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PROBLEM STATEMENT OF DYNAMIC CREEP FOR ISOTROPIC AND ORTHOTROPIC BODIES

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ABSTRACT

Mathematical problem statement of dynamic creep for isotropic and orthotropic bodies is presented in the paper. The cyclic creep-damage theory of Breslavsky-Morachkovsky is used. Numerical methods for the solution of such creep problems is considered, where the mixed variational functional and RFM (Rvachov's Functions method), or finite element method (FEM), are applied. Numerical results of the creep problem analysis for plate with centered hole under tension are given.

INTRODUCTION

It is necessary to take into account the creep phenomena in long-term strength analysis of structural elements with exploitation at elevated temperatures. This process leads to evaluation of creep irreversible strains and damage parameter and finally to predict the fracture. The most of structural members in aviation and space-rocket techniques have such exploitation conditions and are made from materials with anisotropic creep properties. A lot of such elements are used under action of quasi-static loadings and non-stationary physical and mechanical rapidly oscillating fields (aero-hydrodynamic streams with pulsations, etc.). In the paper a problem statement of dynamic creep for isotropic and orthotropic bodies is considered. Solution methods and numerical results are presented.

1. PROBLEM STATEMENT

Let us consider the main statement of dynamic creep-damage problem for solids. In Cartesian coordinate system, x_i, i=1,2,3, one considers a space body with volume *V*, and surface *S*: $S = S_t \bigcup S_u$, where S_t, S_u are surface parts of the body, under acting external distributed forces and boundary conditions.

In standard form the complete system of the initial-boundary-value problem of creep for bodies subjected to action of volume forces, surface forces and oscillating harmonic loading can be written in tensor form (1) as

$$\begin{gathered}\sigma_{ij,j} + f_i = \rho \dot{v}_i,\ \varepsilon_{ij} = 1/2(u_{j,i} + u_{i,j}),\ \varepsilon_{ij} = d_{ijkl}\sigma_{kl} + c_{ij},\ x_i \in V \\ \sigma_{ij} n_j = p_i^0 + \Phi_i(t) - x_i \in S_t,\ u_i - u_i^* = 0 \text{ - } x_i \in S_u \\ u_i(x,0) = v_i(x,0) = c_{ij}(x,0) = 0\end{gathered} \tag{1}$$

The system (1) includes equilibrium, physical, and geometric equations.

It can be taken into account that complete strain tensor consists of a sum of reversible elastic strain and irreversible creep strain tensors. Creep strain tensor can be determined by using creep state equations.

Oscillating fields of external forces can be described as the following singly-periodic and harmonic function:

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$$\Phi_i(t) = p_i \cdot sin(\frac{2\pi}{T}t) \tag{2}$$

where p_i are amplitude values of corresponding surface loads.

Functions $\Phi_i(t)$ are singly-periodic and harmonic functions with cycle *T*, that $T/t_* << 1$, t_* is a rupture time during the creep process.

By using the method of asymptotic expansions by the small parameter $\mu = T/t_*$, and the averaging procedure along the period, we reduce the original initial-boundary-value problem of creep of bodies subjected to the action of harmonic loading, to two correlated initial-boundary-value problems in two time scales (stow *t* and fast ξ) [1].

Equations of the first problem can be written as the following:

$$\begin{gathered} \sigma^0_{ij,j} + f_i = \rho\dot{v}_i\,,\ \varepsilon^0_{ij} = d_{ijkl}\sigma^0_{kl} + c^0_{ij}\,,\ x_i \in V \\ \sigma^0_{ij}n_j = p^0_i \text{ - } x_i \in S_t\,,\ u^0_i - u^*_i = 0 \text{ -} x_i \in S_u \\ u^0_i(0) = v^0_i(0) = 0\,,\ c_{ij}(0) = 0 \end{gathered} \tag{3}$$

where all functions are slowly varying ones in the macroscopic time scale *t*.

Equations of the second problem is written in the form:

$$\begin{gathered} \sigma^1_{ij} = \frac{1}{\mu^2}\rho u^1_{,\xi\xi} \quad \varepsilon^1_{ij} = d_{ijkl}\sigma^1_{kl}\,,\ x_i \in V \\ \sigma_{ij}n_j = \Phi_i(\xi)\,,\ x_i \in S_t; \\ u^1_i(0) = v^1_i(0) = 0,\ c^1_{ij} = 0\,,\ 0 \le \xi \le 1 \end{gathered} \tag{4}$$

where all functions are fast varying ones in the microscopic time scale ξ.

