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STUDY OF THERMOELASTOPLASTIC CONTACT DEFORMATION OF PRODUCTION TOOLING MIXED STRUCTURES

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A calculation technique based on the unified methodological approach is proposed for the analysis of thermoelastoplastic contact deformation of mixed structures. The proposed numerical technique involves the finite element method. The problem of stress-strain state evaluation for one class of mixed shrouded half-hot extrusion dies is considered. Recommendations on production tooling design are given.

Keywords: thermoelastoplastic deformation, contact interaction, strength, stiffness, finite element method, heat transfer, mixed shrouded extrusion die.

Problem Solution Status and Urgency. Development of mechanical engineering requires cost-effective manufacturing industry techniques. Technological processes, which take advantage of material plastic properties, are characterized by high productivity and cost-efficiency and thus find wide application. One of the most effective low-waste technologies is metal forming (MF). This process makes it possible to produce precision workpiece billets with minimal machining allowances or, in some cases, even completed parts [1–4]. However, implementation of the MF techniques, especially for production of items from hardly-deformed metals, is hindered by insufficient strength and reliability of tooling components. This necessitates adequate strength analysis to be performed at the design stage of tooling equipment components [1–4].

The majority of analytical techniques of die extrusion dies are based on the Lame formulas and are reduced to the final stress assessment in a thick-walled cylinder. Thus, in study [5], the Lame problem solution is used for calculation of the required number of shrouds. Since real configurations of extrusion dies may significantly differ from thick-walled cylinders, the simplified solutions can produce large errors. Noteworthy is also the existence of contact interaction zones, effect of which is either neglected or underestimated by analytical techniques.

In order to improve the calculation accuracy and account for various factors neglected within the analytical approaches, numerical techniques for stress-strain state (SSS) evaluation can be applied. The finite element method (FEM) is one of the most widely used numerical techniques. State-of-the-art universal FEM software complexes, such as ANSYS, COSMOS/M, etc., are currently available, which are applicable for the SSS evaluation of practically any structure. There are also special FEM elaborations aimed at design and strength calculation of tooling equipment components [6–8]. However, the latter elaborations are much or less based on simplified schemes of contact interaction, material behavior and temperature distribution. The proposed technique is aimed at the SSS assessment of mixed (composite) structures proper and analysis of contact interaction of their components. Here heat transfer and thermoelstoplastic deformation problems are solved simultaneously, in contrast to a superposition of the respective solutions which is typical for other numerical calculation techniques. We also propose a method for calculation of heat released during plastic deformation and friction which envisages incorporation of heat release sources at the contact boundary.

Mathematical Formulation of the Problem. Consider general mathematical formulation of the problem of thermoelastoplastic contact deformation of mixed structures with axisymmetrical geometry and loading conditions.

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One of the load constituents is a nonuniform thermal field. Consider solution of the stationary thermal conductivity problem used for the analysis of thermal load in half-hot intrusion (HHI) processes [4], which is reduced to determination of the following functional minimum:

$$I = \iint_S \left\{ \frac{K}{2}\left[\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] - Q(r,\ z)T(r,\ z)\right\} rdS + \int_{L_2} q(r,\ z)T(r,\ z)rdL + \int_{L_3} \frac{\alpha}{2}[T^2(r,\ z) - 2T_\infty T(r,\ z)]rdL, \quad (1)$$

where $T(r,\ z)$ is temperature distribution function, $Q(r,\ z)$ is density of internal heat release sources, which is assessed via the work of metal billet plastic deformation and friction-induced heat release, $q(r,\ z)$ is density of heat flow through a certain boundary L_2, α is a coefficient of convective heat transfer through a certain boundary L_3, and T_∞ is the ambient temperature.

