THE BAYESIAN NETWORK AS A DECISION-MAKING MODEL UNDER UNCERTAINTY

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A Bayesian network is a probabilistic graphical model that represents a set of variables and their conditional dependencies via a directed acyclic graph (DAG). It is one of several forms of causal notation. Bayesian networks are ideal for taking an event that occurred and predicting the likelihood that any one of several possible known causes was the contributing factor.

Formally, Bayesian networks are DAGs whose nodes represent variables in the Bayesian sense: they may be observable quantities, latent variables, unknown parameters or hypotheses. Each edge (arc) represents a direct conditional dependency. Each node is associated with a probability function that takes, as input, a particular set of values for the node's parent variables, and gives (as output) the probability (or probability distribution, if applicable) of the variable represented by the node.

A classic example of a DAG is the Bayesian network "Water Sprinkler", which is shown in Fig. 1 with conditional probability tables (CPTs). The special CPT is associated with each node. It contains the conditional probability distribution of the node given its parents in the DAG.

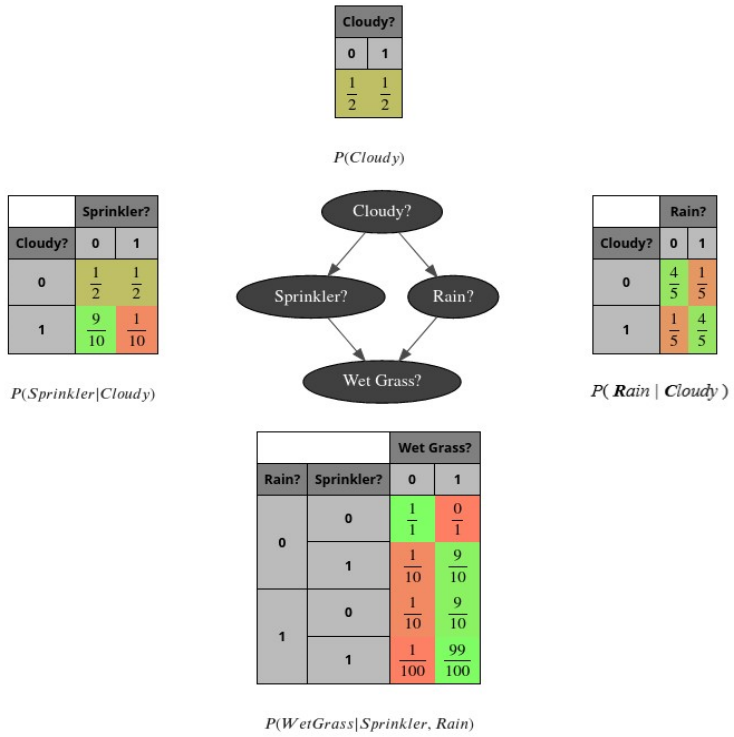

Fig. 1. Bayesian network "Water Sprinkler" with conditional probability tables

Let us use an illustration to enforce the concepts of a Bayesian network. Suppose we want to develop a program that asks questions to the user to find out whether the grass is wet or not. We need to model the dependencies between four variables: whether the sky is cloudy or not, the sprinkler (or more appropriately, its state) whether is on or not, the presence or absence of rain and whether the grass is wet or not. Observe that two events can cause the grass to become wet: an active sprinkler or rain. Rain has a direct effect on the use of the sprinkler (namely that when it rains, the sprinkler usually is not active). This situation can be modeled with a Bayesian network. Each variable has two possible values, 1 (for true) and 0 (for false).

Such a Bayesian network allows to manipulate the joint probability function $P(C,S,R,W)$ using this decomposition (by the chain rule of probability):

$$P(C,S,R,W)=\prod_{k=1}^{4}P\left(N_k \middle| Parents_k\right)=P(C)\cdot P\left(S\middle|C\right)\cdot P\left(R\middle|C\right)\cdot P\left(W\middle|S,R\right),$$

where $k=1, 2, 3, 4$ – number of the node within Bayesian network in Fig. 1, $P\left(N_k \middle| Parents_k\right)$ – conditional probability of the k-th node given its parents; as first node has no parents, its conditional probability is equal to an unconditional one: $P\left(N_1 \middle| Parents_1\right)=P(N_1)$; the letter notations as follows: C is for "Cloudy (true/false)", S – "Sprinkler turned on (true/false)", R – "Raining (true/false)", W – "Wet Grass (true/false)".

Since we are dealing with decision making regarding whether the grass is wet or not under uncertainty, our main concern is to reduce the entropy in the Bayesian network. Reduction is achieved by the procedure of investigating of the most entropic node. By this process means that the user is asked a question and he or she gives an answer to it. After that we obtain a probability distribution based on the user response and set it as evidence to the node. Afterward the Bayesian network is updating, consequently, the probability distributions are refreshed as well, and the most entropic node is searching again. Then, after reducing the entropy in the Bayesian network, we can obtain a probability distribution from the "Wet Grass" node, with the help of which we will be able to determine if the grass is wet or not.

Therefore, Bayesian networks are used in fields that are characterized by inherited uncertainty that may arise due to incomplete understanding of the subject area or incomplete knowledge and in cases when the objective is characterized by randomness.