МІНІСТЕРСТВО ОСВІТИ І НАУКИ, МОЛОДІ ТА СПОРТУ УКРАЇНИ

Національний технічний університет
«Харківський політехнічний інститут»

ПРАКТИКА ПЕРЕКЛАДУ МАТЕМАТИЧНОЇ ТЕРМІНОЛОГІЇ

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Розглянуто труднощі перекладу математичної термінології. Як додатковий матеріал наведено таблиці «Mathematical Signs & Symbols» і «Formulae Reading», англо-український та україно-англійський словники математичних термінів і ключі до завдань з перекладу українською мовою.
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Вступ

Особливість опанування іноземною мовою на денному відділенні полягає в тому, що обсяг самостійної роботи студента з удосконалення мовних навичків і вмінь значно перевищує обсяг практичних аудиторних занять з викладачем.

Для досягнення успіху необхідно вивчати мову з перших днів навчання і займатися систематично.

Самостійна робота студента з вивчення іноземної мови охоплює: запам'ятовування слів англійської мови, розуміння утворення словосполучень, граматичних правил, читання вголос текстів англійською мовою відповідно до правил читання, розуміння текстів, складання запитань та відповідей до текстів; переклад на українську мову (усний і письмовий).

Для досягнення такого рівня оволодіння мовою потрібно систематично тренувати пам'ять вивченням іноземних слів. Необхідно пам'ятати, що здібності розвиваються під час роботи, що усвідомлений матеріал легше запам'ятовується, ніж неусвідомлений, що навички удосконалюються шляхом постійного виконання вправ.

Ціль нашого начально-методичного посібника полягає в оволодінні навичками розуміння та перекладу математичних текстів, ознайомлення з професійною лексикою і математичною термінологією.

Навчально-методичний посібник складається з семи модулів, підсумкового тестування у трьох варіантах, таблиць «Mathematical Signs & Symbols» і «Formulae Reading», англо-українського та україно-англійського словників математичних термінів і ключів до завдань з перекладу на українську мову. Кожен модуль містить два-три тексти англійською мовою і різноманітні вправи на переклад і закріплення нової лексики.

Тексти у навчально-методичному посібнику містять ґрунтовні поняття сучасної алгебри і геометрії.

Навчально-методичний посібник призначається для студентів спеціальності «Переклад».

Contents

Unit 1

Text 1. Read and translate the text into Ukrainian

Essential Vocabulary

add – додавати
addend – доданок
difference – різниця
digit – цифра
divide – ділити
dividend – ділене
division – ділення
divisor – дільник
factor – множник
minuend – зменшуване
multiplicand – множене
quotient – часткове
subtrahend – від'ємник
summand – доданок

Four Basic Operations of Arithmetic

We cannot live a day without numerals. Numbers and numerals are everywhere. In this article you will see number names and numerals. The number names: zero, one, two, three, four, and so on. And here are the corresponding numerals: 0, 1, 2, 4, and so on. In a numeration system[1] numerals are used to represent numbers, and the numerals are grouped in a special way. The numbers used in our numeration system are called digits.

In our Hindu-Arabic system[2] we use only ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to represent any number. We use the same ten digits over and over again in a place-value system[3] whose base is ten.

These digits may be used in various combinations. Thus, for example, 1, 2, and 3 are used to write 123, 213, 132 and so on.

One and the same number could be represented in various ways. For example, take 3. It can be represented as the sum of the numbers 2 and 1 or the difference between the numbers 8 and 5 and so on.

A very simple way to say that each of the numerals names the same number is to write an equation – a mathematical sentence that has an equal sign (=) between these numerals. For example, the sum of the numbers 3 and 4 equals the sum of the numbers 5 and 2. In this case we say: three plus four (3 + 4) is equal to five plus two (5 + 2). One more example of an equation is as follows: the difference between numbers 3 and 1 equals the difference between numbers 6 and 4. That is three minus one (3 – 1) equals six minus four (6 – 4). Another example of an equation is 3 + 5 = 8. In this case you have three numbers. Here you add 3 and 5 and get 8 as a result. 3 and 5 are addends (or summands) and 8 is the sum. There is also a plus (+) sign and a sign of equality (=). They are mathematical symbols.

Now let us turn to the basic operations of arithmetic. There are four basic operations that you all know of. They are addition, subtraction, multiplication and division. In arithmetic an operation is a way of thinking of two numbers and getting one number. We were just considering an operation of addition. An equation like 7 – 2 = 5 represents an operation of subtraction. Here seven is the minuend and two is the subtrahend. As a result of the operation you get five. It is the difference, as you remember from the above[4]. We may say that subtraction is the inverse operation of the addition since 5 + 2 = 7 and 7 – 2 = 5.

The same might be said about division and multiplication, which are also inverse operations.

In multiplication there is a number that must be multiplied. It is the multiplicand. There is also multiplier. It is the number by which we multiply. When we are multiplying the multiplicand by the multiplier we get the product as a result. When two or more numbers are multiplied, each of them is called a factor. In the expression five multiplied by two (5 × 2), the 5 and the 2 will be factors. The multiplicand and the multiplier are names for factors.

In the operation of division there is a number that is divided and it is called the dividend; the number by which we divide is called the divisor. When we are diving the dividend by the divisor we get the quotient. But suppose you are dividing 10 by 3. In this case the divisor will not be contained a whole number of times in the dividend. You will get a part of the dividend left over[5]. This part is called the remainder. In our case the remainder will be 1. Since multiplication and division are inverse operations you may check division by using multiplication.

There are two very important facts that must be remembered about division.

a) The quotient is 0 (zero) whenever the dividend is 0 and the divisor is not 0. That is, 0 : n is equal to 0 for all values of n except n = 0.

b) Division by 0 is meaningless. If you say that you cannot divide by 0 it really means that division by 0 is meaningless. That is, n : 0 is meaningless for all values of n.

Notes

1) numeration system – система числення;
2) Hindu-Arabic system – арабська система;
3) place-value system – позиційна система розрядів;
4) from the above – із викладеного вище;
5) a part of left over – залишок.

Exercise 1.1. Read the equations

45 × 2 = 90; 313 + 53,456 = 53,769; 12 ÷ 4 = 3; 69 – 153 = –84; 67 × 27 = = 1809; 164 + 765 = 929; 113 ÷ 2 = 56.5; 1,297,696 – 976,247 = 321,449; 12,857,346 + 4,957,001 = 17,814,347; 555,555 ÷ 5 = 111,111; 7,123 × 17 = = 121,091; 12,546 – 4,346 = 8,200; 1,239 × 91 = 112,749; 53,562 ÷ 3 = 17,854; 95,678 – 857 = 94,821; 9,011 + 591 = 9,602; 601 × 24 = 14,424.

Exercise 1.2. Read the equations

11,414 ÷ 13 = 878; 890 – 129 = 761; 5,691 – 593 = 5,098; 578 × 24 = = 13,872; 701 + 54 = 755; 1,023 – 899 = 124; 59 × 112 = 6608; 392 ÷ 7 = 56; 456 + 978 = 1,434; 768 – 15 = 753; 546 ÷ 6 = 91; 12,345,917 + 8,010,718 = = 20,356,635; 801 × 11 = 8,811; 5,443 – 3,083 = 2,360; 692 + 949 = 1,641; 106 × 91= 158,997; 1026 ÷ 2 = 513; 4,680 + 728 = 5,408; 2,013 – 1,979 = 34.

