambda_1}^{\lambda_2} b(\lambda) K_1(\lambda) d\lambda \\
 U_2 &= \int_{\lambda_1}^{\lambda_2} b(\lambda) K_2(\lambda) d\lambda \\
 U_3 &= \int_{\lambda_1}^{\lambda_2} b(\lambda) K_3(\lambda) d\lambda
 \end{aligned}
 \tag{1}$$

here $b(\lambda)$ – emission spectrum;

$K_1(\lambda)$, $K_2(\lambda)$, $K_3(\lambda)$ – some weight functions characterizing sensitivity of an eye to emissions with various wave lengths;

λ_1 , λ_2 – the minimum and maximum light emissions wave length, visible an eye.

But T. Young was interested not so much in the psychophysical side of the phenomenon as in the physiological aspect, therefore, from the point of view of psychophysics, his results look like an episode. The most consistent continuation of Newton's ideas was Maxwell, who attempted to experimentally determine a specific kind of weight functions K_1 , K_2 , K_3 , called spectral sensitivity functions of the eye. The first attempt to create an axiomatic theory of the eye was undertaken by Grassmann. Relying on Newton's results, he formulated the laws of color vision. Let's result one of these statements.

The law of additivity. The sum of pairwise equal colors also an essence equal colors.

The law of three-dimensionality. Any four colors are linearly dependent, however there are triples of linearly independent colors.

The law of continuity. To the continuous change of emission correspond the continuous change of color.

Starting from these laws, Schrödinger attempted to derive transformations (1) from them in a purely formal way. However, the imperfection of the mathematical algorithm used by him and the formulation of the laws of vision did not allow him to do this correctly enough. But on the whole, Schroedinger's research should be seen as an essential step in the development of the theory of vision. They have shown a possibility of successful objective study of subjective states of the person and have closely brought his continuers to mathematical description of perceptions [4].

For the first time J.E. Shabanov-Kushnarenko had been gave on the basis of some other ideas the true proof of Schrödinger's assumption [1, 2]. He had been proposed the new definition of Grassman's laws in which concepts of addition of colors and multiplication of color to constant number are not used. As a result is shown, that the mathematical model of color vision can be deduced in the form of conclusion from the new definition of Grassman's laws. For the first time the mathematical description of input and output signals of vision organ for a case of visual pictures of any kind (nonstationary and nonuniform) has been given and the task of mathematical modeling of vision of the person is formulated. The transformation of signals underlying in base of vision spectral sensitivity is established. It described by following formulas:

$$\begin{aligned}
 \alpha_1(x, y, t) &= \int_{\lambda_1}^{\lambda_2} b_\lambda(x, y, t) e_1(\lambda) d\lambda \\
 \alpha_2(x, y, t) &= \int_{\lambda_1}^{\lambda_2} b_\lambda(x, y, t) e_2(\lambda) d\lambda \\
 \alpha_3(x, y, t) &= \int_{\lambda_1}^{\lambda_2} b_\lambda(x, y, t) e_3(\lambda) d\lambda \\
 \bar{S}(x, y, t) &= F[\bar{B}(x, y, t)]
 \end{aligned}
 \tag{3}$$

This is generalization of triple-component vision known model.

All this has opened a way for deep penetration of mathematical methods into research of diverse functions of vision such as inertia and irradiation, static and dynamic adaptation, the phenomena of edge contrast, etc. Thus it was found out, that for the description of all these properties it is necessary to use integrated operators of more special kind. They are operators of convolution, operators with factorial kernels, the sums of integrated operators, etc.

Based on the analysis of existing mathematical methods of vision research, the present work accepts the interpretation of input signals in the form of Hilbert spaces. The main reason for this is as follows. As studies of sensory systems show, the description of the work of sense organs leads us to linear functionals whose general form is known in Hilbert spaces or in spaces of type L_p .

The big attention recently is given for studying of recognition process by the person of color shades and construction of devices for classification of subjects under their color characteristics [7, 8, 9, 10]. However mathematical models of spectral sensitivity of visual system had some blanks in rigidity of argumentation till the present time.

