MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL TECHNICAL UNIVERSITY
KHARKIV POLYTECHNICAL INSTITUTE

GUIDELINES
for the calculation of the individual assignment: "The Laplace transform analysis of transient processes in linear electrical circuits"

for the courses "Theoretical Basics of Electrical Engineering",
"Theory of Electrical and Magnetic Circuits",
"Theory of Electromagnetic Circuits",
"Theory of Electric Circuits and Signals",

for the students studying Electrical Engineering and Computer Sciences

Approved
by editorial and publishing
university council,
record No.1 of 28.01.2022

Kharkiv
NTU "KPI"
2022

The Guidelines are intended for the calculation and graphical presentation of the assignment on the topic "The Laplace transform analysis of transient processes in linear electrical circuits" for the courses "Theoretical Basics of Electrical Engineering", "Theory of Electrical and Magnetic Circuits", "Theory of Electromagnetic Circuits", "Theory of Electric Circuits and Signals", for the students of specialties 141 "Electric Power, Electrical Engineering and Electromechanics", 151 "Automation and computer – integrated technologies", 152 "Metrology and information –measuring equipment", 171 "Electronics", 172 "Telecommunications and radio engineering" / comp. M. M. Rezynkina, I. O. Kostiukov, S. A. Lytvynenko – Kharkiv : NTU "KhPI", 2022.

Compilers: M. M. Rezynkina, I. O. Kostiukov, S. A. Lytvynenko

Department of Theoretical Electrical Engineering

INTRODUCTION

These guidelines are intended to provide students with the basic theoretical provisions, necessary to carry out the analysis of transient processes in electrical circuits by applying the Laplace transform. Due to the convenience of its practical implementation and computational efficiency this method has become a widely spread tool which is commonly used on order to solve a pretty broad range of applied problems in electrical engineering. Thereby the Laplace transform approach for the analysis of transient processes in electrical circuits is usually included in the curriculums for the students studying electrical and electronic engineering.

The presented theoretical provisions are necessary for the analysis of transient processes in electrical circuits with the DC power sources. Current guidelines also provide the examples of analysis of transient processes based on the applying of the Laplace transform. The scope of these guidelines is limited by the analysis of transient processes for the case of electrical circuits with lumped parameters and DC power sources. All mathematical details which concern the derivation of the Laplace transform for the time-domain functions, as well as the proofs of the main properties of the Laplace transform are intestinally omitted. For these details readers are referred to the corresponding specialized mathematical literature. The presented examples of solutions for typical assignments, on the contrary, are provided in details and, therefore, facilitate students ability to implement the Laplace transform approach for the analysis of transient processes in practice.

1. THE LAPLACE TRANSFORM APPROACH FOR THE ANALYSIS OF TRANSIENT PROCESSES IN ELECTRICAL CIRCUITS

1.1. Definition of the Laplace transform

The conventional approach for the analysis of transient processes in electrical circuits usually is based on derivation of solutions for an ordinary differential equation. Therefore, such analysis involves the compulsory determination of the integration constants, based on the previously determined initial conditions. The evaluation of these integration constants is carried out under the stipulation which implies that during the analyzed transient process the flowing through the inductance current, as well as the voltage across the capacitors, remain continuous. For the case of high order differential equation this approach can become pretty sophisticated and cumbersome. The Laplace transform approach, on the contrary, allows us to carry out the analysis of transient processes in electrical circuits without the determination of integration constants. Thereby this approach can considered to be more tractable for many practical applications.

The applying of Laplace integral transform converts the originally determined in the time domain continuous function $f(t)$ in to a function of the auxiliary complex variable p.

$$F(p) = f(t) \qquad (1.1)$$

The function $f(t)$ is supposed to fit the **Dirichlet conditions**: it should have a finite number of maximums and minimums in a finite time interval, as well as the finite number of discontinuities of the first kind. The function $f(t)$ also should be limited:

$$|f(t)| < Ae^{\alpha t},$$

where A and α are positive numbers.

The transition from the original time-domain function $f(t)$ to the function $F(p)$ is accomplished according to the unilateral (one-sided) Laplace transform:

$$F(p) = \int_0^\infty f(t)e^{-pt}dt \qquad (1.2)$$

where p denotes the arbitrarily selected complex variable with its real and imaginary components designated as a and b: $p = a + jb$, and $j = -1^{0.5}$ The time-domain function $f(t)$, which in practical applications of electrical engineering might represent the continuous waveform of current, voltage or the electromotive force, is often called the "original", whereas the defined in the domain of the complex variable p its counterpart function $F(p)$ is often called the "image". By the analysis of (2) it is easy to infer that, depending on the specific type of the integrand function $f(t)$, various values of complex variable p will allow to obtain various counterpart functions $F(p)$. Consequently, instead of employing the defined in the time domain continuous functions $f(t)$ the Laplace transform approach for the analysis of transient processes deals with the functions of the auxiliary complex variable p. The analytically determined for several important for practical applications of electrical engineering time-domain functions and mathematical operators expressions for the Laplace transforms are presented in the Table 1.

Table 1 Several important one-sided Laplace transforms

Time-domain function $f(t)$, or mathematical operator	Corresponding Laplace transform $F(p)$
$df(t)/dt$	$pF(p) - f(0)$
$\int_0^t f(t)dt$	$F(p)/p$
A	A/p
$e^{-\alpha t}$	$1/(p+a)$
t	$1/p^2$

The presented in the Table 1 correspondence between the functions $f(t)$ and $F(p)$ allows us to establish the direct correspondence between the time-domain functions, derivative and antiderivative operators and their counterparts in the domain of complex variable *p*. Taking into account this correspondence it is easy to derive the Laplace transform for the expressions which determine the values of voltage drop across the capacitor and inductance in the time domain. The derived expressions constitute the basis of the Laplace transform approach for the analysis of transient processes and are widely used in practice.

1.2. The Laplace transform for the voltage drop across the elements of electrical circuits and their corresponding parameters.

The correspondence between the relation for the voltage drop across the inductance *L* in the time domain and its Laplace transform can be expressed in the following form:

$$L\frac{di}{dt} = LpI(p) - Li(0) \tag{1.3}$$

where *I*(*p*) is the Laplace transform for the flowing through the inductor current, *i*(0) designates the value of current flowing through the inductance in the beginning of the transient process. This current is determined for the case of $t = 0$.