Exercise 1.3. Translate the text into Ukrainian

The theory of systems of linear equations serves as the foundation for a vast and important division of algebra – linear algebra – to which a good portion of this book is devoted (the first three chapters in particular). The coefficients of equations considered in these three chapters, the values of the unknowns and, generally, all numbers that will be encountered are to be considered real. Incidentally, all the material of these three chapters is readily extendable to the case of arbitrary complex numbers, which are familiar from elementary mathematics.

In contrast to elementary algebra, we will study systems with an arbitrary number of equations and unknowns; at times, the number of equations of a system will not even be assumed to coincide with the number of unknowns. Suppose we have a system of s linear equations in n unknowns. Let us agree to use the following symbolism: the unknowns will be denoted by x and subscripts: $x_1, x_2, \ldots, x_n$; we will consider the equations to be enumerated thus: first, second, …, sth; the coefficient of x_j in the ith equation will be given as a_{ij}. Finally, the constant term of the ith equation will be indicated as b_i.

Exercise 1.4. Translate the text into English

Отримане рівняння визначає параболу. Парабола ділить усю площину на дві частини – внутрішню і зовнішню відносно параболи. Для точок однієї з її частин виконується нерівність $y^2 < 4+4x$, а для іншої – $y^2 > 4+4x$ (на самій параболі $y^2 = 4+4x$).

Щоб встановити, яка з цих двох частин є областю визначення даної функції, тобто задовольняє умову $y^2 < 4+4x$, треба перевірити цю умову для будь-якої однієї точки, що не лежить на параболі. Наприклад, початок координат O (0,0) лежить всередині параболи і задовольняє потрібну умову $0 < 4+4 \times 0$. Таким чином, розглянута область D складається з точок, що містяться всередині параболи. Оскільки сама парабола не належить до області D, границю області – параболу – відмітимо на кресленні пунктиром.

Text 1.2. Read and translate the text into Ukrainian

The teacher asked his students to add the numbers 5, 8 and 6 at the same time. Some of the students first added 5 and 8 and got 13. Next they added 13 and 6 and obtained 19. Other students first added 8 and 6 and got 14 and then added 14 and 5 also getting 19 as a result.

The examples given above illustrate two important characteristics of addition. One of the two characteristics is that addition is an operation applied to only two numbers at a time. For this reason we say that addition is a binary operation. Any operation is called a binary operation when it is applied to only two numbers at a time and gives a single result. The other characteristic shown by the above examples is that the sum is the same regardless of the order of the addition.

The use of parentheses will indicate how the answer, that is the result, was obtained. The first example can be written as follows: 5 + 8 + 6 = (5 + 8) + 6 = = 13 + 6 = 19. The second example can be written as follows: 5 + 8 + 6 = 5 + + (8 + 6) = 5 + 14 = 19. Since the final answers are the same we are making the following statement: (5 + 8) + 6 = 5 + (8 + 6).

Exercise 1.5. Read the equations

294 ÷ 3 = 98; 7,867 – 1,468 = 6,399; 3,913 – 1,098 = 2,815; 1,044 ÷ 12 = = 87; 856 × 9 = 7,704; 35,705 – 33,631 = 2,074; 9,058 + 1,863 = 10,921; 72 ÷ ÷ 4 = 18; 6,794 – 12,553 = –5,759; 295 + 592 = 887; 671 × 25 = 16,775; 815 + + 902 = 1,717; 1,376 ÷ 8 = 172; 479 × 22 = 10,583; 163 × 9 = 1,467; 194 × 1 = = 194; 53 – 93 = –40; 73 × 0 = 0.

Exercise 1.6. Read the equations

532,536,420 – 491,098,004 = 41,438,416; 453 × 78 = 35,334; 36,331 ÷ 47 = = 773; 68,023 – 42,946 = 25,077; 56,786 + 23,458 = 80,244; 26 × 5 = 130; 1,287 ÷ ÷ 13 = 99; 654 + 671 = 1,325; 888,888 ÷ 2 = 444,444; 101 × 16 = 1,616; 47 ÷ 2 = = 23.5; 9,571 + 34,627 = 44,198; 784 – 612 = 172; 865 × 830 = = 717,950; 658 ÷ 7 = 94; 3,543 – 2,769 = 774; 868 + 143 = 1,011; 45 × 3 = 135.

Exercise 1.7. Translate it into Ukrainian

Every polynomial of degree at least one with arbitrary numerical coefficients has at least one root, which in the general case is complex.

This theorem is one of the greatest attainments of the whole of mathematics ands finds application in the most diverse spheres of science. In particular, it is the starting point of everything in the theory of polynomials with numerical coefficients and for this reason it was once called (and sometimes still is) the “fundamental theorem of higher algebra”.

In the proof which we now give, the polynomial $f(x)$ with complex coefficients will be regarded as a complex function of a complex variable x. Thus, x can assume any complex values, or, taking into account the mode of constructing complex numbers given in Sec. 17, the variable x ranges over the complex plane. The values of the function *f(x)* will also be complex numbers.

Unit 2

Text 2.1. Read and translate the text into Ukrainian

Essential Vocabulary

denominator – знаменник
divide (by) – ділити (на)
dividend – ділене
divisor – дільник
equation – рівняння
fraction – дріб
integer – ціле число
multiply (by) – помножити (на)
numerator – чисельник
quotient – часткове
set – множина
solution – розв’язання
solve – розв’язувати
value – величина, значення

Rational Numbers[1]

John has read twice as many books as Jane. John has read 7 books. How many books has Jane read?

This problem is easily translated into the equation $2n = 7$, where n represents the number of books that Jane has read. If we are allowed to use integers, the equation $2n = 7$ has no solution. This is an indication that the set of integers does not meet all of our needs.

If we attempt to solve the equation $2n = 7$, our work might appear as follows:

$2n = 7$, ${}^{2n}/_2 = {}^7/_2$, ${}^2/_2 \times n = {}^7/_2$, $1 \times n = {}^7/_2$, $n = {}^7/_2$.

The symbol, or fraction, ${}^7/_2$ means 7 divided by 2. This is not the name of an integer but involves a pair of integers. It is the name for a rational number. A rational number is the quotient of two integers (dividend and divisor). The rational numbers can be named by fractions. The following fractions name rational numbers: ${}^1/_2$, ${}^8/_3$, ${}^0/_5$, ${}^3/_1$, ${}^9/_4$.

We might define a rational number as any number named by ${}^a/_n$ where a and n name integers and $n \neq 0$.

Let us dwell on fractions in some greater detail.

Every fraction has a numerator and a denominator. The denominator tells you the number of parts of equal size into which some quantity id to be divided. The numerator tells you how many of these parts are to be taken.

Fractions representing values less than 1, like ${}^2/_3$ (two thirds) for example, are called proper fractions[2]. Fractions which name number equal to or greater than 1, like ${}^2/_2$ or ${}^3/_2$, are called improper fractions[3].

There are numerals like $1^1/_2$ (one and one second), which name a whole number and a fractional number. Such numerals are called mixed fractions[4].

Fractions which represent the same fractional number like $^1/_2$, $^2/_4$, $^3/_6$, $^4/_8$, and so on, are called equivalent fractions[5].

