# EVALUATION OF THE EFFICIENCY OF THE VEHICLE WITH VARIOUS INTER-WHEELED DIFFERENTIALS FOR DIFFERENT CLUTCH CONDITIONS ON SIDES IN ACCELERATION REGIME

## ОЦЕНКА ЭФФЕКТИВНОСТИ РАЗГОНА АВТОМОБИЛЯ С РАЗЛИЧНЫМИ МЕЖКОЛЕСНЫМИ ДИФФЕРЕНЦИАЛАМИ ПРИ НЕОДИНАКОВЫХ УСЛОВИЯХ СЦЕПЛЕНИЯ ПО БОРТАМ

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**Abstract:** *In this paper, the dynamics of acceleration and energy costs for cars with various types of inter-wheeled differentials are analyzed. The used dynamic models for drive with: a classical symmetric conical open differential, a fully locked differential, and limited-slip differentials for which the locking torque depends on the load or the relative speed of rotation are described. During the study, the ratios of the adhesion coefficients under the right and left sides were varied. As a result of the work, tasks are formulated for further more detailed analysis of limited-slip differentials.*
***KEYWORDS***: *INTER-WHEELED DIFFERENTIALS, LIMITED-SLIP DIFFERENTIALS, ENERGY EFFICIENCY, ACCELERATION DYNAMICS.*

### 1. Introduction

The need for inter-wheeled differentials arose immediately after the appearance of the first vehicles with a two-wheel drive on the same axle. The most noticeable this need was manifested when the car turns and drives on roads with irregularities. The absence of an inter-wheeled differential in transmissions leads to the appearance of power circulation, unjustifiably large additional loads on the drive axle and wheels, increased fuel consumption, and large tire wear. With the invention of the classical conical symmetrical inter-wheeled differential, these problems were solved. However, other problems associated with the phenomenon of slippage in severe road conditions are appeared. Naturally, there was a large number of technical solutions to this challenge that somehow smoothed out the problem, but did not solve it in a all-inclusive manner.

The simplest and chronologically the first solution to combat slippage of one of the driving wheels was a complete interlock of the differential on demand (manual control). As a rule, the locking is effected by a gear or cam clutch. In this case, the process of turning the clutch on or off requires a complete stop of the vehicle. In severe road conditions, this requires the driver to activate the described device in advance. Such a locking provides maximum traction capabilities of the wheeled propeller, since it allows to transmit up to 100% of the power to any of the wheels, which is able to accept it under the conditions of adhesion to the supporting surface. If the implementation of a complete locking is structurally solved using a friction clutch, it no longer requires the vehicle to stop. This design is relatively easy to automate, but is usually associated with an increase in the size and weight of the differential and the main gear, as well as a decrease in ground clearance. Full locking of the differential requires mandatory shutdown after entering to the road with a hard surface or a dry dirt road.

The second solution to the slip problem was the invention of a whole group of self-locking differentials or limited-slip differentials. Depending on the design, these may be Torque sensitive differentials (HLSD) or Speed sensitive differentials. In recent decades, there have been design solutions in which the degree of blocking is electronically controlled by a given algorithm, as well as drives in which the traction control system directly controls the torque applied to each drive wheel. Especially effectively they work in the case of electric or hydrostatic motor-wheels.

In the scientific and technical literature, the distribution of power between wheels and axles in modern cars is given a lot of attention. There are a large number of publications describing the work of different types of differentials [1–6] and considering a scientific approach to determining their optimal characteristics [7–10]. However, in most cases when analyzing the differentials for cars operating in difficult road conditions and off-road, the issues of proper patency are considered.

The papers [11–14] also consider the efficiency of using differential drives primarily for all-wheel drive vehicles. Especially intensive in this direction works the scientific school headed by Andrei Keller from the South Ural State University (Russian Federation). However, the complex comparative analysis of the dynamics of acceleration and losses on slippage for various types of inter-wheeled differentials has not been found in the literature. This issue, despite the rapid development of electronic control systems and individual electric drives, remains relevant for military wheeled vehicles and all-wheel drive vehicles of multi-purpose use, in which the full lock of differentials with manually operating is still most often used.