Assuming zero initial conditions, i.e. the case of $i(0) = 0$, the Laplace transform for the expression which determines the voltage drop across the inductor in the time domain can be reduced to the following relation:

$$L\frac{di}{dt} = LpI(p) \tag{1.4}$$

The Laplace transform for the voltage drop across capacitor *C* in the time domain can be obtained by using following relation:

$$u_c = \frac{I(p)}{Cp} + \frac{u_c(0)}{p} \qquad (1.5)$$

$u_c(0)$ denotes the value of voltage across the capacitor in the beginning of the transient process i.e. for the case of $t = 0$. The value of $u_c(0)$ should be taken into account for the case if the capacitor has been initially charged to the certain level of voltage before the beginning of the transient process. $I(p)$ is the Laplace transform for the current flowing through the capacitor. Assuming that capacitor has not been charged before the beginning of the transient process, i.e. the case $u_c(0)= 0$ the Laplace transform for the expression which determines the voltage drop across the capacitor in the time domain can be reduced to the following relation:

$$u_c = \frac{I(p)}{Cp} \qquad (1.6)$$

For the case of switching of the analyzed electrical circuit to the source of constant electromotive force E the analysis is carried out by applying the Laplace transform for the constant electromotive force E. The relation for this transform can be expressed according to (1.7):

$$E = Ep^{-1}. \qquad (1.7)$$

It can be noticed that the relation for the Laplace transform of the invariable electromotive force E coincides with the second term of the relation (1.5), which determines the Laplace transform for the voltage drop across the capacitor C in the time domain.

The Laplace transform for the voltage drop across the resistor R can be determined as:

$$U_R(p) = I(p)R. \quad (1.8)$$

In some cases it is convenient to employ the relations for the capacitive and inductive reactance of electrical circuits in the domain of complex variable *p*. Taking into account the previously given relations in the *p*-domain for the voltage drop the capacitor and inductance, these expressions can be easily obtain from (1.4, 1.6) by setting $I(p) = 1$. Thereby, the Laplace transform for the capacitive and inductive reactance of electrical circuits can be correspondingly determined according to the following relations:

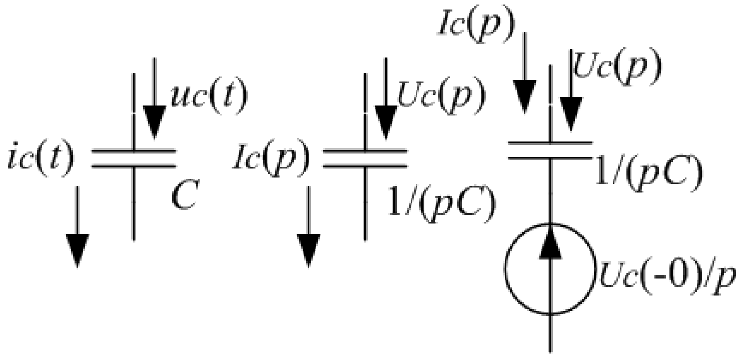
$$z_c = \frac{1}{Cp}; \quad (1.9)$$

$$z_L = pL. \quad (1.10)$$

Therefore the Laplace transform for the capacitive and inductive reactance can be easily obtained from the relations for the capacitive and inductive reactance in *AC* circuits by substituting the multiplier $j\omega$ with the auxiliary complex variable *p*.

1.3. The replacement of the originally analyzed scheme by the corresponding scheme in the domain of complex variable *p*.

The analysis of transient processes which is based on the applying of the Laplace transform usually requires the replacement of the originally analyzed scheme by the corresponding scheme in the domain of complex variable *p*. Such replacement is necessary due to several reasons, such as the previously mentioned discrepancy between the Laplace transform for the capacitive and inductive reactance and corresponding relations for the capacitive and inductive reactance in *AC* circuits. The transition from the originally analyzed scheme to the corresponding scheme in the domain of complex variable *p* should be carried out taking into the initial conditions in the analyzed scheme. Correspondence between the elements of electrical circuits in the time domain and their equivalent after the Laplace transform is presented in Fig. 1 – Fig. 4.



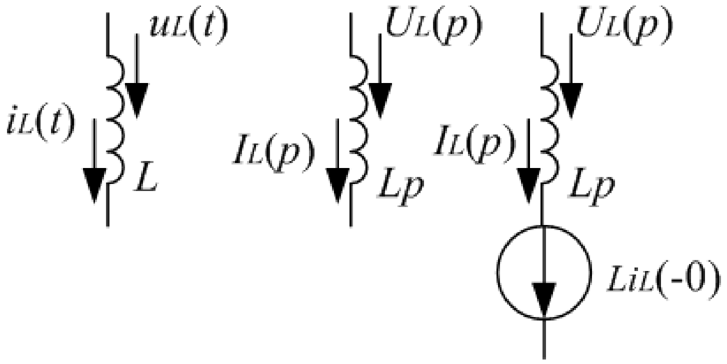
a *b* *c*

Fig. 1.1 The replacement of electrical capacitor by the corresponding element in the domain of a complex variable *p* [1]:

a – capacitor in the initially analyzed electrical circuit

b – corresponding element for the case of zero initial conditions

c – corresponding element for the case of non-zero initial conditions



a *b* *c*

Fig. 1.2 The replacement of inductance by the corresponding element in the domain of complex variable *p* [1]:

a – inductance in the initially analyzed electrical circuit

b – corresponding element for the case of zero initial conditions

c – corresponding element for the case of non-zero initial conditions

Fig. 1.3 The replacement of the idealized source of the electromotive force by the corresponding element in the domain of complex variable *p* [1]

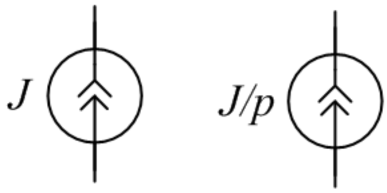


Fig. 1.4 The replacement of the idealized source of current by the corresponding element in the domain of complex variable *p* [1]

According to the presented in Fig. 1.1 – Fig. 1.2 methods of replacement, the non-zero initial conditions are taken into account by the inclusion of the additional sources of the electromotive force. It should be noted that for inductance the direction of the flowing through the replaced element current should coincide with the direction of the included supplementary source of the electromotive force. For capacitor the direction of current flowing through the replaced element should be opposite with respect to the included supplementary source of the electromotive force.

1.4. The relations for Ohm's and Kirchhoff's laws in the domain of complex variable *p*.

The Laplace transform is applicable both to Ohm's and Kirchhoff's laws, which are widely used in practice of the analysis of steady-state processes in AC and DC circuits. However, the transition from the time domain to the domain of the auxiliary complex variable *p* requires the appropriate reformulation of these basic laws. With respect to the Laplace transform these laws can be formulated by applying the presented in Fig. 1.5 part of the electrical circuit.

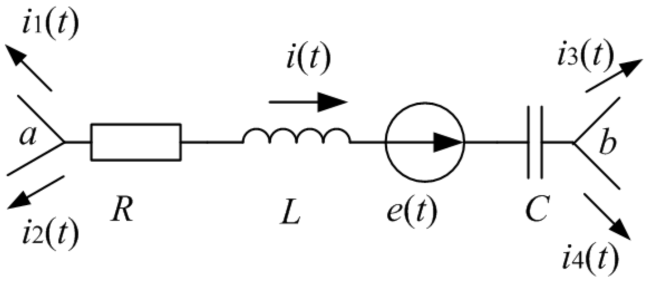


Fig. 1.5 Electric circuit for the formulation of Ohm's law in the domain of complex variable *p* [2].