Conditions of the ideal thermal contact (boundary conditions of the 4th kind) are postulated for the boundary L_4 between separate parts S_i and S_j of the structure:

$$\begin{aligned} &T_i(r,\ z \in L_4) = T_j(r,\ z \in L_4), \\ &K(S_i)\left(\frac{\partial T_i(r,\ z \in L_4)}{\partial r} l_r + \frac{\partial T_i(r,\ z \in L_4)}{\partial z} l_z\right) = K(S_j)\left(\frac{\partial T_j(r,\ z \in L_4)}{\partial r} l_r + \frac{\partial T_j(r,\ z \in L_4)}{\partial z} l_z\right), \end{aligned} \quad (2)$$

where $T_i(r,\ z \in L_4)$ and $T_j(r,\ z \in L_4)$ are the temperatures of mixed parts S_i and S_j at the contact boundary, $K(S_i)$ and $K(S_j)$ are the thermal conductivity coefficients of the respective material parts, and l_r and l_z are direction cosines of the general normal (base tangent) to the contact boundary.

Consider solution of the thermoelastolastic problem in the variational formulation, which is reduced to determination of the minimum of total energy functional (the Lagrangian functional) for the system of interacting solid bodies:

$$\mathrm{E} = \Pi - A_b - A_s, \quad (3)$$

where Π is the potential deformation energy of the elastic system, A_b is work of bulk forces, and A_s is work of surface forces.

Substitution of the formulas for the potential energy and work of surface forces derived for the axisymmetrical problem with account of thermal deformations into the functional yields

$$\mathrm{E} = \iint_S [\sigma_r(\varepsilon_r - \alpha\Delta T) + \sigma_z(\varepsilon_z - \alpha\Delta T) + \sigma_\theta(\varepsilon_\theta - \alpha\Delta T) + \tau_{rz}\gamma_{rz}]rdS - 2\pi\int_{L_r} P_r u_r dL_r + 2\pi\int_{L_z} P_z u_z dL_z, \quad (4)$$

where u_r and u_z are displacements of the solid body points in the direction of axes r and z.

For description of the process of elastoplastic deformation of the billet-extrusion die system we have used the equations the Il'yushin theory of small elastoplastic deformations in the form of variable elastic parameters [9, 10].

The functional minimum condition (4) is attained, if the following condition is met

$$\delta(\Pi - A_b - A_s) = 0. \quad (5)$$

Peculiarities of the Contact Problem Solution. In the general case, mating of points at the contacting surfaces is described by the following nonequalities:

$$u_n^{m-1} + u_n^{m+1} - \delta_{0n}^m \le 0, \qquad \sigma_{nn}^m \le 0, \quad (6)$$

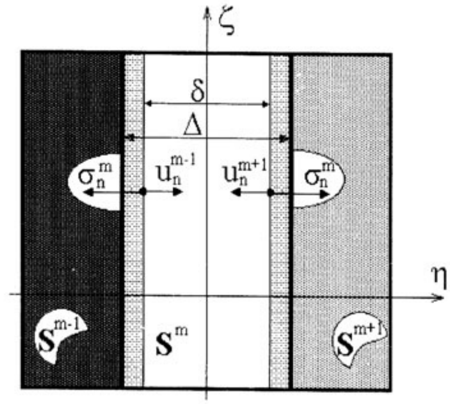


Fig. 1. Contact layer model.

where u_n^{m-1} and u_n^{m+1} are normal displacements of points at the contacting surfaces, δ_{0n}^m is the initial positive or negative allowance, and σ_{nn}^m are normal stresses at the contacting surfaces.

The physical meaning of the first condition in (6) is “nonpenetration” of contacting bodies. Selection of positive allowance for the contact implies contact stress consideration. Mechanisms of contact interaction between the respective points of contacting surfaces of structure parts are simulated by introduction of a contact layer with special properties. Within possible contact zone this layer is postulated to have special properties, which allows one to reduce the “external nonlinearity” envisaged by conditions (6) for the contact layer “internal nonlinearity,” as well as to analyze the interaction of bodies separated by a layer with known nonlinear characteristics (see Fig. 1). Simulation allows one to describe such interaction mechanisms of contacting bodies as grip (cohesion):

$$\sigma_{n\tau}^{m-1} = \sigma_{n\tau}^{m+1}, \qquad u_{n\tau}^{m-1} = u_{n\tau}^{m+1}, \tag{7}$$

slip:

$$\sigma_{n\tau}^{m-1} = \sigma_{n\tau}^{m+1} = 0, \tag{8}$$

dry friction, etc.