We have already seen that if we multiply a whole number by 1 we shall leave the number unchanged. The same is true if fractions since when we multiply both integers named in a fraction by the same number we simply produce another name for the fractional number. For example, $1 \cdot {}^1/_2 = {}^1/_2$. We can also use the idea that 1 can be expressed as a fraction in various ways: $^2/_2$, $^3/_3$, $^4/_4$ and so on.

Notes

1) rational number – раціональне число;
2) proper fraction – правильний дріб;
3) improper fraction – неправильний дріб;
4) mixed fraction – змішаний дріб;
5) equivalent fraction – еквівалентний дріб.

Exercise 2.1. Read the equations

$34^1/_2 + 1^3/_4 = 36^1/_4$; $^1/_4 + 1^1/_2 = 1^3/_4$; $^2/_3 + {}^1/_6 = {}^5/_6$; $5^3/_7 - 2^5/_{14} = 3^1/_{14}$; $12^3/_4 + 3^1/_4 = 16$; $1 \times {}^1/_3 = {}^1/_3$; $2 \times {}^1/_3 = {}^2/_6$; $12^2/_{13} - 2^5/_{13} = 9^{10}/_{13}$; $^7/_{21} \div 7 = {}^1/_3$; $56^1/_6 + 4^1/_3 = 60^1/_2$; $^4/_6 - {}^1/_6 = {}^1/_2$; $34 \times {}^1/_2 = 17$; $0 \times {}^1/_7 = 0$; $12^1/_3 - 5^2/_9 = 7^1/_9$; $5^1/_9 + {}^1/_9 = 5^2/_9$; $4^1/_2 + 3^1/_2 = 8$; $^7/_8 - 4 = -3^1/_8$; $675^6/_{13} + 4^2/_{26} = 679^{14}/_{26}$; $12 \times {}^1/_3 = 4$; $21^3/_4 - 3^1/_2 = 18^1/_4$; $4^1/_{27} + 9 = 13^1/_{27}$; $^5/_{11} - {}^2/_{11} = {}^3/_{11}$; $5 - {}^5/_{11} = 4^6/_{11}$; $2^1/_9 + 1^1/_3 = 3^4/_9$.

Exercise 2.2.Translate the text into Ukrainian

The number of distinct arrangements of n symbols is equal to the product $1 \cdot 2 \ldots n$, denoted by n! (read "n factorial"). Indeed, the general form of an arrangement of n symbols is $i_1, i_2, \ldots, i_n$, where each of the i_s is one of the numbers 1, 2, …, n, without repetitions. Use any one of the numbers 1, 2, …, n for i_1; this yields n distinct possibilities. But if i_1 has been chosen, then for i_2 we can only take one of the remaining $n - 1$ numbers; that is, the number of different ways of choosing the symbols i_1 and i_2 is equal to the product $n(n - 1)$ and so on.

Thus, the number of arrangements of n symbols for $n = 2$ is $2! = 2$ (the arrangements 12 and 24; in examples where $n \leq 9$, we do not separate the symbols by commas); for $n = 3$ this number is $3! = 6$, for $n = 4$ it is $4! = 24$. As n increases, the number of arrangements increases very fast: for $n = 5$ it is $5! = 120$, and for $n = 10$ it is already 3,628,800.

If in a certain arrangement we interchange any two symbols (not necessarily adjacent) and leave all the remaining ones fixed, we obtain a new arrangement. This operation is called a transposition.

All $n!$ arrangements of n symbols may be ordered so that each is obtained from the preceding one via a single transposition; any arrangement can serve as the starting point.

Exercise 2.3. Translate the text into English

Теорема Коші для рівняння $y' = f(x, y)$ стверджує, що якщо у деякій області функції $f(x, y)$ і $f'_y(x, y)$ неперервні, то через кожну точку цієї області проходить тільки одна інтегральна крива цього рівняння. Точки, у яких не виконуються умови теореми Коші, називаються особливими точками диференціального рівняння. Криву лінію, всі точки якої є особливими, називають особливою кривою рівняння. Якщо особлива крива лінія є у той же час інтегральною для рівняння, то вона називається особливою інтегральною кривою або особливим рішенням.

Про те, що відбувається в особливій точці, заздалегідь сказати нічого не можна: через неї може проходити кілька інтегральних кривих або не проходити жодної.

Text 2.2. Read and translate the text into Ukrainian

Essential Vocabulary

digit – цифра
numeral – число
decimal – десятинний (дріб)
fraction – дріб
fractional – дробовий
numerator – чисельник
power – степінь
denominator – знаменник
multiply – множити
product – добуток
multiplication – множення
division – ділення
divisor – дільник
integer – ціле число
addition – додавання
add – додавати

Decimal Numbers

In our numeration system we use ten numerals called digits. These digits are used over and over again in various combinations. Suppose, you have been given numerals 1, 2, 3 and asked to write all possible combinations of these digits. You may write 123, 132, 213 and so on. The position in which each digit is written affects its value.

To read 529,248,650,396, you must say: five hundred twenty-nine billion, two hundred forty-eight million, six hundred fifty thousand, three hundred ninety-six. Notice that a coma separates each group or period.

But suppose you have been given a numeral 587.9 where 9 has been separated from 587 by a point, but not by a comma. The numeral 587 names a whole number. The sign (.) is called a decimal point. All digits to the left of the decimal point represent whole numbers. All digits to the right of the decimal point represent fractional parts of 1.

The place-value position at the right of the ones place is called tenths. You obtain a tenth by dividing 1 by 10. Such numerals like 587.9 are called decimals.

You read .2 as two tenths. To read .0054 you skip two zeroes and say fifty four ten thousandths.

Decimals like .333…, or .242424…, are called repeating decimals[1]. In a repeating decimal the same numeral or the same set of numerals is repeated over and over again indefinitely.

We can express rational numbers[2] as decimal numerals. See how it may be done.

$$^{31}/_{100} = 0.31$$
$$^{4}/_{25} = {}^{4}/_{25} \times 4 = {}^{16}/_{100} = 0.16$$

The digits to the right of the decimal point name the numerator of the fraction, and the number of such digits indicates the power of 10, which is the denominator. For example, .217 denotes a numerator 217 and a denominator of 10^3 (ten cubed) or 1000.

In our development of rational numbers we have named them by fractional numerals. We know that rational numbers can just as well be named by decimal numerals. As you might expect, calculations with decimal numerals give the same results as calculations with the corresponding fractional numerals.

Before performing addition with fractional numerals, the fractions must have a common denominator. This is also true of decimal numerals.

When multiplying with fractions, we find the product of the numerators and the product of denominators. The same procedure is used in multiplication with decimals.

Division of numbers in decimal form is more difficult to learn because there is no such simple pattern as has been observed for multiplication.

Yet, we can introduce a procedure that reduces all decimal-division situations to one standard situation, namely the situation where the divisor is an integer. If we do so well shall see that there exists a simple algorithm that will take care of all possible division cases.

In operating with decimal numbers you will see that the arithmetic of numbers in decimal form is in full agreement with the arithmetic of numbers in fractional form.

You only have to use your knowledge of fractional numbers.

Take addition, for example. Each step of addition in fractional form has a corresponding step in decimal form.