The value of voltage drop across the points *ab* in for the presented in Fig. 5 electrical circuit can be expressed as [2]:

$$u_{ab} = iR + L\frac{di}{dt} + u_c(0) + \frac{1}{C}\int_0^t i(t)dt - e(t) \tag{1.11}$$

Taking into account the determined according to (1.3, 1.5) relations for the voltage drop across the capacitor and inductance in the domain of the auxiliary complex variable *p*, the Laplace transform for the voltage drop across the points *ab* can be expressed as [2]:

$$U_{ab}(p) = I(p)(p + pL + \frac{1}{pC}) - Li(0) + \frac{U_c(0)}{p} - E(p) \tag{1.12}$$

The total value of current flowing in the analyzed part of the electrical circuit can be derived from (1.12). This current can be expressed as [2]:

$$I(p) = \frac{U_{ab}(p) + Li(0) - \dfrac{U_c(0)}{p} + E(p)}{Z(p)} \tag{1.13}$$

where $Z(p)$ denotes the Laplace transform for the impedance of the analyzed circuit:

$$Z(p) = R + pL + \frac{1}{pC} \tag{1.14}$$

For the case of zero initial conditions (1.13) can be reduced to [2]:

$$I(p) = \frac{U_{ab}(p)}{Z(p)} \tag{1.15}$$

Therefore the Laplace transform for the current flowing in the arbitrarily selected brunch of the analyzed circuit can be determined by employing Ohm's law formulated in terms of parameters of the analyzed circuit in p domain.

With respect to the node a of the presented in Fig. 1.5 part of the electrical; circuit the Kirchhoff's current law can be expressed as [2];

$$I_1(p) + I(p) + I_2(p) = 0 \tag{1.16}$$

where $I_1(p)$, $I(p)$ and $I_2(p)$ denote the Laplace transforms for the currents $i_1(t)$, $i(t)$, $i_2(t)$.

The Kirchhoff's voltage law can be expressed as [2]:

$$\sum I_k(p)Z_k(p) = \sum E_k(p) \tag{1.17}$$

The algebraic sum of the operational voltage drops is equal to an algebraic sum of the operational EMFs in the loop, including additional EMFs, which takes into account nonzero initial conditions.

As Ohm's and Kirchhoff's laws are applicable in the operational form, all methods of analyzed of electrical circuits can be used in the operational form for determination the Laplace images: mesh method and nodal potentials method, the superposition method, the equivalent generator method.

1.5. The transition from the domain of the auxiliary complex variable *p* to the time domain.

After the derivation of the expression for unknown currents and voltages in the analyzed scheme the Laplace transform-based analysis of transient processes in electrical circuits implies the compulsory reverse transition to the time domain. In practice the inverse transition from the domain of complex variable *p* to the time domain can be made by means of applying the inverse Laplace transform, by using the specialized tables, which allow to establish the correspondence between some functions in the *p* domain and their counterparts *f*(*t*) in the time domain, or by employing the decomposition theorem. The presented in these guidelines examples of solutions for typical problems are based on the applying of the decomposition theorem. In general form this approach for the inverse transition from the *p* domain to the time domain can be expressed as:

$$\frac{N(p)}{M(p)} = \sum_{n=1}^{M} \frac{N(p_n)}{M'(p_n)} e^{p_n t}. \qquad (1.18)$$

where $N(p)$ and $M(p)$ are the general designations for the numerator and denominator of the derived expression in the *p* domain, *M* denotes the total number of roots of the denominator $M(p)$, all $N(p_n)$ denote the values of the numerator $N(p)$ determined for the roots p_n of the denominator $M(p)$ and finally all $M'(p_n)$ denote the values of the derivative, determined for the denominator $M(p)$ with respect to the auxiliary variable *p*, calculated for the roots p_n.

The example of practical applying of the decomposition theorem can be given with respect to the determined according to (1.19) relation in the *p* domain for the current flowing through the analyzed circuit.

$$i_2(p) = \frac{p^2 M U_c C_1 C_2}{ap^4 + bp^3 + cp^2 + dp + 1}. \tag{1.19}$$

where all parameters *a*, *b*, *c*, *d* can be determined by applying the following expressions:

$$a = L_2 C_2 L_1 C_1 - M^2 C_1 C_2, \tag{1.20}$$

$$b = L_2 C_2 R_1 C_1 + R_2 C_2 L_1 C_1, \tag{1.21}$$

$$c = L_2 C_2 + R_2 C_2 R_1 C_1 + L_1 C_1, \tag{1.22}$$

$$d = R_2 C_2 + R_1 C_1. \tag{1.23}$$

All the values of L_1, L_2, M, R_2, C_1 C_2, which determine the parameters *a*, *b*, *c*, *d* are given in the Table 2

Table 2

R_1	R_2	L_1	L_2	C_1	C_2	M
2.79	1.57	$186 \cdot 10^{-6}$	$126 \cdot 10^{-6}$	$4.5 \cdot 10^{-6}$	$6.128 \cdot 10^{-9}$	1.13310^{-4}

According to the decomposition theorem the inverse transition from the *p* domain to the time domain can be made according to the following relation:

$$i_2(t) = \sum_{n=1}^{4} \frac{N(p_n)}{M'(p_n)} e^{p_n t}. \tag{24}$$

where all $N(p_n)$ are the values of the numerator in (1.19) calculated for the values of *p* which correspond to the roots p_n of the polynomial from the denominator (1.19). All $M'(p_n)$ are the values of the derivative with respect to the parameter *p* from the polynomial in the denominator of the expression (1.19). The values of this derivative should be calculated for the roots of this denominator.

Since for the polynomial of the 4th order, the denominator of (1.19) will have 4 roots, the transition to the time domain for our example will require taking into account of 4 terms in (1.19), i.e. $M = 4$.

For the case of the determined according to (19) relation, the expressions for $N(p)$ and $M(p)$ can be determined according to:

$$N(p) = p^2 M U_c C_1 C_2 \tag{1.25}$$

$$M(p) = ap^4 + bp^3 + cp^2 + dp + 1 \tag{1.26}$$

$$\frac{dM(p)}{dp} = 4ap^3 + 3bp^2 + 2cp + d \tag{1.27}$$

All the required calculations for the transition to the time domain by employing the decomposition theorem are presented in the Table 3.