Friction interaction conditions are described by the laws interrelating tangential and normal stress constituents at the contact surface in form of the Coulomb law:

$$|\sigma_{n\tau}| = f_n \sigma_{nn} \tag{9}$$

or the Siebel law:

$$|\sigma_{n\tau}| = f_m \sigma_m, \tag{10}$$

where σ_m is the material yield stress and f_m is friction coefficient.

According to recommendations available in literature, Eq. (9) should be applied for the analysis of metal forming tooling equipment that involve processes with prevailing tensile stresses under condition $\sigma_m \geq |\sigma_n|$. For the analysis of pressing and die forging processes with high negative value of the average stress, the Siebel formula (10) should be applied.

Since axisymmetric models applicable to die forging processes are used for the SSS calculation of MF extrusion dies, friction between billet and extrusion die operating surface is described by the Siebel law, while friction between extrusion die parts – by the Coulomb law. Testing of software programs used for realization of the above technique of contact problem solution, has been performed earlier [11, 12]. Incorporation of the contact layer notion also provides effective solution of the thermal conductivity problem by way of attributing special thermo-physical properties to the contact elements. The results of studies on selection of contact layer parameters for solution of the thermal conductivity problem are described elsewhere [13].

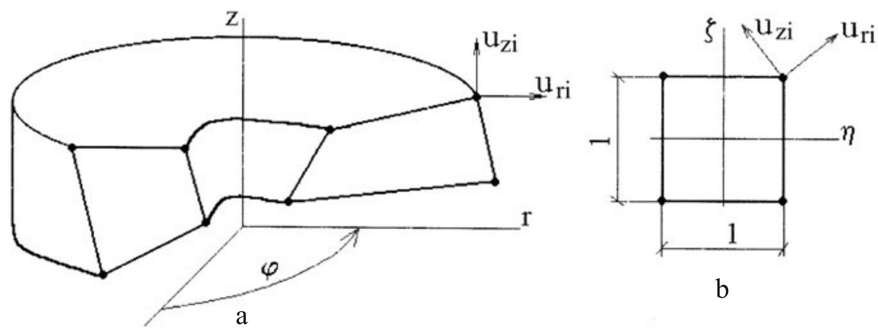


Fig. 2. Basic plane (axisymmetric) finite element in the global (a) and local (b) coordinate systems.

Numerical Calculation Technique. As the basic axisymmetric finite element, we use isoparametric 4-nodal plane element with bilinear approximation of displacements and temperature within the element shown in Fig. 2 in the global and local coordinate systems. In the local one, the finite element is a unit square (Fig. 2b) with its center coinciding with the origin of coordinates.

Distribution laws for displacements (u_r, u_z) and temperature (T) within the finite element with the given restrictions take the following form:

$$\begin{cases} u_r(\eta, \zeta) = b_1 + b_2\eta + b_3\zeta + b_4\eta\zeta, \\ u_z(\eta, \zeta) = a_1 + a_2\eta + a_3\zeta + a_4\eta\zeta, \\ T(\eta, \zeta) = c_1 + c_2\eta + c_3\zeta + c_4\eta\zeta. \end{cases} \tag{11}$$