Suppose you are to find the sum of, say, .26 and 2.18. You can change the decimal numerals, if necessary, so that they denote a common denominator. We may write .26 = .260 or 2.18 = 2.180. Then we add the numbers just as we have added integers and denote the common denominator in the sum by proper placement of the decimal point.

We only have to write the decimals so that all the decimal points lie on the same vertical line. This keeps each digit in its proper place-value position.

Since zero is the identity element of addition it is unnecessary to write .26 as .260, or 2.18 as 2.180 if you are careful to align the decimal points, as appropriate.

Notes

1) repeating decimal – періодичний десятковий дріб;
2) rational number – раціональне число.

Exercise 2.4. Read the equations

24.45 + 12.78 = 37.23; 56.0594 × 5.4 = 302, 72076; 10.378 + 11.397 = = 21.775; 618,569,303.34 – 78,491,004.912 = 540,078,298.428; 901.153 + + 12.278 = 913.431; 34.59 × 8.4 = 290.556; 891.57 × 10.45 = 9,316.9065; 695.12÷ 6 = 115.85(3); 256 ÷ 5 = 51.3; 4,463.67 + 3,439.346 = 7,903.016; 343.378 × 24 = 8,241.072; 2497 ÷ 23 ≈ 108.565217; 34.67 – 27.356 = 7.314; 67 – 32 ≠ 134.

Exercise 2.5. Read the equations

78.567 + 39.695 = 118.262; 56 × 11.47 = 642.32; 67.363 – 46.36567 = = 20,99733; 35,458,345.5678 + 3,450,612.4671 = 38,908,958.0349; 45.9801 × × 61.4 = 2,823.17814; 96 ÷ 7 ≈ 13.714286; 61 – 35.69132 = 25.30868; 76.678 + + 893.4564 = 970.1344; 805.56 ÷ 4 = 201.39; 53.9603 + 45.7825 = 99.7428; 90.491 × 41.4 = 3,746.3274; 71,567.674 – 70,084.459 = 1,483.215.

Exercise 2.6.Translate the text into Ukrainian

Suppose we have nonzero polynomials $f(x)$ and $\varphi(x)$ with complex coefficients. If the remainder after dividing $f(x)$ by $\varphi(x)$ is zero, we then say that $f(x)$ is divisible by $\varphi(x)$. Here, the polynomial $\varphi(x)$ is called the divisor of the polynomial $f(x)$.

The polynomial $\varphi(x)$ is a divisor of the polynomial $f(x)$ if and only if there exists a polynomial $\psi(x)$ such that satisfies the equation

$$f(x) = \varphi(x)\,\psi(x) \qquad (1)$$

Indeed, if $\varphi(x)$ is a divisor of $f(x)$, then for $\psi(x)$ we should take the quotient of $f(x)$ divided by $\varphi(x)$. Conversely, let there be a polynomial $\psi(x)$ witch satisfies (1). From the proof given in the preceding section on the uniqueness of the polynomials $q(x)$ and $r(x)$ which satisfy the equation

$f(x) = \varphi(x)\, q(x) + r(x)$

and the condition that the degree of *r (x)* be less than the degree of *φ (x)*, it follows in our case that the quotient of *f (x)* by *φ (x)* is equal to *ψ (x)*, and the remainder is zero.

Unit 3

Text 3.1. Read and translate the text into Ukrainian

Essential Vocabulary

add – додавати
addition – додавання
calculus – числення
differential – диференціальний
division – ділення
equation – рівняння
identity – тотожність
infinity – нескінченність
integral – інтегральний
multiplication – множення
multiply – помножити
roduct – добуток

A Short Introduction to the New Math

Many who has been out of school for a number of years find, if they want to refresh their knowledge of mathematics, that there has been a great change, a sort of mathematical revolution while they were away from school. The old, classical math has had its face lifted and has taken on a new look which modern instructors claim is a great improvement.

In the classical math often taught in high-school courses, many simple truths were taken for granted and there was a failure to analyze these truths to find out why they are true and under what particular conditions they might not be true.

During the past centuries, great, world-shaking theories were born, notably the Maxwell electromagnetic theory, the theory of relativity, and the concept of differential and integral calculus[1]. And all these extremely important doctrines came about as a result of questioning and continually asking WHY?

The result obtained using the New Math agree, of course, with those obtained using the old, classical math, but the method of the former is much more thorough and therefore more satisfactory to the student who has never before studied math. The New Math teaches a student to think a problem through rather try to recall tricks of manipulation.

Let’s take a simple example of the two methods:

We all learned that if $x^2 - 4 = 0$, x must equal either 2 or –2. Either of these numerical replacements for the letter x makes the statement meaningful. This is so elementary it hardly needs comment. But just how did we arrive at this ± 2? Did we actually “transpose” the – 4 to the other side of the equal sign where it became + 4, the equation becoming $x^2 = 4$ and x becoming ± 2? Any child might well ask, “Why do we change signs when we “transpose” from one side to the other in an equation?” This, of course, is a sensible question. In the New Math this is dealt with before the child asks the question. We say:

If $x^2 - 4 = 0$, then by adding +4 to both sides of the equation we get $x^2 - 4 + + 4 = 0 + 4$.

Next we show that –4 and +4 cancel each other and that 0 + 4 = 4.

Then $x^2 + 0 = 4$ or $x^2 = +4$. Thus $x = \pm 2$.

As a matter of fact, it is not at all difficult to demonstrate that we solved our little problem by making use of some of the eleven laws that form the foundation of arithmetic. Yes, that is a truly startling fact – and a truly startling discovery. Numbers are one of the most basic of the great ideas of mathematics. And believe it or not, eleven laws – not an infinity of manipulative devices – are the tools available to us when we want to solve problems. These are the eleven laws of real numbers:

1. The Closure Law of Addition. The sum of any two real numbers is a unique real number. For example, the sum of any 10 and 117 is 127.

2. The Commutative Law of Addition. The order in which we add is trivial. For example, the sum of 3 and 4 is 7; the sum of 4 and 3 is also 7.

3. The Associative Law of Addition. Since addition is defined for pairs of numbers, the addition of three numbers depends on our first adding any two of the numbers and then adding their sum to the third number; the order in which we do this is trivial. For example, when 3, 4 and 5 are added in three different orders, the same sum is obtained.

3 + 4 = 7, 7 + 5 = 12
4 + 5 = 9, 9 + 3 = 12
3 + 5 = 8, 8 + 4 = 12

4. The Identity Law for Addition. The number zero is the additive identity, for the addition of it to any other number leaves the second number unchanged. For example, the sum of 0 and 9 is 9.

5. The Inverse Law for Addition. The sum of any number and its negative[2] is zero. For example, the sum of 5 and – 5 is 0.

6. The Closure Law for Multiplication. The product of any two real numbers[3] is a unique real number. For example, the product of 117 and 10 is 1,170.

7. The Commutative Law of Multiplication. The order in which we multiply is trivial. For example, the product of 3 and 4 is the same as the product of 4 and 3.

8. The Associative Law of Multiplication. Since multiplication is defined for pairs of numbers, the multiplication of three numbers depends on our first multiplying two of the numbers and then multiplying their product by the third number; the order in which we do this is trivial. For example:

$3 \times 4 = 12$, $12 \times 5 = 60$
$3 \times 5 = 15$, $15 \times 4 = 60$
$4 \times 5 = 20$, $20 \times 3 = 60$

9. The Identity Law for Multiplication. The number one is the multiplicative identity, for the product of it and any other number leaves the second number unchanged. For example, the product of 1 and 7 is 7.