### 2. Materials research.

#### 2.1 Formulation of the Problem

In this paper, the authors posed the task of performing a comparative analysis of the classical (open) conical symmetric differentials, of the limited-slip differentials, of the traction control systems (TCS) based on anti-lock brake systems (ABS) and of the completely locked differentials, from the point of view of the dynamics of the acceleration of the machine, the magnitude of the slip losses and the magnitude of the moment of resistance to turning. The study was carried out for vehicles under conditions where dry and pure asphalt concrete is located under one of the sides of the machine, and under other side the coupling properties of the road surface change from dry asphalt to an icy road. The car was considered as all-wheel driven, but the type of the inter-axle differential and the method of power distribution between the axes at this stage of the research was adopted rigid (blocked). In this case, the wheel propeller on each of the axles can realize the traction force proportional to the coupling weight. This was done to assess the effect on the investigated factors of the imbalance of the adhesion coefficients under the sides and the specific power for any single axis.

#### 2.2 The Description of the Mathematical Model of the Vehicle Acceleration and the Calculation Assumptions

**Accepted Assumptions, Baseline Data and Variable Parameters.**

1) The entire weight of the all-wheel-drive vehicle is reduced to one axis and is considered constant during the entire acceleration time. Vibration processes in the suspension of the vehicle are not considered.

2) We considered a plane model of vehicle rectilinear motion on a horizontal surface during acceleration without lateral displacement and skidding.

3) The change in the instantaneous radius of the driving wheel with respect to the static radius under the influence of the driving torque and centrifugal forces is not taken into account.
4) The resistance of air during acceleration is not taken into account, since acceleration is considered up to 10 m / s.
5) We accept that the left wheel is always in better conditions of grip with the road than the right one.

The work examines the acceleration from the place of the all-wheel drive wheeled armored personnel carrier BTR-4, which is in conditions when the adhesion coefficient of wheels with the road under left side meets the dry clean asphalt $\varphi_l = 0{,}8$, and under right side the adhesion coefficient varies in different arrivals from dry asphalt $\varphi_r = 0{,}8$ to the icy road $\varphi_r = 0{,}1$.

The middle coefficient of resistance to movement (drag coefficient) also changed in various races from dry clean asphalt ( $f_m = 0{,}02$ for tires with adjustable pressure) to wet dirt road $f_m = 0{,}08$.

The drive to the differential was stepless with constant power, which was limited to the maximum torque with a value providing a specific traction force equal to unity. This limited the slippage of both wheels during the acceleration process.

For carrying out the calculations, the data needed were taken using the example of the wheeled armored personnel carrier BTR-4, namely:

– armored vehicle weight $G$ = 24 tons;

– static wheel radius at normal tire pressure $R_w$ = 0,525 m;

– reduced moment of inertia of a wheel with a wheel gear and a semiaxis $I_{l(r)}$=1,5 kg·m2;

– moment of inertia of the drive (engine and transmission) in the first gear $I_{in}$=5 kg·m2;

– the maximum torque on the differential case when the specific thrust force is limited to unity $M_{in}^{\max}$ = 7275 Nm.

**Structure of the Mathematical Model.**

The calculation scheme for modeling the process of accelerating the machine is shown in Fig. 1. Here the following forces and torques are indicated:
$G_M$ – weight of the armored personnel carrier, per axle;
$N_r$= $N_r$= $G_M$ /2 – normal reaction of soil (road) in the contact spot of one wheel;
$P_{Dl}$ and $P_{Dr}$ – The traction forces realized on the left and right driving wheels that calculated as functions of the coefficients of slipping and grip on each of the wheels $P_{Di} = f(\sigma_i, \varphi_i)$;
$P_{Rl}$ and $P_{Rr}$ – rolling resistance forces on the left and right driving wheels, calculated as the product of the mean under the right and left sides of the drag coefficient fm and normal ground reaction $N_i$: $P_{Rl} = P_{Rr} = N_i f_m$, where $f_m = (f_l+f_r)/2$;
$M_{in}$ – input (driving) torque on the differential case;
$\omega_{in}$ – angular velocity of the input link (differential case);
$I_{in}$ – the moment of inertia of the differential case and the associated rotating parts of the drive;
$M_l$ and $M_r$ – torque on the left and right driving wheels;
$\omega_l$ and $\omega_r$ – angular speeds of rotation of the left and right driving wheels;
$I_l$ and $I_r$ – moment of inertia of the left and right driving wheels with a wheel gear and a semiaxis;
$V_M$ – linear speed of the machine along the coordinate axis $X$.