Table 3

	p_n	$N(p_n)$	$M'(p_n)$	$N(p_n)/M'(p_n)$	$e^{p_n t}$ for the case of $t = 1\cdot 10^{-4}$ s
p_1	$10^6\cdot(-0.0229 + j1.6924)$	$-8.947\cdot 10^{-5} - j2.418\cdot 10^{-6}$	$0.0001 - j0.0028$	$0.0003 - j\,0.0316$	$0.0935 - j0.0398$
p_2	$10^6\cdot(-0.0229 - j1.6924)$	$-8.947\cdot 10^{-5} + j2.418\cdot 10^{-6}$	$0.0001 + j0.0028$	$0.0003 + j0.0316$	$0.0935 + j0.0398$
p_3	$10^6\cdot(-0.0075 + j0.0337)$	$10^{-8}\cdot(-3.3804 - j1.5796)$	$-2.0455\cdot 10^{-8} + j5.6469\cdot 10^{-5}$	$-2.7951\cdot 10^{-4} + j5.9874\cdot 10^{-4}$	$-0.4600 - j0.1087$
p_4	$10^6\cdot(-0.0075 - j0.0337)$	$10^{-8}\cdot(-3.3804 + j1.5796)$	$-2.0455\cdot 10^{-8} - j5.6469\cdot 10^{-5}$	$-2.7951\cdot 10^{-4} - j5.9874\cdot 10^{-4}$	$-0.4600 + j0.1087$

According to the presented typical scheme of applying the decomposition theorem the relation for the current $i_2(t)$ can be expressed as:

$$i_2(t) = \sum_{n=1}^{4} \frac{N(p_n)}{M'(p_n)} e^{p_n t}. \qquad (1.28)$$

All the necessary for calculations values of $N(p_1)$.. $N(p_2)$, $M'(p_1)$.. $M'(p_4)$, as well as the values of $N(p_1)/M'(p_1)$.. $N(p_4)/M'(p_4)$, are presented in the Table 3. The imaginary components of $N(p_1)/M'(p_1)$.. $N(p_4)/M'(p_4)$ offset each other. The same compensation remains valid for the imaginary components of the multipliers $e^{p_1 t}$.. $e^{p_4 t}$. Thereby despite the complex conjugate roots of the polynomial $M(p)$ the derived relation for $i_2(t)$ does not have any imaginary component.

1.6. The sequence of procedures for the analysis of transient processes in electrical circuits by applying the Laplace transform.

The analysis of transient processes by applying the Laplace transform approach usually contains the following sequence of procedures:

- the evaluation of the initial conditions for the analyzed circuit. The evaluation of the initial conditions implies the calculation of the flowing through the inductors currents and voltages across the capacitors

- the transition from the analyzed scheme to its counterpart in the domain of complex variable p. For this step all elements of the initial circuit should be replaced by their presented in Fig. 1.1 – Fig. 1.4 equivalent. The non-zero initial conditions should be taken into account by the inclusion of supplementary sources of the electromotive force. This transition is carried out for the analyzed circuit after the switching.

- derivation of the expressions for the unknown currents and voltages in the domain of complex variable p. This derivation can be accomplished by employing the Ohm's and Kirchhoff's laws formulated with respect to the domain of complex variable p.

- the transition from the domain of complex variable p to the time domain. This transition can be made by means of applying the decomposition theorem.

2. EXAMPLES OF SOLUTIONS FOR TYPICAL PROBLEMS

<u>Example no. 1</u>

Determine the flowing in the presented in Fig. 2.1 current, caused by the discharge of the initially charged up to the value of 10 V capacitor *C*. All calculations for the following parameters of the analyzed circuit: *L* = 10 mH, *R* = 1000 Ohm, C = 1 μF.

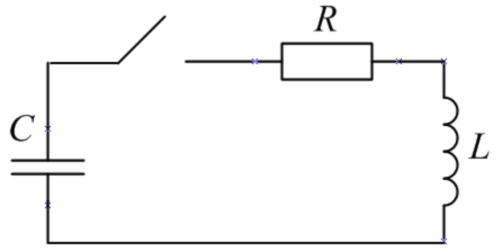


Fig. 2.1 Analyzed electrical circuit.

The analysis of transient process should be carried out under the non-zero initial conditions for the capacitor *C* and zero initial conditions for the inductance *L*. For this case the equivalent circuit in the *p* domain is presented in Fig. 2.2.

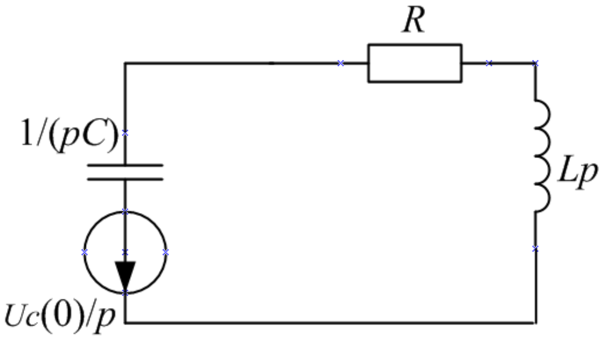


Fig. 2.2 The equivalent electrical scheme for the presented in Fig. 2.1 circuit

For the presented in Fig. 2.2 circuit the input impedance can be expresses as:

$$Z(p) = R + pL + \frac{1}{pC} \tag{2.1}$$

Taking into account the derived expression (2.1), the flowing in the analyzed circuit current in the *p* domain can be expressed as:

$$i(p)=\frac{-\dfrac{U_C(0)}{p}}{R+pL+\dfrac{1}{pC}}=-\frac{U_C C}{p^2LC+pRC+1} \tag{2.2}$$

For the derived expression $N(p)$ and $M(p)$ can be defined as:

$$N(p) = EC, \tag{2.3}$$

$$M(p) = p^2LC+pRC+1. \tag{2.4}$$

The roots of the polynomial $M(p)$ are equal to: $p_1 = -1010.2$, $p_2 = -98989.9$. The derivative from $M(p)$ can be expressed as:

$$\frac{dM(p)}{dp}=2LCp+RC \tag{2.5}$$

By substituting the obtained roots in (2.4) and employing the decomposition theorem the relation for the flowing in the discharge circuit current can be expressed as:

$$i(t)=-\frac{E}{R-2020.4L}e^{-1010.2t}+\frac{E}{197979.59L-R}e^{-98989.8t} \tag{2.6}$$

<u>Example no. 2</u>

Determine the Laplace transform for the current $I(p)$ flowing in the presented in Fig. 2.3 analyzed circuit.