The authors of the paper have some the developed methods and devices realizing them for colorimetric measurements [7, 8]. These methods have been based on studying of the materials presented in [11, 12, 13]. The Basic element of color shades recognition devices is the sensor of color which to each emission assigns three signals, proportional to color coordinates. For transformation of light energy in electric it is usually used three photodetectors. It's spectral characteristics should be in the field of a visible section of a spectrum and reproduce one of curves of color addition. As it is difficult to select photodetectors with such performances so before them usually install the light filters which have been selected so that resultant performance of a photodetector and a light filter $f(\lambda)$ coincided with one of addition curves.

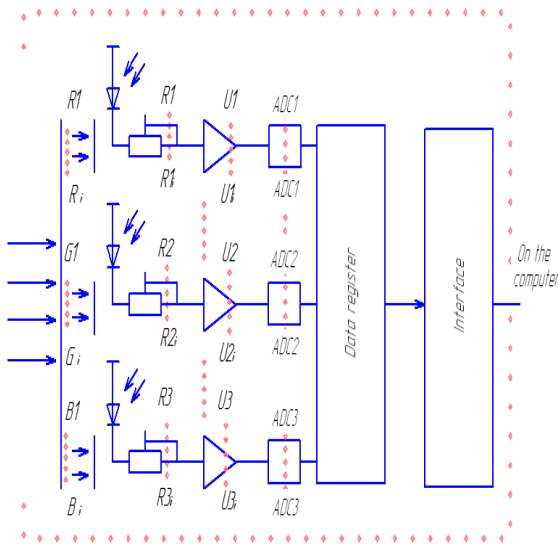


Fig. 1 – Principle of operation of the color sensor

For this purpose it is necessary to realize individual adjustment of a spectral characteristic of a light filter that is rather labor-intensive and diligent operation. For this reason regard color sensors in which uses not three, and more quantity of photodetectors that gives the possibility to reproduce the given curve of color addition by systematic transformations of signals and to simplify tuning process essentially, and also to raise high-speed performance and accuracy of measuring. The schematic illustration of such color sensor (fig. 1) and process of its experimental tuning [3] is observed in paper. However, practical implementation of this process calls some difficulties and occupies a lot of time. It can be sped up and simplified essentially by computer calculation of approximating coefficients. We'll consider this problem in more details. Principle of operation of the color sensor (fig. 1). Emission $b(\lambda)$ moves on an entry of photodetectors $\Phi_1, \Phi_2, \dots, \Phi_n$. Electrical signals from these photodetectors arrive through adjusting resistors R_1, R_2, R_3 on operational amplifiers U_1, U_2, U_3 , after which – on the analogue-digital converter, and further, through the data register and the interface – on the computer. Signal can be given on an entry of each of channels with the various transmission factor determined from expression

$$u_{0j} = a_{ji} u_i = \frac{R_{0i}}{R_{ji}} u_i \quad (4)$$

where u_i – output signal of i -th photodetector ($i = 1, \dots, n$);
 a_{ji} – output signal of j -th operational amplifier ($j=1, 2, 3$)

R_{ji} – i -th resistor resistance on input of j -th operational amplifier;

R_{0i} – feedback resistor resistance of y -th operational amplifier.

The current appearing in the circuit of each photodiode by reason of emission $b(\lambda)$ can be determined from expression

$$I_i = k \int_{\lambda_{i1}}^{\lambda_{i2}} b(\lambda) f_i(\lambda) d\lambda \quad (5)$$

where $f_i(\lambda)$ – a resultant spectral characteristic of i -th photodetector and a light filter;

$\lambda_{i1}, \lambda_{i2}$ – emissions waves lengths correspond to the beginning and the ending of a transmission range of i -th light filter;

k – coefficient of proportionality.