10. The Inverse Law for Multiplication. The product of any number (except zero) and its reciprocal[4] is one. For example, the product of 3 and $^1/_3$ is 1; the product of 9 and $^1/_9$ is 1; the product of $^7/_8$ and $^8/_7$ is 1. Division of a number by zero is meaningless.

11. The Distributive Law. Multiplication "distributes" across addition. For example:

$6 \times (4 + 5) = 6 \times 9 = 54$

$6 \times (4 + 5) = (6 \times 4) + (6 \times 5) = 24 + 30 = 54.$

Notes

1) differential and integral calculus – диференційне та інтегральне числення;

2) negative – від'ємна величина;

3) real number – речове число;

4) reciprocal – обернена величина.

Exercise 3.1. Read the equations

$56 \div 45 = 1.2(4)$; $787 \times 71 = 55{,}877$; $16 \times {}^1/_2 = 8$; $0 \times {}^1/_{112} = 0$; $658{,}238 + 283{,}608 = 941{,}846$; $502.469 - 173.909 = 328.56$; $10^1/_3 - 5^2/_9 = 5^1/_9$; $^1/_4 + 1^3/_4 = 2$; $3{,}198 \div 41 = 78$; $^5/_6 - {}^1/_6 = {}^2/_3$; $1^2/_9 + 1^1/_3 = 2^5/_9$; $3{,}715.3032 + 4{,}189.3789 = 7{,}904.6821$; $^3/_6 - {}^1/_6 = {}^1/_3$; $2 \times {}^1/_2 = {}^2/_4$; $1^1/_3 - 1^1/_6 = {}^1/_6$; $27 \times {}^1/_3 = 9$; $^8/_{17} - {}^2/_{17} = {}^6/_{17}$; $369 \times 7 = 2{,}583$; $52.5 \div 3 = 17.5$; $5^1/_6 + 3^1/_6 = 8^1/_3$.

Exercise 3.2. Translate the text into Ukrainian

In the system of columns of matrix *A* we have thus found a maximal linearly independent subsystem consisting of *r* columns. This is proof that the rank of matrix *A* is equal to *r*, and it completes the proof of the rank theorem.

This theorem provides a practical method for computing the rank of a matrix and therefore for settling the question of the existence of linear dependence in a given system of vectors; forming a matrix for which the given vectors serve as columns and computing the rank of the matrix, we find the maximum number of linearly independent vectors of our system.

The method of finding the rank of a matrix based on the rank theorem requires computing a finite but perhaps very large number of minors of the matrix. The following remark suggests a way of substantially simplifying this procedure. If the reader will again look through the proof of the rank theorem, he will notice that in the proof we did not take advantage of the fact that all minors of order $(r + 1)$ of matrix *A* are equal to zero; actually, we used only those minors of order (r +1) which border the given nonzero rth-order minor D (that is, those which contain it completely within themselves); for this reason, from the fact that only these minors are equal to zero it follows that r is the maximum number of linearly independent columns of matrix A; this implies that all minors of order (r +1) of this matrix are zero.

Exercise 3.3. Translate the text into English

Якщо коефіцієнти та права частина диференціального рівняння

$y^{(n)} + \alpha_1 (x) y^{(n-1)} + \ldots + \alpha_n (x) y = f(x)$

розкладаються у степеневий ряд за степенями $x - \alpha$, що є збіжними у деякому околі точки $x = \alpha$, то розв'язання цього рівняння, що задовольняє початкові умови

$y(\alpha) = y_0, \ y'(\alpha) = y'_0, \ \ldots, \ y^{(n-1)}(\alpha) = y_0^{(n-1)},$

розкладається у степеневий ряд за степенями $x - \alpha$, що є збіжними, принаймні, у меншому з інтервалів збіжності рядів для коефіцієнтів і правої частини диференціального рівняння.

Для приблизного розв'язання диференціального рівняння за допомогою степеневих рядів застосовуються два методи: порівняння коефіцієнтів і послідовне диференціювання.

Rules of Reading

Involution or Raise to power

3^2 is read «three to the second power» or «three squared».

5^3 is read «five to the third power», «five to power three», or «five cubed».

x^2 (x is called «the base of the power», 2 is called «an exponent or index of the power»).

Evolution

$\sqrt{9} = 3$ is read: «the square root of nine is three».

$\sqrt[3]{27} = 3$ is read: «the cube root of twenty seven is three».

$\sqrt{\ }$ is called «the radical sign» or «the sign of the root»; «to extract the root of» is translated «добувати корінь з».

Exercise 3.4. Read the equations

$\sqrt{4} = 2$; $5^2 = 25$; $\sqrt[3]{8} = 2$; $\sqrt[3]{125} = 5$; $7^2 = 49$; $3^6 = 729$; $\sqrt[5]{371{,}293} = 13$; $9^2 = 81$; $\sqrt{144} = 12$; $17^5 = 1{,}419{,}857$; $\sqrt[4]{256} = 4$; $\sqrt[3]{571{,}787} = 83$; $1^{51} = 1$; $2^9 = 512$; $\sqrt{1{,}024} = 32$; $7^3 = 343$; $5^4 = 625$; $\sqrt[4]{16} = 2$; $\sqrt[6]{729} = 3$; $8^6 = 262{,}144$; $9^5 = 59{,}049$; $\sqrt{218{,}089} = 467$; $561^3 = 176{,}558{,}481$; $213^5 = 438{,}427{,}732{,}293$; $\sqrt{49} = 7$; $5^3 = 125$; $\sqrt[4]{81} = 3$; $\sqrt[5]{7776} = 6$; $7^{11} = 1{,}977{,}326{,}743$; $\sqrt[3]{2{,}744} = 14$; $4^7 = 16{,}384$.

Exercise 3.5. Read the equations

87,451 – 6^6 = 87,451 – 46,656 = 40,795; $^3\sqrt{}$6,859 + 5^2 = 19 + 25 = 44; 7,106 + 89^2 = 7,106 + 7,921 = 15,027; $^4\sqrt{}$84,934,656 + 2^{12} = 96 + 4,096 = 4,192; 78^2 + 14^3 = 6,084 + 2,744 = 8,828; 8^3 + 7^2 – 5^4 = 512 + 49 – 625 = -64; √529,984 – 3^6 = 728 – 729 = 1; 7^9 –308,730 = 823,543 – 308,730 = 514,813; 908 – 12^3 = 908 – 1,728 = –820.

Text 3.2. Read and translate the text into Ukrainian

Essential Vocabulary

minuend – зменшуване	subtraction – віднімання
subtract – віднімати	subtrahend – віднімаюче

Example of False Arguments

Someone stubbornly asserted that 45 – 45 = 45.

In support of this, he argues thus:

We write the subtrahend in the form of a sum of the consecutive natural numbers from 1 to 9 and the minuend in the form of the sum of the same numbers, but we take them in the opposite order (from 9 to 1).

Locating the subtrahend under the minuend:

1) 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1;

2) 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9.