Equations of the systems (3, 4) must be expanded by state equation of dynamic creep. From the creep state equation we can determine irreversible creep strains and damage parameter.

In early works for isotropic materials it has been used the Rabotnov-Kachanov theory with a scalar damage or continuity parameters [5]. However such theory can't be used to describe the anisotropic creep. Vector and tensor models for a presentation of the damage parameter for anisotropic creep are suggested by many famous scientists (Rabotnov Yu.M., Kachanov L.M., Shesterikov S.A., Murakami S., Betten J. and others). Physical interpretation of the damage parameter in vector or tensor form is the density in microvolume of different defects in the form of voids, microcracks, vacancies, dislocation processes etc. Defects in material accumulate at elevated temperature due to the creep process. This fact is proved by experimental data in metallurgical science.

State equations of dynamic creep for isotropic material have been written [1-3] as

$$\dot{c}_{ij} = \frac{3}{2}\cdot\frac{B\sigma_i^{n-1}\left(1+H(A)\right)}{\left(1-\omega^r\right)^l}\cdot s_{ij}\,,\ \dot{\omega} = \frac{B\sigma_e^m\left(1+K(A)\right)}{\left(1-\omega^r\right)^k}\,,\ \omega(0)=0,\ \omega(t_*)=1 \tag{5}$$

$$H(A) = \frac{n(n-1)}{4}A^2\left(1+\frac{(n-2)(n-3)}{16}A^2\right)$$

$$K(A) = \frac{m(m-1)}{4}A^2\left(1+\frac{(m-2)(m-3)}{16}A^2\right),\ A = \frac{\sigma_i^a}{\sigma_i}$$

State equations of dynamic creep for orthotropic materials can be presented

$$\underline{\dot{c}} = b_{1111}^{(N+1)/2} \frac{\bar{\sigma}_2^{N-1}\left(1+H(A)\right)}{(1-\omega)^N}[B]\underline{\sigma},\ \dot{\omega} = d_{1111}^{k/2} \frac{\sigma_{*2}^{k}\left(1+G(A)\right)}{(1-\omega)^{k+S}},\ \omega(0)=0,\ \omega(t_*)=1 \tag{6}$$

$$\underline{\dot{\varpi}} = d_{1111}^{k/2} \frac{\sigma_{*2}^{k-2}\left(1+G(A)\right)}{(1-\omega)^{k+S-1}}[D]\underline{\sigma} \tag{7}$$

Where

$\underline{\dot{c}} = \left(\dot{c}_{11}, \dot{c}_{22}, 2\dot{c}_{12}\right)^T$, $\underline{\sigma} = \left(\sigma_{11}, \sigma_{22}, \sigma_{12}\right)^T$, $\underline{\dot{\varpi}} = \left(\dot{\varpi}_{11}, \dot{\varpi}_{22}, 2\dot{\varpi}_{12}\right)^T$ - are respectively vectors of the creep strain rate, stresses and damage parameter;

$\bar{\sigma}_2^2 = \underline{\sigma}^T[B]\underline{\sigma},\ \sigma_{*2}^2 = \underline{\sigma}^T[D]\underline{\sigma}$ - are the invariants of stress tenzors;

$A = \dfrac{\sigma_i^a}{\bar{\sigma}_2}$ - is the asymmetry parameter of stress;

σ_i^a - is the intensity of the amplitude of stress;

$\dot{\omega} = \underline{\sigma}^T\underline{\dot{\varpi}}$ - is the specific power dissipation due to damages.

