A 4D COMBINED BOUNDARY PROBLEM FOR LOADING-TYPE BIANCHI HYPERBOLIC EQUATIONS WITH NON-SMOOTH COEFFICIENTS

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This paper addresses a four-dimensional (4D) combined boundary value problem for a loading-type Bianchi hyperbolic equation with non-smooth coefficients.

The problem is formulated with non-classical boundary conditions of a unique structure—three-dimensional (3D) conditions in the middle and one-dimensional (1D) conditions in the geometrical middle—which notably do not require compatibility (matching) conditions. It is substantiated that these non-classical conditions are equivalent to classical 4D boundary conditions when solutions are considered in isotropic Sobolev spaces.

The studied equation generalizes numerous classical and model equations of mathematical physics, such as the Laplace, telegraph, and string vibration equations, along with their 3D and 4D analogues, including wave and Bianchi-type systems.

The presence of non-smooth coefficients complicates the application of classical methods, as no formal adjoint equation exists.

Consequently, standard tools like integration by parts and Riemann function methods are inapplicable.

The research emphasizes the importance of this class of problems in modeling real-world processes such as fluid filtration in fractured media, moisture migration in soils, thermal neutron diffusion, and complex biological phenomena. By substantiating the equivalence of classical and non-classical combined boundary conditions in a fourth-order setting, the paper contributes to the theory of hyperbolic equations and their applications in applied mathematics and physics.

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