Three generalized speeds are identified in the model:
$\omega_{in}$ – angular velocity of the input link (differential case);
$\Delta\omega$ – the difference between the angular velocities of the right and left semi-axles or driving wheels (in the absence of wheel gears), determined by formula $\Delta\omega = \omega_r - \omega_l$;
$V_M$ – linear speed of the machine along the coordinate axis $X$.

For these generalized velocities, according to the d'Alembert principle (Newton's second law), differential equations are constructed that describe the accelerated motion of these masses:

(1) $$\frac{d\omega_{in}}{dt} = \frac{M_{in} - M_l - M_r}{I_{in} + I_l + I_r};$$

(2) $$\frac{d\Delta\omega}{dt} = \frac{(M_l - M_r)\eta_{cg}^2 - M_{fr}}{I_l + I_r};$$

(3) $$\frac{dV_M}{dt} = \frac{P_{Dl} + P_{Dr} - P_{Rl} - P_{Rr}}{G_M / g},$$

where $\eta_{cg}$ – coefficient of efficiency of differential with conical straight teeth ( $\eta_{cg} \approx 0{,}95...0{,}96$ ); $M_{fr}$ – additional frictional torque, formed on the limited-slip differentials or on brake mechanisms of ABS to prevent intensive slippage.

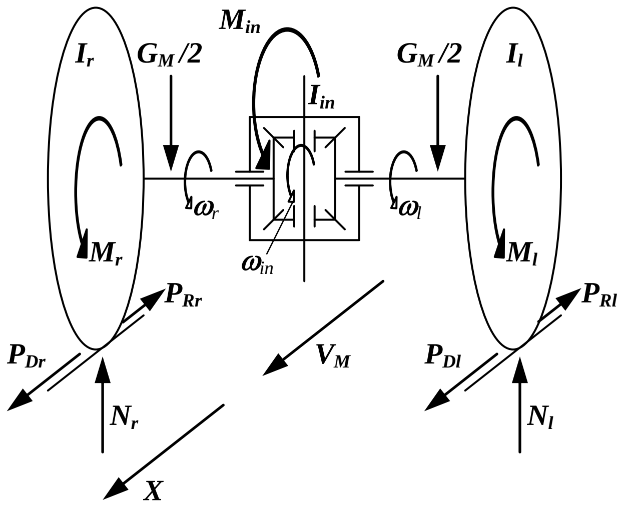


Figure 1 – Calculation scheme for modeling the process of the machine accelerating

Regardless of the type of modeled differential, the torque input is applied to the system input, which is calculated according to the following algorithm:

– for a specific power and current angular velocity given in a particular experiment, the input torque is determined for one of the four axes

$$M_{in} = \frac{N_S G_M}{4\omega_{in}};$$

– if $M_{in} > M_{in}^{\max}$, then we accept $M_{in} = M_{in}^{\max}$;

– to determine the traction forces $P_{Dl}$ and $P_{Dr}$, we first calculate the theoretical and actual speeds of the wheels along the sides and the corresponding skid coefficients:

$$\sigma_l = \frac{(\omega_{in} - 0{,}5\Delta\omega)R_w - V_M}{(\omega_{in} - 0{,}5\Delta\omega)R_w} \text{ and } \sigma_r = \frac{(\omega_{in} + 0{,}5\Delta\omega)R_w - V_M}{(\omega_{in} + 0{,}5\Delta\omega)R_w},$$

and then, using known dependencies $P_D = f(\sigma, \varphi)$, we determine the traction forces $P_{Dl}$ and $P_{Dr}$ as a function of the corresponding skid coefficients and the current values of the adhesion coefficients $\varphi_l$ and $\varphi_r$;

– then the torques on the wheels $M_l$ and $M_r$ are determined by multiplying the corresponding traction forces by the radius of the driving wheel $R_w$.

In the simulation of the blocked inter-wheeled differential in equation (2), it is assumed that $\Delta\omega = 0$ and $\frac{d\Delta\omega}{dt} = 0$.

When modelling the limited-slip differentials, the additional frictional moment is determined depending on the type of differential or traction control systems as follows.

For differentials in which the degree of blockage depends on the load (torque sensitive differentials):

$$M_{fr} = M_{fr0} + k_M (M_l + M_r),$$

where $M_{fr0}$ – initial value of additional locking torque; $k_M$ – coefficient of proportionality.

For differentials in which the degree of blockage depends on the difference in the speed of the wheels rotation $M_{fr} = k_\omega \Delta\omega$ or $M_{fr} = k_\omega \Delta\omega^2$ depending on the type of differential, where $k_\omega$ – corresponding coefficient of proportionality.