Then voltage on output of each photodetector will be equal

$$u_i = I_i R_{H_i} = k R_{H_i} \int_{\lambda_{i1}}^{\lambda_{i2}} b(\lambda) f_i(\lambda) d\lambda \quad (6)$$

where R_{H_i} – a load resistance in the circuit of i -th photodetector.

As signals move simultaneously on input of the operational amplifier from all photodetectors, so its output voltage will be determined according to the formula

$$\begin{aligned} u_{0j} &= \sum_{i=1}^n a_{ij} u_i = \sum_{i=1}^n a_{ij} k R_{H_i} \int_{\lambda_{i1}}^{\lambda_{i2}} b(\lambda) f_i(\lambda) d\lambda = \\ &= k R_{H_i} \int_{\lambda_{i1}}^{\lambda_{i2}} b(\lambda) \left(\sum_{i=1}^n a_{ij} f_i(\lambda) \right) d\lambda \end{aligned} \quad (7)$$

where λ_1, λ_2 – waves lengths of the lower and upper boundary lines of a light band.

From last expression it is visible, that the spectral characteristic of each of three signals transiting channels is determined from a relationship

$$J_i(\lambda) = \sum_{i=1}^n a_{ij} f_i(\lambda) \quad (8)$$

Thus, we have: functions system $\{f_i(\lambda)\}_{i=1}^n$ is given on segment $[\lambda_1, \lambda_2]$. It is required to discover such values of coefficients $a_{ij} (i=1, \dots, n)$ in expression (5), defining adjustments of the resistors (fig. 1), that differences between $e_i(\lambda)$ – human eye spectral response addition curves and $J_i(\lambda); (J=1, 2, 3)$ were minimal.

The mean-square deviation should be taken as the degree of polynomial (5) deviation from given function $e_i(\lambda)$ in sense of the metric of this space if we observe these functions as elements of Hilbert space $L_2[\lambda_1, \lambda_2]$

$$\delta = \int_{\lambda_1}^{\lambda_2} (J_i(\lambda) \cdot e_i(\lambda))^2 d\lambda \quad (9)$$

For mean-square deviation definition we'll observe

a linear combination $\sum_{i=1}^n a_{ij} f_i(\lambda)$ for which

$$\int_{\lambda_1}^{\lambda_2} \left(e_j(\lambda) - \sum_{i=1}^n a_{ij} f_i(\lambda) \right) f_m(\lambda) d\lambda = 0, m=1, \dots, n \quad (10)$$

Its existence follows from single valued coefficients a_{ji} solving from system of the linear equations

$$\begin{aligned} \int_{\lambda_1}^{\lambda_2} \left(e_j(\lambda) - \sum_{i=1}^n a_{ij} f_i(\lambda) \right)^2 d\lambda &= \int_{\lambda_1}^{\lambda_2} \left(e_j(\lambda) - \sum_{i=1}^n a_{ij} f_i(\lambda) + \sum_{i=1}^n (a_{ij} - a_{ji}) f_i(\lambda) \right)^2 d\lambda = \\ &= \int_{\lambda_1}^{\lambda_2} \left(e_j(\lambda) - \sum_{i=1}^n a_{ji} f_i(\lambda) \right)^2 d\lambda + \sum_{i=1}^n \left((a_{ij} - a_{ji}) f_i(\lambda) \right)^2 \geq \int_{\lambda_1}^{\lambda_2} \left(e_j(\lambda) - \sum_{i=1}^n a_{ji} f_i(\lambda) \right)^2 d\lambda \end{aligned}$$

Our statement follows from the obtained inequality. We approximate integrals of system (11) on any point set $(\lambda_1, \lambda_2, \dots, \lambda_{p-1}, \lambda_p) \in [\lambda_1, \lambda_2]$

Then

$$\int_{\lambda_1}^{\lambda_2} \left(f_i(\lambda) f_m(\lambda) d\lambda \right) = \sum_{k=1}^n (f_k(\lambda_k) f_m(\lambda_k) d\lambda_k) \quad (12)$$