As follows from Eqs. (11), displacements and temperature within the finite element can be described as

$$u_r(r, z) = \sum_{i=1}^{4} u_{ri}\varphi_i(\eta, \zeta), \qquad u_z(r, z) = \sum_{i=1}^{4} u_{zi}\varphi_i(\eta, \zeta), \qquad T(r, z) = \sum_{i=1}^{4} T_i\varphi_i(\eta, \zeta), \tag{12}$$

where u_{ri} and u_{zi} are values of nodal displacements, T_i are nodal temperature values, and $\varphi_i(\eta, \zeta)$ are coordinate functions of the finite element derived using the following formulas:

$$\begin{aligned} &\varphi_1(\eta, \zeta) = (\eta - 0.5)(\zeta - 0.5), \qquad &&\varphi_2(\eta, \zeta) = -(\eta + 0.5)(\zeta - 0.5), \\ &\varphi_3(\eta, \zeta) = -(\eta - 0.5)(\zeta + 0.5), \qquad &&\varphi_4(\eta, \zeta) = (\eta + 0.5)(\zeta + 0.5). \end{aligned} \tag{13}$$

It is known that choice of distribution law of the derived functions (displacement, temperature, etc.) within element in many respects controls the intrinsic convergence of the approximate FEM-based solution to the precise one with reduction of finite element dimensions. Here the preset functions of displacements and temperature should satisfy the basic convergence criteria [14].

The above technique of solving thermoelastoplastic mixed contact axisymmetrical problems has been implemented into a FEM-based SPACE-T software complex module [12].

In studies [11–13], the results calculated via the above software complex have been compared to the results of strength studies of production tooling equipment. Their satisfactory correlation proves the applicability of the proposed technique to solving practical problems.

Calculation Examples. Using the above technique and the elaborated software, we have performed calculations and elaborated recommendations on design of production tooling equipment for half-hot extrusion of axisymmetric cylinder-conical bushings. The calculated results are given below.

During elaboration and analysis of calculation schemes for technological operation tooling components one should bear in mind that no unified calculation scheme can be apriori introduced. In each particular case, structures differ by geometry, topology, material and a number of other parameters. In case of die forging, extrusion die is a set

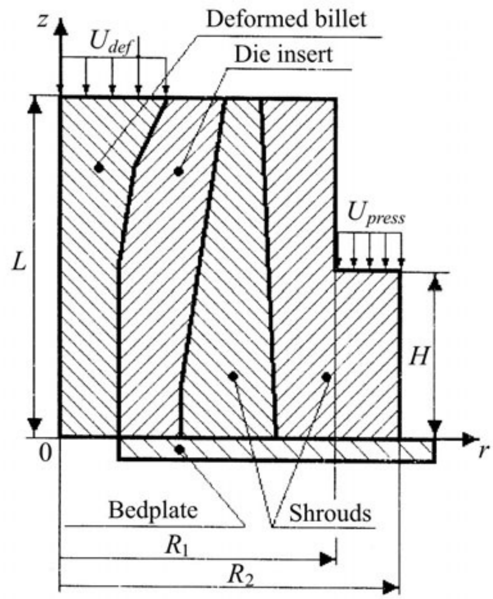


Fig. 3. Generalized calculation scheme of die extrusion die and billet.

of rotation bodies with a common axis, which are enveloped by the second order surfaces: cylindrical, conical and their combinations.

Figure 3 shows a generalized calculation scheme of the stress-strain state (SSS) of multilayer shrouded extrusion die for die forging of axisymmetric billets. Typical components of half-hot extrusion extrusion dies under study are die insert and shrouds, number of which is determined by technological requirements. Die insert, which provides the required shape of product, has stringent strength and stiffness requirements mainly related to the geometrical invariance of working surface during billet's deformation. Thus, extrusion die surface invariance requirement implies formulation of strength-based extrusion die workability as a combination of the following nonequalities:

$$\{\sigma_i < \sigma_m^{ins}\} \cup \{U_{\max} < U^{ins}\}, \tag{14}$$

where σ_i is stress intensity of the insert working surface points, σ_m^{ins} is the yield stress of the insert material, $U_{\max}$ is the total maximal displacement of the insert working surface points, and U^{ins} are technologically allowable displacements of the die insert.