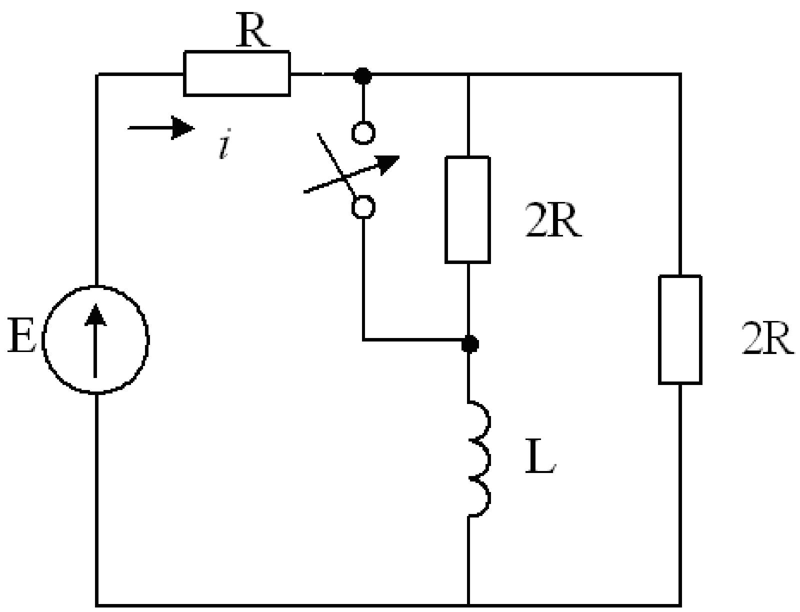


Fig. 2.3 The analyzed electrical circuit for the example no. 2

Before the switching ($t < 0$), taking into account that for the case of DC circuits $u_L = 0$ the flowing through the inductance current i_L can be expressed by using the relations (2.7 – 2.9):

$$R_{eq} = R + \frac{2R \cdot 2R}{4R} = 2R \tag{2.7}$$

$$i = \frac{E}{2R} \tag{2.8}$$

$$i_L = \frac{1}{2} i = \frac{E}{4R} \tag{2.9}$$

Since the flowing through the inductance current i_L is not equal to zero, for our particular case we have to take into account the non-zero initial conditions for the inductance L, i.e. $i_L(0) = E/4R$.

Taking into account the non-zero initial conditions and the presented in Fig. 1.1 – Fig. 1.4 correspondence between the elements of electrical circuits and their counterparts in the *p* domain the equivalent circuit for the originally analyzed scheme can be built according to the presented in Fig. 9 circuit.

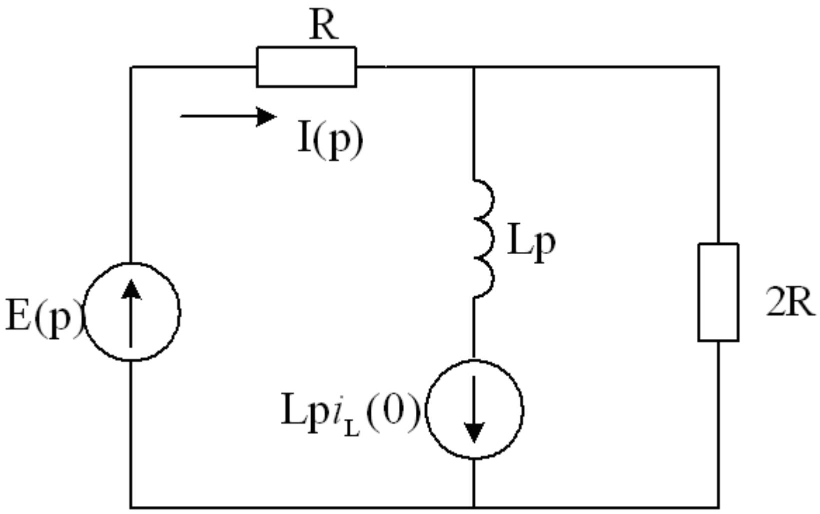


Fig. 2.4 The equivalent electrical scheme for the presented in Fig. 2.3 circuit

By using the principle of superposition the current $I(p)$ can be expressed as the sum of contributions caused by the the sourse of the electromotive force E(p) and the supplementary source $Lpi(0)$.

$$I(p) = I^{'}(p) + I^{''}(p) = \frac{\frac{E}{p}}{R + \frac{Lp \cdot 2R}{Lp + 2R}} + \frac{Li_L(0)}{Lp + \frac{R \cdot 2R}{3R}} \cdot \frac{2R}{3R} = \frac{E(3Lp + 4R)}{2Rp(3Lp + 2R)} \qquad (2.10)$$

where $I'(p)$ denotes the component of current $I(p)$ determined by the electromotive force E/p whereas $I''(p)$ denotes the component of current $I(p)$ determined by the electromotive force $Lpi_L(0)$.

Example no. 3

Determine the $i_L(t)$, $u_L(t)$ for the presented in Fig. 2.4 electrical circuit. The parameters of the analyzed circuit are: E=77 V, R=10 Ohm, L=70 mH.

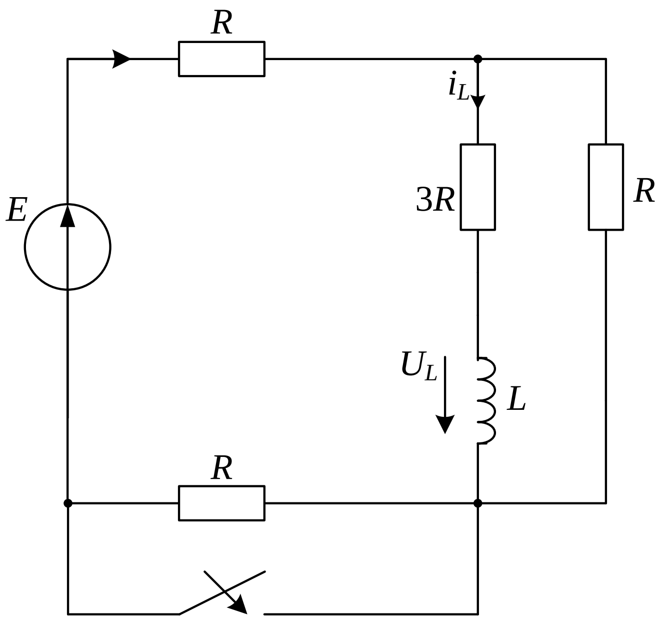


Fig. 2.4 The analyzed electrical circuit for the example no. 3

In order to solve the problem by using the Laplace transform approach, it is necessary to draw up an equivalent operator circuit. The additional sources of the electromotive force must be included if before the switching there was a current in the inductance or voltage across the capacitor.

Before the switching ($t < 0$), taking into account that for the case of DC circuits $u_L = 0$ and the flowing through the inductance current i_L can be expressed by using the relations (2.11 – 2.13)

$$R_{eq}=R+3R\cdot R/4R+R=27{,}5 \text{ Ohm}. \quad (2.11)$$

$$i=E/R_{eq}=77/27{,}5=2{,}8 \text{ A} \quad (2.12)$$

$$i_L(0)=i\cdot R/(3R+R)=0{,}7 \text{ A} \quad (2.13)$$

Since the flowing through the inductance current i_L is not equal to zero, for our particular case we have to take into account the non-zero initial conditions for the inductance L.

The equivalent circuit for the originally analyzed scheme can be built according to the presented in Fig. 2.5 circuit.