Let us call

$$\sum_{k=1}^n f_i(\lambda_k) f_m(\lambda_k) = (f_i, f_m) \quad (13)$$

Then system (11) can be written over

$$\left\{ \begin{aligned} a_{j1}(f_1, f_1) + \dots + a_{jn}(f_n, f_1) &= (e_j, f_1) \\ a_{j1}(f_1, f_2) + \dots + a_{jn}(f_n, f_2) &= (e_j, f_2) \\ &\dots \\ a_{j1}(f_1, f_n) + \dots + a_{jn}(f_n, f_n) &= (e_j, f_n) \end{aligned} \right. \quad (14)$$

In that case minimum value δ in expression (9) will determined from a relationship

$$\delta = \sum_{k=1}^P \left(\sum_{i=1}^n a_{ji} f_i(\lambda_k) - e_j(\lambda_k) \right)^2 \quad (15)$$

Considering, that a_{ji} – the solution of system (14), we'll get finally

$$\sum_{i=1}^n a_{ij} \int_{\lambda_1}^{\lambda_2} \left(f_i(\lambda) f_m(\lambda) d\lambda \right) = \int_{\lambda_1}^{\lambda_2} \left(e_i(\lambda) f_m(\lambda) d\lambda \right) \quad (11)$$

System matrix is Gram matrix for set $\{f_i(\lambda)\}_{i=1}^n$ of linearly independent functions so its determinant is not equal to null. It is possible to show, that expression (9) attains the minimum on this linear combination.

For this purpose we take any another linear combination $\sum_{i=1}^n a_{ij} f_i(\lambda)$. Expression (9) taking into account (10) is for it

$$\delta = \frac{(f_1, f_1) \dots (f_n, f_1)(e_j, f_1) (f_1, f_n) \dots (f_n, f_n)(e_j, f_n) \dots (e_j, f_n) \dots (e_j, f_1)(e_j, e_j)}{(f_1, f_1) \dots (f_n, f_1) \dots (f_1, f_n) \dots (f_n, f_n)} \quad (16)$$

The solution of system (14) can be easily implemented on the computer.

Conclusions. Authors have carried out the analysis of existing theoretical and experimental designs in the field of vision modeling. The practical concept of an applied colorimetry has been proposed. The structure is developed and the measurer of color performances of the reflected light is made. It supplies automation, digital processing of the colorimetric measuring and covered by patents of Ukraine.

References

1. Щербаківський М.Г., Хорошайло Ю.Е., Краснікова О.Л. Особливості визначення кольору в криміналістичних дослідженнях // Научний журнал – №1 (38), 2011.
2. Лук'яничук А.И. Цвет в промышленности –М.: Логос, 2002, –396 с.
3. Хорошайло Ю.Е., Сезонова И.К., Колошенко К.А., Хорошайло Е.Ю. Измерение // Сборник статей. Контрольно-измерительные приборы и автоматика – №1, 2005.– С. 30.
4. Марк Д. Фершильд. Модели цветового восприятия (2 изд.) / пер. с англ. А. Шадрина, 2006 –437 с.
5. Шабанов-Кушнарченко Ю.П. Аксиоматическое построение модели цветового зрения / В сб. «Проблемы бионики» – вып.4. Изд-во ХГУ, 1970.
6. Шабанов-Кушнарченко Ю.П. Математическое моделирование некоторых функций человеческого зрения. Докт. дис., Харьков, 1970.
7. Цифровий датчик для вимірювання кольору. Патент України на корисну модель UA №107317 МПК G01J 3/46 (2006.01) Україна, Ю.Е. Хорошайло, С.Х. Семенов, В.В. Лимаренко. 25.05.2016.