When using TCS based on ABS, the algorithm for determining the additional frictional moment is as follows:

– if $\Delta\omega < \Delta\omega_0$, then $M_{fr} = 0$;

– if $\Delta\omega > \Delta\omega_0$ and $\frac{d\Delta\omega}{dt} > 0$, then there is an increase in the frictional moment with a high speed in proportion to time $M_{fr} = k_t t$;

– if $\Delta\omega > \Delta\omega_0$ and $\frac{d\Delta\omega}{dt} < 0$, then the value of the frictional moment is fixed from the previous calculation step and does not change.

It should be noted that when using TCS based on ABS, in equation (1) the additional friction moment is also added, since it additionally loads the drive:

$$\frac{d\omega_{in}}{dt} = \frac{M_{in} - M_l - M_r - M_{fr}}{I_{in} + I_l + I_r}.$$

The acceleration of the machine was carried out to a speed of 10 m/s, while at each step of solving the system of differential equations the work performed by the drive was calculated, summing over the entire period of acceleration.

## *2.3. Calculations results*

In fig. Figures 2-5 show the calculation results for the various inter-wheeled differentials, described above, for the BTR-4 armored personnel carrier. On all graphs, the horizontal scale is presented in relative units $\Delta\varphi = \frac{\varphi_r}{\varphi_l} \in [0{,}125;\ 0{,}5]$, characterizing the balance of adhesion coefficients under the sides. The right boundary of the range was calculated to $\Delta\varphi = 1$, but due to the low information content on this site was limited to $\Delta\varphi = 0{,}5$.

Calculations were carried out for the following types and characteristics of differentials: *w*_10, *w*_20 and *w*_40 – differentials in which the locking factor depends on the difference in wheel speeds with coefficients, respectively: $k_\omega = 10,\ 20,\ 40$ Nm·s; *qw*_05, *qw*_1, *qw*_15 and *qw*_2 – differentials in which the locking factor depends on the square of the difference in wheel speeds with coefficients, respectively: $k_\omega = 0{,}5;\ 1{,}0;\ 1{,}5;\ 2{,}0$ Nm·s$^2$; *M*_02, *M*_04 and *M*_06 – differentials in which the locking factor depends on the load with coefficients, respectively: $k_M = 0{,}2;\ 0{,}4;\ 0{,}6$; TCS – traction control system based on ABS with $\Delta\omega_0 = 5$ s$^{-1}$, $k_t = 20000$ Nm/s.

In all graphs, the results are presented in relative units relative to the indicators obtained for the blocked differential. Since the blocked differential has the best performance in straight-line motion, all the values shown in the graphs are greater than one.

The graphs do not show curves for an open conic differential, since they differ significantly from other indicators and have a bad effect on the scale. For average coefficients of resistance to movement $f_m = 0{,}02$ and $f_m = 0{,}08$, the relative excess of the acceleration time to 10 m/s with an open differential is 1,91 and 5,19 respectively. And the relative excess of energy costs for the same conditions is 2.13 and 5.85 respectively.

All the charts in the work, as mentioned earlier, were built for the wheeled armored personnel carrier BTR-4, which as standard has a specific power of 14.58 kW / t (19.83 hp / t).

This moment is significant, since the change in specific power, both in the smaller and in the larger direction, substantially distorts the obtained graphical dependencies not only in absolute, but also in relative dimension.

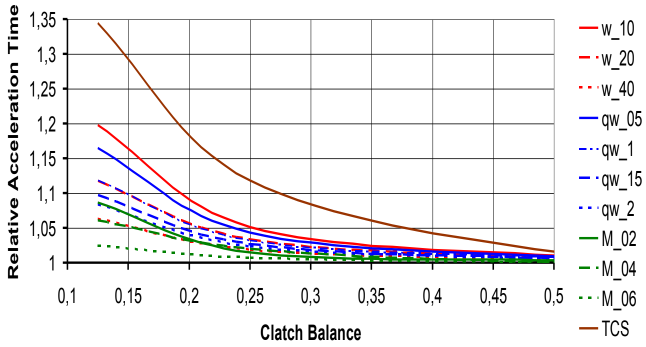


Figure 2 – Comparison of the relative acceleration time during the acceleration up to 10 m/s of armored personnel carriers with different types of differentials on a road with an average drag coefficient $f_m = 0{,}02$