The deformed billet interacts, in addition to the die insert, with die punch and pusher. In the extrusion die SSS analysis, die punch action on billet can be conventionally replaced by pressure P_{def} applied along the normal to the billet's upper edge [4]. In our opinion, more realistic simulation of the die punch action can be achieved by a kinemetic set of the respective values of the die upper edge displacements, which is given by the following condition

$$u_z(r, z)|_{z=L} = U_0, \tag{15}$$

where U_0 is the preset vertical displacement of the die upper edge.

The preset modeling of the punch action is satisfied by the following condition of the billet lower edge free support in vertial direction

$$u_z(r, z)|_{z=0} = 0. \tag{16}$$

The bedplate is disregarded in calculations. Therefore, free support conditions (16) are preset in points of the die insert surface and shrouds with coordinates $z = 0$. In order to exclude or minimize the radial displacements of the extrusion die components, the external shroud is usually fixed to the bedplate via bolted connection. Taking this into

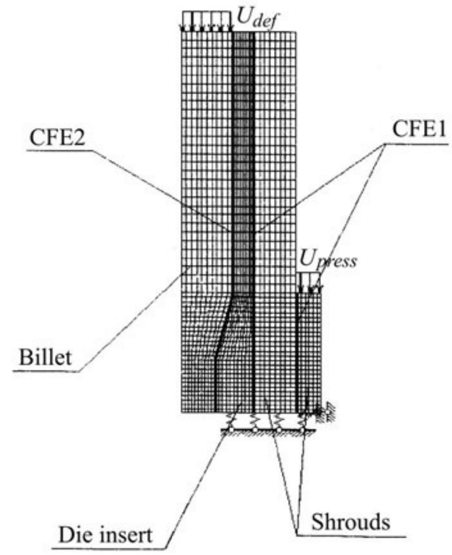


Fig. 4. FEM-model of composite extrusion die.

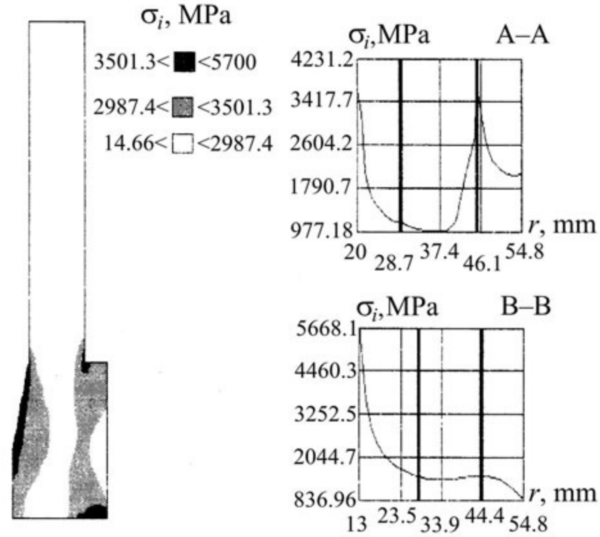


Fig. 5. Stress distribution in extrusion die.

consideration, points of the external shrouds with coordinate $z=0$, in addition to condition (16), should satisfy the following condition:

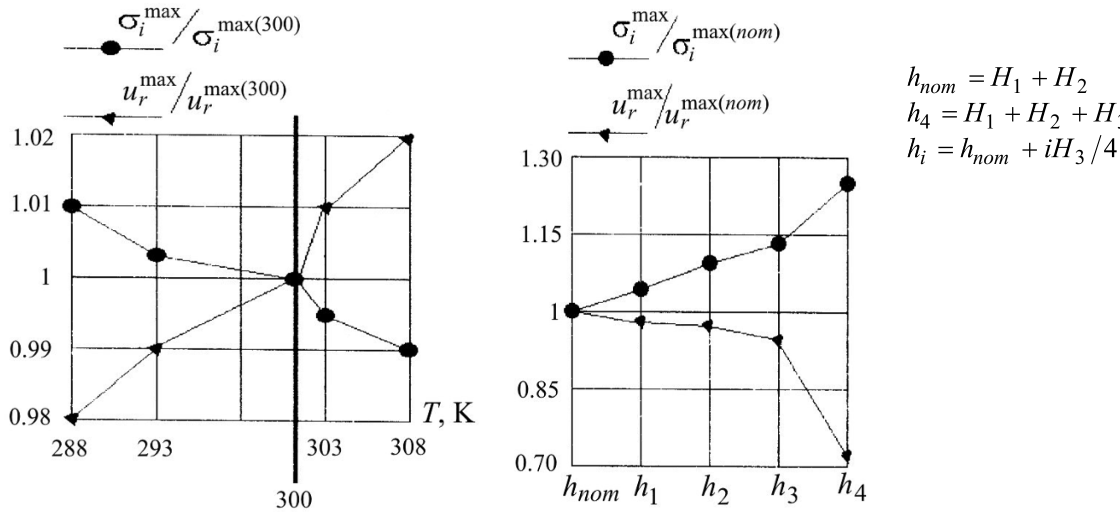
$$u_r(r,\ z)|_{z=0}=0. \tag{17}$$

FEM calculation scheme for the extrusion die is shown in Fig. 4. The extrusion die pretension is accounted for in calculations by setting displacements (U_{zat}) of the external shroud upper edge. One-sided contact conditions are specified by incorporation of contact finite element (CFE) layers, which are taken into account in simulation of negative allowance between matrix components and Coulomb friction (CFE1), as well as Siebel friction between the insert working surface and billet (CFE2). Extrusion die strength calculations have been performed in view of nonuniform thermal field controlled by the preheated billet temperature, heat release during its plastic deformation, billet-extrusion die friction and cooling temperature at the outer surface of the external shroud. We considered various temperature distribution laws corresponding to the extrusion process start and steady-state extrusion thermal modes. The thermal field in steady-state modes has been assessed by solving stationary heat transfer problem. Stress intensity in critical cross sections turned out to be higher at the start of extrusion process than during further extrusion operation cycle in steady-state thermal modes.

Figure 5 depicts stress distribution in the extrusion die meridional cross section and stress variation by thickness in critical cross sections.

The SSS analysis has revealed the following. Stress concentration zone is located at the point of contact of the external shroud upper edge with the inner shroud. Radial displacements of the die insert inner surface induced by upper edge zone heating are quite high and can exceed the allowable ones under certain conditions. In order to minimize these, we have performed calculations, where cooling temperature and external shroud dimensions were varied. The respective calculated results are plotted in Fig. 6.

One can see that at cooling temperature of 298–303 K the maximal displacements does not exceed technologically allowable deflections of the radial dimensions. Calculation results with variation of the external shroud height indicate show that increasing the latter is quite inefficient for stress reduction. Although increase of the external shroud height to the level of the total extrusion die height leads to significant reduction (by 30%) of the maximal displacements, this implies design problems with “high shroud” fixation. In case of critically high value of shroud height, the maximal stress intensity value increases by 25%, which implies structure stiffness reduction and temperature variation increase by the extrusion die thickness. In order to minimize the stress concentration factor, we have considered the extrusion die option with the external shroud height exceeding the line of action of the maximal load at the operation surface. One of general recommendations on the similar class of extrusion dies envisages as



a b

Fig. 6. Maximal radial displacements and stress intensities of the die insert vs cooling temperature (a) and external shroud height (b).

smooth as possible shape of the shroud components. The initial geometric stress raisers should be relocated into the area outside the extrusion die maximal load application zone.

The performed calculations make it possible to provide particular recommendations on design and elaboration of new technological metal forming equipment. The following guidelines for further refinement of the proposed technique an its application for the strength calculations of dies with high plastic deformations during plastic forming of billets, as well as for high-speed (pulse) treatment of billets can be formulated:

(i) refinement of contact interaction simulation (more intricate friction models and nonideal thermal contact);

(ii) application of more adequate theories of billet material nonlinear behavior (incremental theories of plasticity);

(iii) account of the dynamic constituent during billet deformation.

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