Particularly, for transversally-isotropic materials matrixes of material properties are

$$[B] = \begin{pmatrix} 1 & \beta_{12} & 0 \\ \beta_{21} & \beta_{22} & 0 \\ 0 & 0 & 4\beta \end{pmatrix},\quad [D] = \begin{pmatrix} 1 & \delta_{12} & 0 \\ \delta_{21} & \delta_{22} & 0 \\ 0 & 0 & 4\delta \end{pmatrix}$$

$$\beta_{12} = -\frac{1}{2}, \beta_{22} = \frac{b_{2222}}{b_{1111}}, 4\beta = \frac{b_{1212}}{b_{1111}},\quad \delta_{12} = -\frac{1}{2}, \delta_{22} = \frac{d_{2222}}{d_{1111}}, 4\delta = \frac{d_{1212}}{d_{1111}}$$

A detailed conclusion and assumption of the state equations for anisotropic creep taking into account the damage parameter are presented in [1,7,8].

2. METHOD OF SOLUTION

The complete system of initial-boundary-value problem of dynamic creep for bodies (3, 4, 6, 7) can be solved by using two methods.

The first one is based on the variational statement by using variational-structural method of the R – functions theory [4]. The second one is FEM [2].

The variation principle for the mixed functional is given. It is formulated on independently varied functions of strains and stress for known creep strains at arbitrary time moment.

To solve the boundary-initial value creep problem (3) the mixed variational functional can be written as the following:

$$R_{u\sigma} = \iiint_V \left[\frac{1}{2}\sigma_{ij}\left(u_{i,j} + u_{j,i}\right) - \sigma_{ij}C_{ij} - \Lambda(\sigma_{ij}) - \left(f_i - \rho\dot{v}_i\right)u_i \right] dV - \\ -\iint_{S_t} p_i^0 u_i dS - \iint_{S_u} n_i\sigma_{ij}(u_j - u_j^*) dS \tag{8}$$

The first variation of the functional (8) can be presented as

$$\delta R_{u\sigma} = \iiint_V \left[\delta\sigma_{ij}\left\{\frac{1}{2}\left(u_{j,i} + u_{i,j}\right) - d_{ijkl}\sigma_{kl} - c_{ij}\right\} - \delta u_i\left(\sigma_{ij,j} + f_i - \rho\dot{v}_i\right) \right] dV \\ -\iint_{S_u} n_i\delta\sigma_{ij}\left(u_j - u_j^*\right) dS - \iint_{S_t}\left(\sigma_{ij}n_i - p_j^0\right)\delta u_j dS \tag{9}$$

Numerical method for a solution of the boundary – initial value creep problems (3) is used, the Runge–Kutta–Merson and RFM methods are applied [4].

By using the variational-structural theory of R – functions or FEM we have complete resolving system of equations in a standard form:

$$[K]\{u\}=\{F\}+\{F^{cr}\} \tag{10}$$

The second problem is considered on the "fast" scale and corresponds to forced oscillations of an elastic body under the action of harmonic loading. For determining of amplitude values of stresses in such case we must to solve the next system of equations:

$$\left([K]-\Omega^2[M]\right)\{q_*^1\}=\{p_*^1\} \tag{11}$$

where $\{q_*^1\}$ is the amplitude values vector of nodal displacement (FEM) or free components in structures (variational-structural method) under action of forced harmonic oscillation of a body with eigen frequency Ω; $[K]$, $[M]$ are stiffness and mass matrixes of a body; $\{p_*^1\}$ is the amplitude values vector of loads. Methods for solution of the equations in the form (11) are well known [1-4].

3. NUMERICAL RESULTS

Let us consider a thin-walled rectangular plate with centered hole (Fig. 1) under tension along the axis OX. The plate is made from materials with transversally-isotropic properties during creep process.

Creep problem for the rectangular plate under tension of axial force S = 30 MPa, is presented (Fig. 1). Geometric parameters of the plate are a side 2b=0.2 m, and the centered hole equal to 0.02 m.

In a case when the oscillation component of loading is absent (A = 0), the relations (6,7) turn into ordinary relations of the anisotropic creep theory $H(A)=G(A)=0$.

Numerical results are given in ANSYS software and were compared with known results at initial time [6]. Type of finite elements PLANE 183 has been taken, number of elements is equal 461 FE. Plate material is the aluminium alloy D16AT at 275^0C with the following mechanical properties: elasticity module $E=65$ GPa, Poisson's ratio $\nu=0.42$. The numerical values of material constants of state equations and damage (6, 7) are obtained by materials creep curves processing for D16AT at 275^0 and have been given in the paper [7].