Now we calculate the difference. With this aim we successively subtract the numbers of the second line from the numbers on the first line, beginning by subtracting the 9. Since 9 cannot be subtracted from 1, we take a Unitfrom the two, we have: 11 – 9 = 2. In a similar way we obtain the differences of the number 11 and 8, 12 and 7, 13 and 6, 14 and 5 respectively, and 3, 5, 7 and 9. Carrying out the subtractions of four from five, three from seven, two from eight, and finally one from nine, we obtain the following results: 1, 4, 6, 8.

Thus:

_ 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
  1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
  8 + 6 + 4 + 1 + 9 + 7 + 5 + 3 + 2

It is not hard to establish, that 8 + 6 + 4 + 1 + 9 + 7 + 5 + 3 + 2 = 45.

Thus 45 – 45 = 45.

Exercise 3.6. Translate the text into Ukrainian

The Sturm theorem completely resolves the question of the number of real roots of a polynomial, but it has one essential defect and that is the cumbersome computations involved in constructing a Sturm sequence, as the reader could see after performing all the computations of the first example above. We now prove two theorems, which do not yield the exact number of real roots but only bound the number from above. These theorems are employed after a graph has been used to bound the number of real roots from below and at times enable us to find the exact number of real roots without resorting to the Sturm method.

Suppose we have an *n*th-degree polynomial $f(x)$ with real coefficients; we assume it can have multiple roots. Let us consider a sequence of its consecutive derivatives:

$$f(x) = f^{(0)}(x),\ f'(x),\ f''(x),\ \ldots,\ f^{(n-1)}(x),\ f^{(n)}(x) \qquad (1)$$

of which the last one is equal to the leading coefficient α 0 of $f(x)$ multiplied by $n!$ and for this reason preserves sign at all times. If a real number c is not a root of any one of the polynomials of the sequence (1), then by $S(c)$ we denote the number of variations in sign in the ordered sequence of numbers

$$f(c), f'(c), f''(c), \ldots, f^{(n-1)}(c), f^{(n)}(c).$$

Unit 4

Text 4.1. Read and translate the text into Ukrainian

Essential Vocabulary

area – площа
product – добуток
equation – рівняння
factor – множник
integer – ціле число
value – значення
negative number – від’ємне число
positive number – додатне число
radical sign – знак кореня
right triangle – прямокутний трикутник
square root – квадратний корінь

Square Root

It is not particularly useful to know the areas of the squares on the sides of a right triangle, but the Pythagorean Property is very useful if we can use it to find the length of a side of a triangle. When the Pythagorean Property is expressed in the form $c^2 = a^2 + b^2$, we can replace any two of the letters with the measures of two sides of a right triangle. The resulting equation can then be solved to find the measure of the third side of the triangle. For example, suppose the measures of the shorter sides of a right triangle are 3 units1 and 4 units and we wish to find the measure of the longer side. The Pythagorean Property could be used as shown below:

$c^2 = a^2 + b^2$, $c^2 = 3^2 + 4^2$, $c^2 = 9 + 16$, $c^2 = 25$.

You will know the number represented by c if you can find a number which, when used as a factor twice, gives a product of 25. Of course, 5×5 = 25, so $c = 5$ and 5 is called the positive square root of 25. If a number is a product of two equal factors, then either of the equal factors is called a square root of the number. When we say that y is the square root of K we merely mean that $y^2 = K$. For example, 2 is a square root of 4 because $2^2 = 4$. The product of two negative numbers being a positive number, -2 is also a square root of 4 because $(-2)^2 = 4$. The following symbol √ called a radical sign is used to denote the positive square root of a number. That is $\sqrt{K}$ means the positive square root of K. Therefore, $\sqrt{4} = 2$ and $\sqrt{25} = 5$. But suppose you wish to find the $\sqrt{20}$. There is no integer whose square is 20, which is obvious from the following computation. $4^2 = 16$ so $\sqrt{16} = 4$; $a^2 = 20$ so $4 < a < 5$, so $\sqrt{25} = 5$. $\sqrt{20}$ is greater than 4 but less than 5. You might try to get a closer approximation[2] of $\sqrt{20}$ by squaring some numbers between 4 and 5. Since $\sqrt{20}$ is about as near to 4^2 as to 5^2, suppose we square 4.4 and 4.5.

$4.4^2 = 19.36 \quad a^2 = 20 \quad 4.5^2 = 20.25$

Since $19.36 < 20 < 20.25$ we know that $4.4 < a < 4.5$. 20 being nearer to 20.25 than to 19.36, we might guess that $\sqrt{20}$ is nearer to 4.5 than to 4.4. Of course, in order to make sure that $\sqrt{20} = 4.5$, to the nearest tenth, you might select values between 4.4 and 4.5, square them, and check the results. You could continue the process indefinitely and never get the exact value of 20. As a matter of fact, $\sqrt{20}$ represents an irrational number which can only be expressed approximately as rational number. Therefore we say that $\sqrt{20} = 4.5$ approximately to the nearest tenth.

Notes

1) units – одиниці вимірювання;
2) approximation – приблизне значення.

Exercise 4.1. Read the equations

$7^2 = 49$; $\sqrt{25} = 5$; $9^3 = 729$; $11^3 = 1{,}331$; $6^2 = 36$; $\sqrt[3]{343} = 7$; $\sqrt[13]{8{,}192} = 2$; $18^6 = 34{,}012{,}224$; $\sqrt{904{,}401} = 951$; $26^4 = 456{,}976$; $\sqrt[8]{6{,}561} = 3$; $\sqrt[4]{256} = 4$; $9^2 =$ $= 81$; $\sqrt[7]{279{,}936} = 6$; $8^9 = 134{,}217{,}728$; $6^3 = 216$; $\sqrt{100} = 10$; $\sqrt[5]{161{,}051} = 11$; $314^3 = 30{,}959{,}144$; $23.12^2 = 534.5344$; $8.9^3 = 704.969$; $\sqrt[5]{32{,}768} + \sqrt[3]{2{,}744} = 8 +$ $+ 14 = 22$; $13^3 + 4^6 = 2{,}197 + 4{,}096 = 6{,}293$; $19^4 - 77{,}403 = 130{,}321 - 77{,}403 =$ $= 52{,}918$.

Exercise 4.2. Read the equations

$23^3 = 12{,}167$; $\sqrt[5]{243} = 3$; $9^4 - 9^3 = 6{,}561 - 729 = 5{,}832$; $8^4 = 4{,}096$; $7^5 =$ $= 16{,}807$; $2^9 - 3^5 = 512 - 243 = 269$; $3^{11} - 5^6 = 177{,}147 - 15{,}625 = 161{,}522$; $\sqrt[3]{531{,}441} = 81$; $\sqrt[4]{2{,}401} = 7$; $102^2 + \sqrt[2]{9{,}801} = 10{,}404 + 99 = 10{,}503$; $73^4 =$ $= 28{,}398{,}241$; $\sqrt[5]{3{,}125} = 5$; $34.77^2 = 1{,}208.9529$; $19^6 = 47{,}045{,}881$; $\sqrt[4]{28{,}561} =$ $= 13$; $106{,}890 - 46^3 = 106{,}890 - 97{,}336 = 9{,}554$; $55^4 = 9{,}150{,}625$.

Exercise 4.3. Translate the text into Ukrainian

Every finite Abelian group G that is not a zero group can be decomposed into a direct sum of primary cyclic subgroups.

