

Atomic Functions: the History of the Formation, Development and Practical Application

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Abstract

Atomic functions are infinitely differentiable compactly supported solutions of functional differential equations of a special type. After the first successful building of the functions performed by VL Rvachev and VA Rvachev in the 70s of the previous century, different classes of the atomic functions of one and several variables were studied, which have found application in the solution of various problems of mathematical analysis and mathematical modeling of practical problems.

Generalization of atomic functions to the case of several variables associated with the expansion of their possible application to solving boundary value problems in partial derivatives had been considered, in particular, and the development of new methods for the numerical solution of such tasks. Mathematical tools based on atomic functions of several variables have the necessary properties of universality and locality, to be requested in the practice of numerical solutions of boundary value problems. The study of functional differential equations, which are used for their formation other differential operators, for example, Laplace, Helmholtz, biharmonic operators et al., leads to the construction of the special form of atomic functions.

The atomic functions form the classes radial basis functions that allow you to develop on their basis meshless scheme of solving boundary value problems. In comparison with the known radial basis functions atomic radial basis functions have advantages, namely, are infinitely smooth, satisfy the functional-differential equation, effectively computable, have explicit formulas for the calculation of the Fourier transform.

Keywords

atomic functions, radial basis functions, meshless methods

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The need for a generalization of atomic functions to the case of several variables emerged in the study of the possibility of their use in combination with the method of R -functions in the implementation of variational methods for solving partial mathematical physics. They are a natural generalization to the case of several variables which can be implemented on the basis of the usual tensor product of one-dimensional atomic functions. Trying to create an extension of atomic functions to the case of several variables was made earlier, but it shows no evidence of a number of necessary conditions for the existence of finite solutions of some functional-differential equations. Multidimensional generalizations of atomic functions are the result of a decision from the list of current problems in the theory of atomic functions on the construction of compactly supported solutions of functional differential equations of the special form.