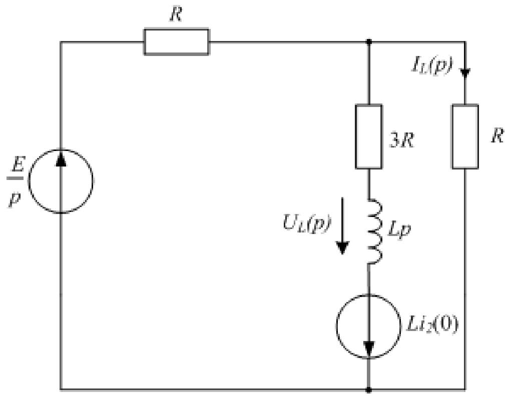


Fig. 2.5 The equivalent electrical scheme for the presented in Fig. 2.4 circuit

The analysis of the presented in Fig. 11 equivalent scheme can be made by using the superposition principle. The contributions of the included in the presented in Fig. 11 electrical scheme sources of the electromotive force to the flowing in the analyzed scheme currents will be considered separately.

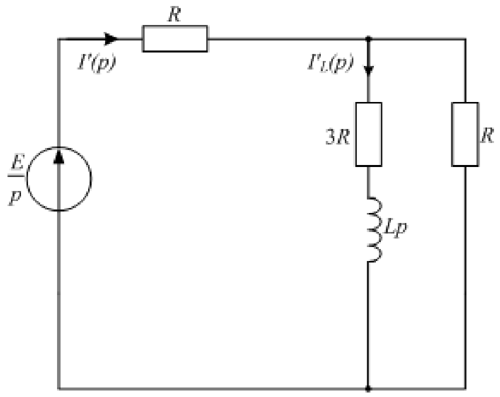


Fig. 2.6 Electrical circuit for the analysis of contribution of the source of electromotive force *E*/*p* to the flowing in the analyzed scheme currents

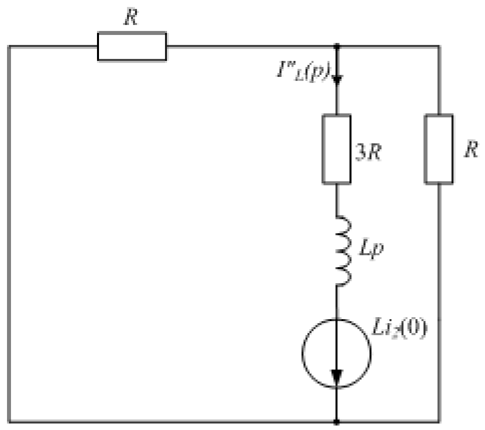


Fig. 2.7 Electrical circuit for the analysis of contribution of the source of electromotive force $Li_L(-0)$ to the flowing in the analyzed scheme currents

Taking into account the presented in the Fig. 2.7 – Fig, 2.7 circuits we are going to carry out our analysis by considering the contributions of the electromotive forces *E*/*p* and $Li_L(-0)$. For the presented in Fig. 2.6 scheme the unknown flowing through the inductance current can be calculated according to (2.14, 2.15):

$$I'(p) = \frac{\frac{E}{p}}{R + \frac{R(3R+Lp)}{3R+Lp+R}} = \frac{E(Lp+4R)}{2LRp^2 + 7R^2 p} \tag{2.14}$$

where *I'*(*p*) denotes the component of the total current *I*(*p*), determined by the electromotive force *E/p*.

$$I''_L(p) = \frac{E(Lp+4R)}{2LRp^2 + 7R^2 p} \cdot \frac{R}{R+3R+pL} = \frac{E}{2Lp^2 + 7Rp} \tag{2.15}$$

where $I'_L(p)$ denotes the component of the flowing through the inductance current *I*(*p*) determined by the electromotive force *E/p*. For the presented in Fig. 2.7 scheme the unknown flowing through the inductance current can be expressed as:

$$I''_L(p) = \frac{Li_L(0)}{Lp + 3R + \frac{R^2}{R+R}} \cdot = \frac{2Li_L(0)}{2Lp+7R} \tag{2.16}$$

The unknown current flowing through the inductance due to the combined affect of *E*/*p* and $Li_L(-0)$ can be expressed as:

$$I_L(p) = I'(p) + I''(p) = \frac{E}{2Lp^2 + 7Rp} + \frac{2Li_L(0)}{2Lp+7R} = \frac{2Li_l(0)p + E}{2Lp^2 + 7Rp} \tag{2.17}$$

The expression for the flowing through the inductance current $i_L(t)$ can be determined by using the decomposition theorem.

$$i_L(t) = \sum_{n=1}^{M} \frac{N(p_n)}{M'(p_n)} e^{p_n t} \tag{2.18}$$

In the determined according to (2.17) relation for the flowing through the inductance current the polynomials *N*(*p*) and *M*(*p*) can be expressed as follows:

$$N(p) = E + 2Li_L(0)p \tag{2.19}$$

$$M(p) = 2Lp^2 + 7Rp \tag{2.20}$$

In our case the roots of the polynomial $M(p)$ are equal to: $p_1 = 0$, $p_2 = -500$. And the derivative from $M(p)$ can be expressed as:

$$Q(p) = \frac{dM(p)}{dp} = 4Lp + 7R \tag{2.21}$$

All the necessary in order to implement the decomposition theorem parameters are determined according to:

$$N(p_1) = E \tag{2.22}$$

$$N(p_2) = 28 \tag{2.23}$$

$$Q(p_1) = 70 \tag{2.24}$$

$$Q(p_2) = -70 \tag{2.25}$$

By substituting the calculated parameters $N(p_1)$, $N(p_2)$, $Q(p_1)$, $Q(p_2)$ in (2.18) the relation for the flowing through the inductance current can be expressed as:

$$i_L(t) = N(p_1)/Q(p_1)\cdot e^{p_1 t} + N(p_2)/Q(p_2)\cdot e^{p_2 t} = 77/70\cdot e^{0\cdot t} + 28/{-70}\cdot e^{-500t} \tag{2.18}$$

THE LIST OF QUESTIONS FOR SELF-TESTING

1. The replacement of the originally analyzed scheme by the corresponding scheme in the domain of complex variable *p*.

2. The Laplace transform for the values of voltage drop across the elements of electrical circuits and their corresponding parameters.

3. The decomposition theorem.

4. The sequence of the procedures for the analysis of transient processes in electrical circuits by applying the Laplace transform.

5. The affect of initial conditions on replacement of the originally analyzed scheme by the corresponding scheme in the domain of complex variable *p*.