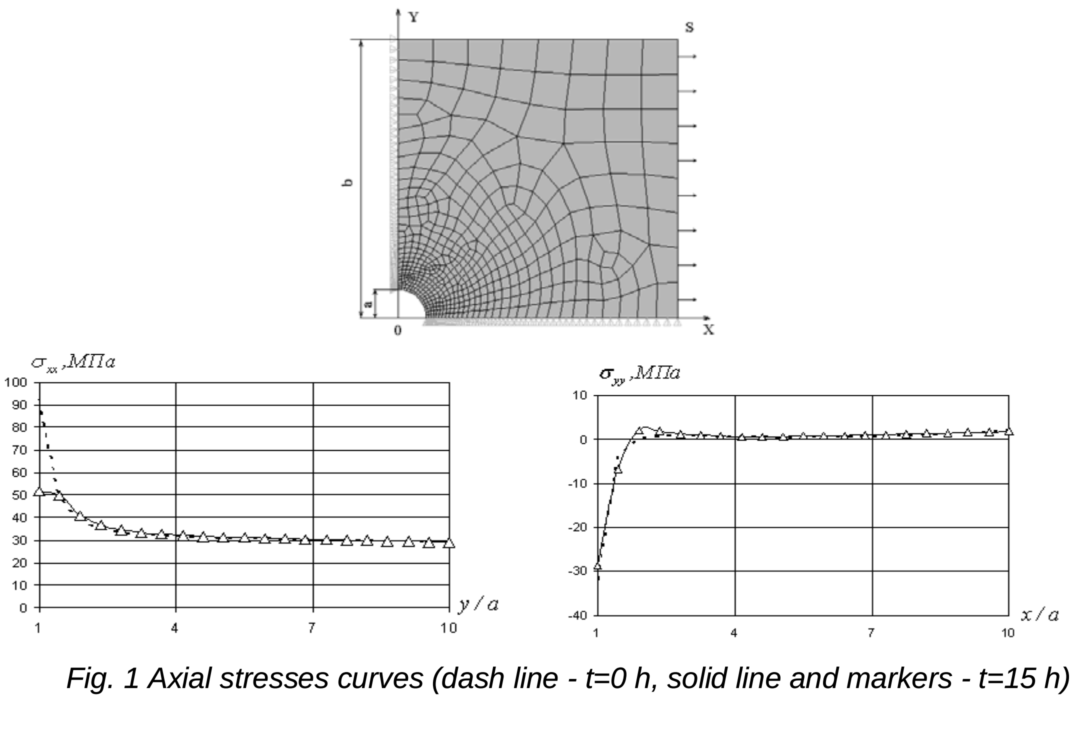

Fig. 1 Axial stresses curves (dash line - t=0 h, solid line and markers - t=15 h)

As an example of calculation, we consider the anisotropic creep of rectangular plate depending on the angle orientation of main axes of anisotropic material properties (θ°) to axis OX during the creep process.

At the beginning the calculation have been done for the case of θ°=0. In the Fig. 1 we can see a distribution of axes stresses and its change in time.

In the table 1 comparisons of maximum values of stress intensity and creep strains from anisotropy axes orientations are given.

Table 1 Maximum values of stress intensity and creep strains

Anisotropy axes orientation, θ°	Maximum value of stress intensity σ_i, *MPa*	Maximum value of creep strain intensity ε_i, %
0	56.283	0.9754
30	64.53	0.64
45	66.3	0.51
90	55.987	1.2

CONCLUSIONS

The general statement of boundary-initial problems of dynamic anisotropic creep for bodies is given in the paper. The state equations in incremental form of dynamic anisotropic creep for isotropic and transversally-isotropic materials with damage parameter have been shown. Methods of analysis to solve these problems are presented on the basis of RFM – variational-structure and FEM methods. FEM solutions of anisotropic creep for plates with centered hole under tension load have been done. It is established that if a plate is made from transversally-isotropic materials, then in a case of coincidence of anisotropic properties axis with tension axis one has the maximum long-term durability, intensity of stresses redistribution near the plate hole and an increasing level of creep strains. For the angle of main anisotropic axis orientation θ°=30^0, 45^0 we can see in the plate a less intensity of stress redistribution and accumulating of creep strains.

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