We begin the proof of this theorem with the remark that in the group G there will inevitably be nonzero elements of prime power orders. Indeed, if some nonzero element x of G has order l, $l x = 0$ and if p^k, $k > 0$, is a power of the prime p such that divides the number l,

$l = p^k m$

then the element $m x$ is different from zero and has order p^k.

Let

$p^1, p^2, \ldots, p^s$ (1)

be all distinct primes, some powers of which serve as the order of certain elements of the group G. Denote any such number by p and the set of elements of G having powers of p as their orders by P.

The set P is a subgroup of the group G. Indeed, P includes the element 0 since its order is $1 = p^0$. Furthermore, if $p^k x = 0$, then $p^k (-x) = 0$ as well. Finally, if $p^k x = 0$, $p^l y = 0$ and if, say, $k \geq l$, then

$p^k (x + y) = 0$

Thus, either the number p^k or a divisor of this number, at any rate some power of p, serves as the order of the element $x + y$.

Exercise 4.4. Translate the text into English

Змінна u називається функцією незалежних змінних $x_1, x_2, \ldots, x_n$, якщо кожній множині значень $(x_1, x_2, \ldots, x_n)$ цих змінних у даній області їх змінювань згідно з певним правилом або законом відповідає одне або декілька значень величини u. Записуємо $u = f(x_1, x_2, \ldots, x_n)$ або $u = u(x_1, x_2, \ldots, x_n)$. У випадку функції трьох змінних функцію u записують таким чином: $u = f(x, y, z)$, а у випадку функції двох змінних – $u = f(x, y)$ або $z = = f(x, y)$, $z = z(x, y)$.

Множина n величин $x_1, x_2, \ldots, x_n$ називається “точкою” в області змінювань цих змінних, і ми маємо справу зі значенням функції u в цій точці. Якщо функція задана аналітичним виразом (формулою) без будь-яких додаткових умов, то область існування її аналітичного виразу вважається областю її визначення, тобто множина тих точок, в яких даний аналітичний вираз є визначеним і набуває тільки дійсних та кінцевих значень.

Text 4.2. Read and translate the text into Ukrainian

Essential Vocabulary

angle – кут

set – множина

Set Theory[1]

If you stop to think of all the new things that have been developed since you can remember, you will agree with the often-repeated comment that you live in a changing world. Nothing can live and be useful without becoming adjusted to the conditions around it. This is as true of mathematics and other areas of knowledge as it is of plants and people. It is generally agreed that the basic truths and most of the fundamental processes of arithmetic and algebra are not likely to change a great deal. Nevertheless, better understanding of the meaning of these relationships and operations can sometimes be gained by studying them from a different viewpoint or expressing them in different terms and symbols.

One of the most recent approaches to the meanings of algebra is based on the idea of sets of things. Everyone has used the word set at some time to mean a group or series of things to be used together or for a single purpose as a set of dishes, a set of tools and so on. You may have played or at least you may have heard of – a set of tennis, meaning a particular series of games considered as a group.

The meaning of sets in algebra is very similar to the general use of the word. That is, a set means any collection of objects, persons, or ideas that is so defined or limited that one can always tell whether or not a given object or idea belongs to that collection. In other words, a collection of stamps, matchboxes and the like can be called a set if the contents of the collection is limited to the objects described in the name of the collection. Moreover, you are a member of several sets. The first set of which you became a member was your family. Then when you started to school you became a member of your class, and so on.

A set is usually represented by a capital letter. Any object or idea that belongs to a set is called an element of the set[2]. All elements of a set are to be enclosed in braces { } so that there should be no misunderstanding of what is included in the set.

Sets of mathematical objects or ideas are usually collections of particular numbers, points, lines, angles, and so on. As you work with mathematical sets, remember that the term “set” means a collection which includes all of the numbers, points, or lines that satisfy the stated conditions for membership and which does not include any numbers, points, or lines, that do not satisfy the given conditions.

Notes

1) set theory – теорія множин;
2) an element of the set – елемент множини;

Exercise 4.5. Translate the text into Ukrainian

Suppose we have arbitrary polynomials *f (x)* and *g(x)*. The polynomial *φ (x)* is called the common divisor of *f (x)* and *g(x*) if it is a divisor of each of them. Property V (see above) shows that the common divisors of the polynomials *f (x)* and *g (x)* include all polynomials of degree zero. If there are no other common divisors of these two polynomials, then the polynomials are called relatively prime.

But in the general case, the polynomials *f (x)* and *g (x)* may have divisors, which depend on *x*; we wish to introduce the concept of the greatest common divisor of these polynomials.

It would be inconvenient to take a definition stating that the greatest common divisor of the polynomials *f (x)* and *g (x)* is their common divisor of highest degree. On the other hand, as yet we do not know whether *f (x)* and *g (x)* have many different common divisors of highest degree, which differ not only in a zero-degree factor. In other words, isn't this definition too indeterminate? On the other hand, the reader will recall from elementary arithmetic the problem of finding the greatest common divisor of integers and also that the greatest common divisor 6 of the integers 12 and 18 is not only the greatest among the common divisors of these numbers but is even divisible by any other of their common divisors; the other common divisors of 12 and 18 are 1, 2, 3, -1, -2, -3.

Unit 5

Text 5.1. Read and translate the text into Ukrainian

Essential Vocabulary

equality – рівність
inequality – нерівність
non-equality – нерівність
set – множина

Solution Sets

If each element of a set makes a given system true, the set is called the solution set for the statement. You have worked with statements of equality as well as with statements called inequalities. You are sure to remember that the simple fact that one number is not equal to another can be expressed by the symbol of non-equality ≠. For example, $5 - 2 \neq 4$. However, to know which of two unequal numbers is the larger we need a more exact description. In this case we use more specific symbols < or >. Therefore a mathematical sentence[1] like $5 \neq 3$ expresses the general inequality, $5 > 3$ stating the specific condition that 5 is greater than 3 and $3 < 5$ showing the specific condition in which 3 is less than 5. In dealing with mathematical sentences you are to choose the correct symbol for a statement to be true. If you choose the incorrect symbol, the statement is false. Similar statements in which the symbol of relationship is given, but one of the required quantities is missing, are called open sentences. Thus, the sentence «… is a planet whose orbit around the Sun is smaller than the Earth's orbit» is an open sentence. If you write «Jupiter» in the blank you have not made a true statement. The solution set for this open sentence is {Venus, Mercury}, since both of these planets satisfy the two conditions of the required set. That is, the orbits of both planets do lie inside the Earth's orbit and these are the only known planets that do travel closer to the Sun than the Earth does.

Mathematical sentences may also be written as open sentence. Thus to write $5 + \ldots = 8$ is to write an open sentence whose solution set is {3} since this is the only number that will make this a true statement. The open sentence $5 + \ldots > 8$, however, can be completed with any number greater than 3. Therefore, the solution set for this sentence can be written {all numbers greater than 3}. When the numbers from which you are to choose a solution set are limited to a particular group of numbers, it sometimes happens that there is no solution to satisfy a given statement That is, there is no natural number[2] that will make the two statements $5 + \ldots = 3$ and $5 + \ldots < 3$ true statements. Therefore, if you are limited to natural numbers the solution for such statements is said to be an empty set[3], which is indicated by the symbol Ø.