6. The relation for the Ohm’s law in the domain of complex variable *p*.

7. The relations for the Kirchhoff’s current and voltage laws in the domain of complex variable *p*.

3. ASSIGNMENTS FOR THE INDIVIDUAL ANALYSIS

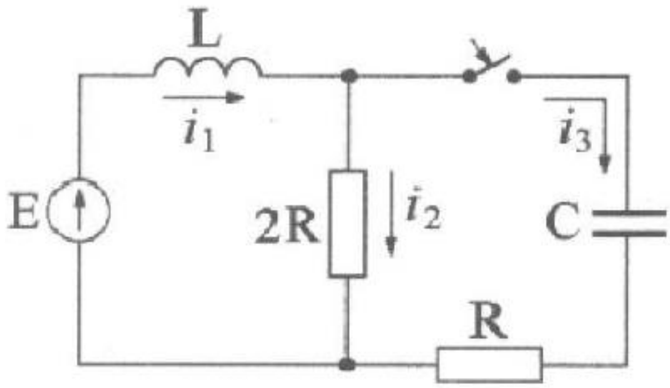


Fig. 3.1 – Electrical circuit for the assignments no. 1- 5

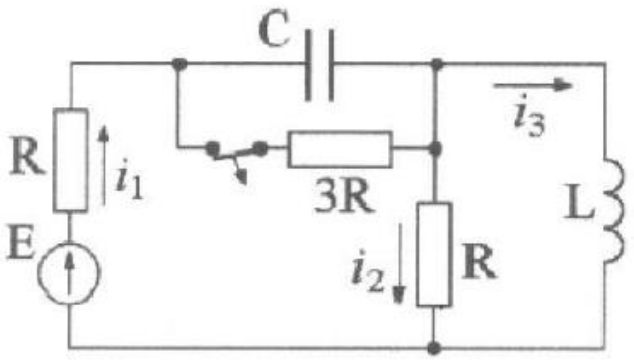


Fig. 3.2 – Electrical circuit for the assignments no. 6 - 10

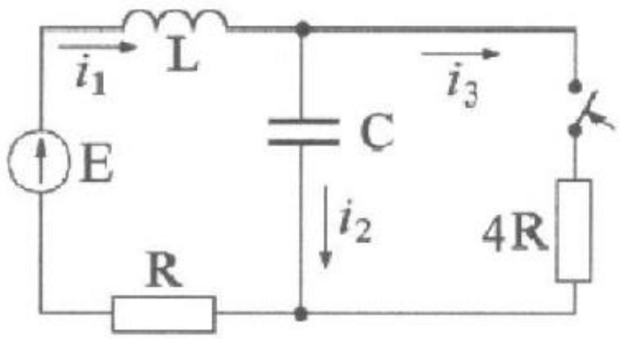


Fig. 3.3 – Electrical circuit for the assignments no. 11- 15

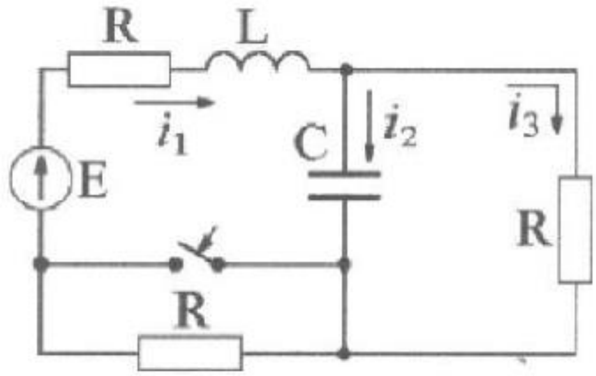


Fig. 3.4 – Electrical circuit for the assignments no. 16- 20

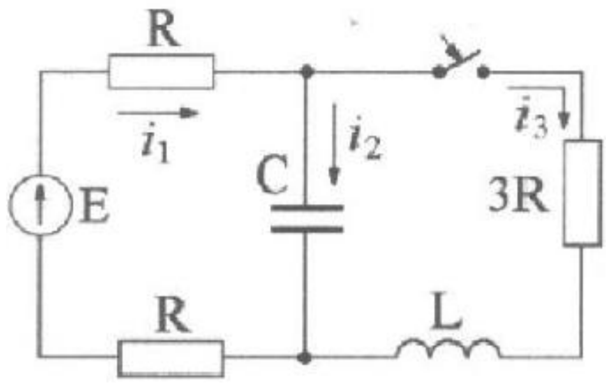


Fig. 3.5 – Electrical circuit for the assignments no. 21- 25

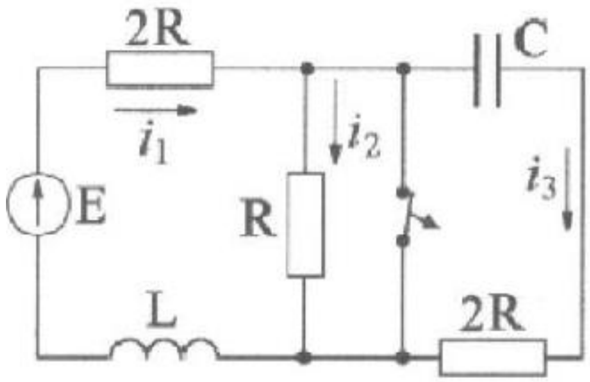


Fig. 3.6 – Electrical circuit for the assignments no. 26- 30

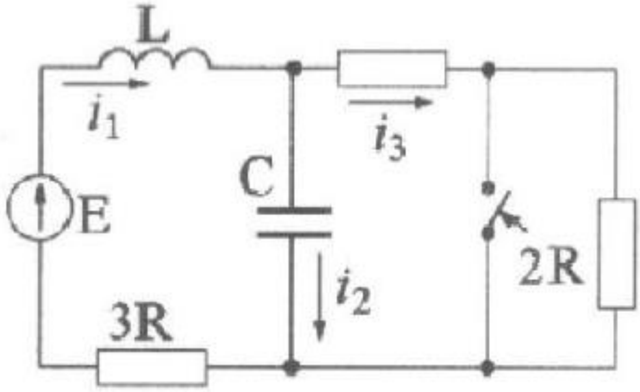


Fig. 3.7 – Electrical circuit for the assignments no. 31 - 35

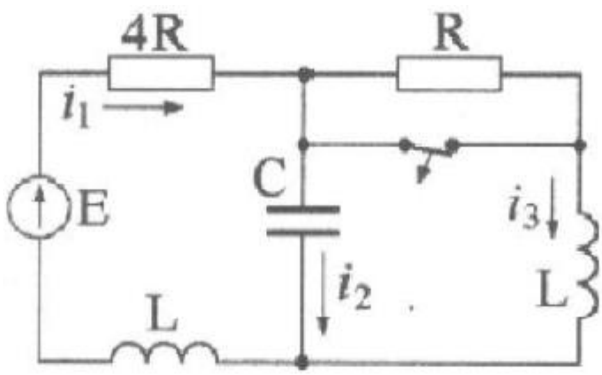


Fig. 3.8 – Electrical circuit for the assignments no. 36- 40

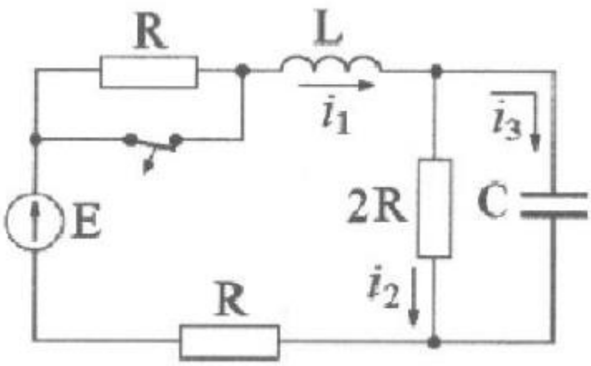


Fig. 3.9 – Electrical circuit for the assignments no. 41- 45

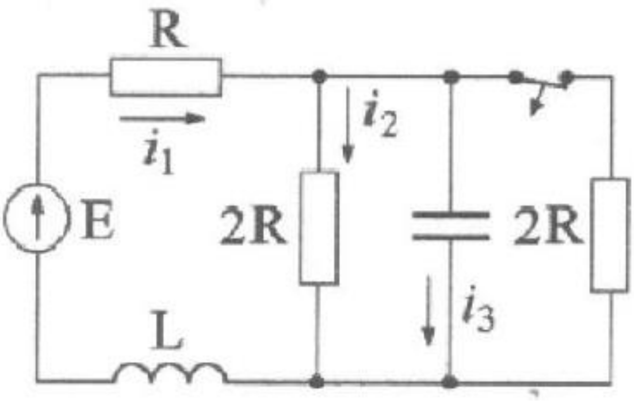


Fig. 3.10 – Electrical circuit for the assignments no. 46- 50

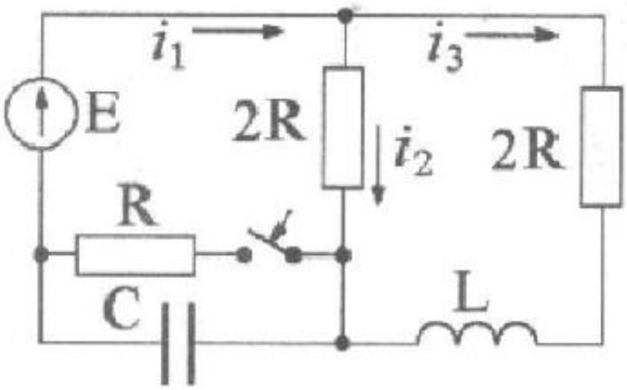


Fig. 3.11 – Electrical circuit for the assignments no. 51- 55

TABLE 3.1 – Parameters of the analyzed circuits

Assignment no.	E, Volt	R, Ohm	L, mH	C, μF	The unknown value
1	160	40	20	33.3	$i_1(t)$
2	600	300	33.3	20	$i_2(t)$
3	600	150	600	10	$i_3(t)$
4	270	54	13.3	50	$u_L(t)$
5	240	60	50	13.33	$u_C(t)$
6	120	24	50	1000	$i_2(t)$
7	240	24	100	500	$i_3(t)$
8	500	100	200	250	$u_L(t)$
9	150	60	500	100	$u_C(t)$
10	250	500	400	125	$i_1(t)$
11	100	20	200	100	$i_3(t)$
12	320	32	100	200	$u_L(t)$
13	30	6	50	400	$u_C(t)$
14	60	120	400	50	$i_2(t)$
15	50	5	80	1000	$i_1(t)$
16	300	50	200	1000	$u_L(t)$
17	60	20	400	500	$u_C(t)$
18	180	300	800	250	$i_1(t)$
19	120	40	500	400	$i_2(t)$
20	600	500	1000	200	$i_3(t)$
21	150	10	100	100	$u_C(t)$
22	30	10	200	50	$i_1(t)$
23	75	25	500	200	$i_2(t)$
24	75	500	1000	10	$i_3(t)$
25	400	75	500	20	$u_L(t)$
26	180	30	250	100	$i_1(t)$