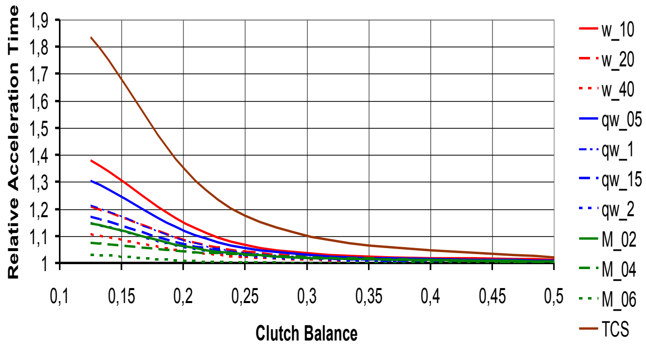


Figure 3 – Comparison of the relative acceleration time during the acceleration up to 10 m/s of armored personnel carriers with different types of differentials on a road with an average drag coefficient $f_m = 0{,}08$

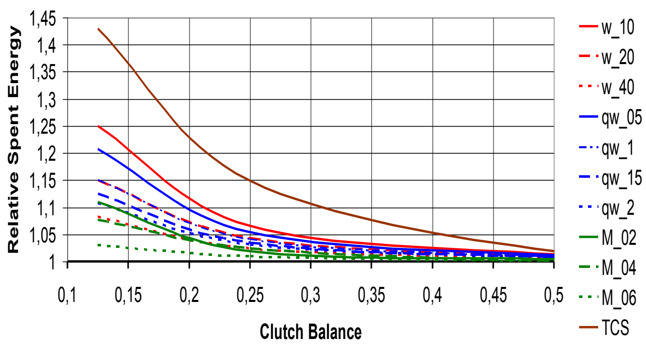


Figure 4 – Comparison of the relative spent energy during the acceleration up to 10 m/s of armored personnel carriers with different types of differentials on a road with an average drag coefficient $f_m = 0{,}02$

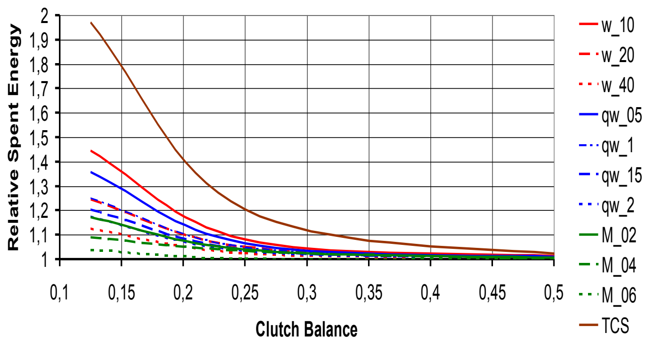


Figure 5 – Comparison of the relative spent energy during the acceleration up to 10 m/s of armored personnel carriers with different types of differentials on a road with an average drag coefficient $f_m = 0{,}08$

## 3. CONCLUSIONS

In accordance with the goals and objectives set for the results of the work done, the following conclusions can be drawn:

1) Non-blocked open conic differentials and fully-locked differentials were considered only to determine the boundaries of the worst and best overclocking performance indicators.

2) The comparative data obtained are given for the average values of the system adjustment factors. Therefore, for final conclusions about the advisability of using a particular limited-slip differentials or TCS based on ABS, taking into account the influence of the differential on the controllability of the machine, it is necessary to optimize by the setting factors.

3) From the preliminary consideration of the problem, it can be stated that all the limited-slip differentials in terms of the efficiency of straight-line acceleration are close to the characteristics of the locked differential in the following sequence:

– fully-locked differentials;

– differentials in which the locking factor depends on the difference in wheel speeds;

– differentials in which the locking factor depends on the square of the difference in wheel speeds;

– differentials in which the locking factor depends on the load;

– TCS – traction control system based on ABS;

– open conic differentials.

4) Differentials with TCS based on ABS are most appropriate for relatively small values of imbalance of adhesion coefficients along the sides of the machine and to integrate them with systems for maintaining directional stability, which allow, depending on the situation, to change the values of the coefficients of the system settings.

5) Limited-slip differentials, in which the blocking moment depends on the square of the difference in the speed of rotation of the wheels, can have high energy parameters during acceleration with minimal negative impact on the controllability of the machine and at the same time rely not on electronic control systems, but on their own internal automatism.

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