8. Цифровой колориметр. Патент України на корисну модель UA No112299 МПК G01J 3/46 (2006.01) Україна Ю.Е. Хорошайло, С.Х. Семенов, В.В. Лимаренко.
9. ISO 11664-1: 2007 (CIE 014-1/Ye: 2006) Колориметрия. Ч. 1. олоиметрические наблюдения по стандарту CIE / Колориметры. Ч. 1.
10. Форсайт Д. А., Понс Х., Компьютерное зрение. Современный подход / пер. с англ. –М.: Издательский дом «Вильямс», 2004. – 928 с.
11. Зыков И.Е. Нормативно-методическое обеспечение колориметрии высокого разрешения / И.Е. Зыков, Е.Н. Савкова // Приборы и системы. Управление, контроль, диагностика. – 2012. –№5. –С. 32–38.
12. Горбунова Е.В., Чертов А.Н. Типовые расчеты по колориметрии источников излучения –Санкт-Петербург. СПб.: Университет ИТМО, 2014. 90 с.
13. Годен Ж Колориметрия при видеобработке –Тенносфера, 2008. 328 с.
5. Shabanov-Kushnarenko YU.P. Aksiometricheskoye postroyeniye modeli tsvetovogo zreniya [Axiometric construction of the color vision model]. V sb.» Problemy bioniki», vyp.4. Izd-vo KHGU, 1970.
6. Shabanov-Kushnarenko YU.P. Matematicheskoye modelirovaniye nekotorykh funktsiy chelovecheskogo zreniya [Mathematical modeling of some functions of human vision]. Dokt. dis., Kharkov, 1970.
7. Tsyfrovyi datchyk dlya vymiryuvannya kol'oru. Patent Ukrainy na korysnu model' UA No107317 МПК G01J 3/46 (2006.01) Ukrayina, YU.YE. Khoroshaylo, S.H. Semenov, V.V. Lymarenko. 25.05.2016.
8. Tsyfrovyi kolorymetr. Patent Ukrainy na korysnu model' UA No112299 МПК G01J 3/46 (2006.01) Ukrayina YU. YE. Khoroshaylo, S.H. Semenov, V.V. Lymarenko.
9. ISO 11664-1: 2007 (CIE 014-1/Ye: 2006) Kolorimetriya. Chast' 1. Kolorimetricheskiye nablyudeniya po standartu CIE [Colorimetry. Part 1. Colorimetric observations according to the CIE standard].
10. Forsayt D. A., Pons ZH., Komp'yuternoye zreniye. Sovremennyy podkhod [Computer vision. The modern approach] : per. s angl. – Moscow: Izdatel'skiy dom «Vil'yams», 2004. – 928 p.
11. Zuykov I.Ye. Normativno-metodicheskoye obespecheniye kolorimetrii vysokogo razresheniya [Normative and methodological support of high-resolution colorimetry]/ I.Ye. Zuykov, Ye.N. Savkova // Pribory i sistemy. Upravleniye, kontrol', diagnostika. – 2012. – No5. – P. 32–38.
12. Gorbunova Ye.V., Chertov A.N. Tipovyye raschety po kolorimetrii istochnikov izlucheniya [Typical calculations for the colorimetry of radiation sources] – Sankt-Peterburg. SPb.: Universitet ITMO, 2014. 90 p.
13. Goden ZH. Kolorimetriya pri videobrabotke [Colorimetry for video processing]. – Tennesfera, 2008. 328 p.

References (transliterated)

1. Shcherbakovskiy M.G., Khoroshaylo YU.Ye., Krasnikova O.L. Osobnosti opredeleniya tsveta v kriminalisticheskikh issledovaniyakh [Features of color determination in forensic studies] // Nauchnyy zhurnal No1 (38), 2011.
2. Luk'yanchuk A.I. Tsvet v promyshlennosti [Color in the industry]. – М.: Logos, 2002, – 396 p.
3. Khoroshaylo YU.Ye., Sezonova I.K., Kolyushenkov K.A., Khoroshaylo Ye.YU. Izmereniye tsveta [Color measurement] // Sbornik statey. Kontrol'no-izmeritel'nyye pribory i avtomatika No1, 2005., P. 30.
4. Mark D. Fershil'd. Modeli tsvetovogo vospriyatiya (vtoroye izdaniye) [Models of color perception (second edition)] : per. s angl. A. Shadrina, 2006 – 437 p.

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