27	72	6	100	250	$i_2(t)$
28	120	10	500	50	$i_3(t)$
29	10	3.33	50	500	$u_L(t)$
30	24	1	25	1000	$u_C(t)$
31	75	5	10	100	$i_2(t)$
32	60	40	20	50	$i_3(t)$
33	150	50	50	5	$u_L(t)$
34	600	20	100	10	$u_C(t)$
35	900	600	1000	1	$i_1(t)$
36	200	100	50	100	$i_3(t)$
37	400	20	100	50	$u_L(t)$
38	640	16	200	25	$u_C(t)$
39	200	10	25	200	$i_1(t)$
40	80	20	40	125	$i_2(t)$
41	600	50	500	100	$u_L(t)$
42	48	2	100	500	$u_C(t)$
43	300	12.5	200	250	$i_1(t)$
44	60	50	250	200	$i_2(t)$
45	300	50	400	125	$i_3(t)$
46	30	5	100	60	$u_C(t)$
47	600	50	200	30	$i_1(t)$
48	120	10	120	50	$i_2(t)$
49	240	100	240	25	$i_3(t)$
50	450	75	60	100	$u_L(t)$
51	90	15	40	100	$i_1(t)$
52	36	3	20	200	$i_2(t)$
53	600	50	80	50	$i_3(t)$
54	300	50	400	10	$u_L(t)$
55	120	200	800	20	$u_C(t)$

REFERENCES

1. Кубрик Б.И., Литвиненко С.А., Сосина Е.В. Методические указания к выполнению расчетной работы по теме «Операторный метод расчета переходных процессов в линейных электрических цепях» для студентов электротехнических факультетов– Харьков. : НТУ «ХПИ», 2016. – 36 с.

2. Бессонов Л. А. Теоретические основы электротехники. Электрические цепи : учебник / Л. А. Бессонов. – Москва : Гардарики, 2007. – 701 с.

Навчальне видання

Методичні вказівки
для виконання розрахунково-графічної роботи за темою
«Перетворення Лапласа для розрахунку перехідних процесів
в лінійних електричних колах»

з дисциплін «Теоретичні основи електротехніки», «Теорія електричних та магнітних кіл», «Теорія електричних кіл», «Теорія електричних кіл та сигналів», «»Теорія електромагнітних кіл»
для студентів електротехнічних та комп'ютерних спеціальностей.

Укладачі:
РЕЗИНКІНА Марина Михайлівна
КОСТЮКОВ Іван Олександрович
ЛИТВИНЕНКО Світлана Анатоліївна

Відповідальний за випуск проф. РЕЗИНКІНА Марина
Роботу рекомендував до друку доц. Олександр ЧЕПЕЛЮК
Комп'ютерна верстка Світлана ЛИТВИНЕНКО
В авторській редакції

План 2022, поз.123

Підп. до друку ________. Формат 60×84 1/16. Папір офсетний.
Riso-друк. Гарнітура Times New Roman. Ум. друк. арк. 0,75.
Наклад 50 прим. Зам. № ______. Ціна договірна.

Видавець Видавничий центр НТУ «ХПІ».
Свідоцтво про державну реєстрацію ДК № 5478 від 21.08.2017 р.
61002, Харків, вул. Кирпичова, 2

Електронний варіант