Notes

1) mathematical sentence – математичне речення;
2) natural number – натуральне число;
3) empty set – пуста множина.

Exercise 5.1. Translate the text into Ukrainian

The concept of an *n*-dimensional linear space does not by any means fully generalize the concept of a plane or three-dimensional Euclidean space: in the *n*-dimensional case, for $n > 3$, neither the length of a vector nor the angle between vectors is defined and it is therefore impossible to develop the rich geometrical theory so familiar to the reader for $n = 2$ and $n = 3$. It turns out, however, that we can rectify the situation in the following manner.

From analytic geometry we know that for two-dimensional (a plane) and three-dimensional space we can introduce the concept of scalar multiplication of vectors. It is defined by means of the lengths of the vectors and the angle between them; it appears, however, that both the length of a vector and the angle between vectors can, in turn, be expressed in terms of scalar products. We will therefore define the concept of scalar multiplication for any *n*-dimensional linear space.

Exercise 5.2. Translate the text into English

Якщо функція $z = f(x, y) \geq 0$, то подвійний інтеграл від цієї функції дорівнює об'єму циліндричного тіла, що обмежене знизу областю *D*, зверху поверхнею $z = f(x, y)$ і циліндричною поверхнею, напрямна якої є границя області *D*, а твірні – паралельні осі *Oz*.

Основні властивості подвійного інтегралу подібні до відповідних властивостей визначеного інтегралу.

Обчислення подвійного інтегралу зводиться до повторного обчислення двох визначених інтегралів. Припустимо, що область інтегрування *D* перетинається будь-якою лінією, що паралельна осі *Oy*, не більше ніж у двох точках.

Таку область будемо називати правильною, або простою областю у напрямку осі *Ox*. Нехай на границі області *F* сама ліва точка *A*, а сама права – *B*. Позначимо їх абсциси через a, b. Точки *A* і *B* ділять контур *F* на нижню частину *ACB*, рівняння якої $y = y_1(x)$, та верхню частину, рівняння якої $y = y_2(x)$.

Text 5.2. Read and translate the text into Ukrainian

Essential Vocabulary

angle – кут
ellipsoid – еліпсоїд
parallelogram – паралелограм
plane – площина
point – точка
quadrilateral – чотирикутник
rectangle – прямокутник
side – сторона

Something about Euclidean and Non-Euclidean Geometries

It is interesting to note that the existence of the special quadrilaterals discussed above is based upon the so-called parallel postulate[1] of Euclidean geometry. This postulate is now usually stated as follows: Through a point not on line L, there is no more than one line parallel to L. Without assuming that there exists at least one parallel to a given line through a point not on the given line, we could not state the definition of the special quadrilaterals which have given pairs of parallel sides. Without the assumption that there exists no more than one parallel to a given line through a point not on the given line, we could not deduce the conclusion we have stated for the special quadrilaterals. An important aspect of geometry (or any other area of mathematics) as a deductive system is that the conclusions, which may be drawn, are consequences of the assumptions, which have been made. The assumptions made for the geometry we have been considering so far are essentially those made by Euclid in Elements. In the nineteenth century, the famous mathematicians Lobachevsky, Bolyai and Riemann developed non-Euclidean geometries. As already stated, Euclid assumed that through a given point not on a given line there is no more than one parallel to the given line. We Know of Lobachevsky and Bolyai having assumed independently of one another that through a given point not on a given line there is more than one line parallel to the given line. Riemann assumed that through a given point not on a given line there is no line parallel to the given line. These variations of the parallel postulate have led to the creation of non-Euclidean geometries, which are as internally consistent[2] as Euclidean geometry. However, the conclusions drawn in non-Euclidean geometries are often completely inconsistent with Euclidean conclusions. For example, according to Euclidean geometry parallelograms and rectangles (in the sense of a parallelogram with four 90-degree angles) exist; according to the geometries of Lobachevsky and Bolyai parallelograms exist but rectangles do not; according to the geometry of Riemann neither parallelograms nor rectangles exist. It should be borne in mind that the conclusions of non-Euclidean geometry are just as valid as those of Euclidean geometry, even though the conclusions of non-Euclidean geometry contradict those of Euclidean geometry. This paradoxical situation becomes intuitively clear when one realizes that any deductive system begins with undefined terms. Although the mathematician forms intuitive images of the concepts to which the undefined terms refer, these images are not logical necessities. That is, the reason for forming these intuitive images is not only to help our reasoning within a certain deductive system. They are not logically a part of the deductive system. Thus, the intuitive images corresponding to the undefined terms straight line and plane are not the same for Euclidean and non-Euclidean geometries. For example, the plane of Euclid is a flat surface; the plane of Lobachevsky is a saddle-shaped or pseudo-spherical surface; the plane of Riemann is an ellipsoid or spherical surface.

Notes

1) parallel postulate – аксіома про паралельність;
2) internally consistent – внутрішньо-послідовні.

Exercise 5.3. Translate the text into Ukrainian

If a matrix *A* with elements from the field *P* can be reduced to a Jordan normal form, i.e., is similar to a Jordan matrix, then, as follows from the theorem that was proved above, the Jordan normal form is determined uniquely for matrix *A* to within the order of the Jordan submatrices on the principal diagonal. The condition that allows a matrix *A* to be so reduced is given in the following theorem, the proof of which offers a practical procedure for finding a Jordan matrix similar to *A* if such a Jordan matrix exists. Note that reducibility over the field *P* means that all the elements of the matrix undergoing transformation are in *P*.

Matrix *A* with elements in the field *P* can be reduced over *P* to the Jordan normal form if and only if all the characteristic roots of *A* lie in the base field *P* itself.

Indeed, if matrix *A* is similar to the Jordan matrix *J*, these two matrices have the same characteristic roots. However, the characteristic roots of *J* are easily found: since the determinant of the matrix $J - \lambda E$ is equal to the product of its elements on the principal diagonal, the polynomial $|J - \lambda E|$ can be factored over *P* into linear factors and its roots are numbers (and only these numbers) on the principal diagonal of *J*.

Unit 6

Text 6.1. Read and translate the text into Ukrainian

Essential Vocabulary

cone – конус
cube – куб
cuboid – прямокутний паралелепіпед
cylinder – циліндр
edge – грань; ребро
intersect – перетинати(ся)
pyramid – піраміда
rectangular – прямокутний
triangular – трикутний
vertex – вершина

Anything, which takes up space, is spoken of as a solid. Thus each page of this book is a solid, however thin the paper may be. The word solid as used here must not be confused with the word solid, which is used as opposed to liquid and gas.

Most solids are irregular in shape, e.g. a pebble in a stream, a cloud in the sky. Geometry deals with the shape, size, and position of solids, which are regular in shape, e.g. a ball, a matchbox, a pencil.

The more common regular solids are: cube, cuboid or rectangular prism, triangular prism, square pyramid, cylinder, cone, sphere.

Solids are bounded by surfaces. These surfaces separate the solids from the surrounding space. Surfaces are of two kinds: plane and curved. The surfaces of a cube, rectangular prism, and pyramid are plane surfaces, while the surface of a sphere is curved. A sheet of paper, e.g. a leaf of a book, may represent a surface, but even the thinnest sheet of paper will be a geometrical solid, since it has length, breadth, and thickness.

Surface intersected in lines are bounded by lines. Lines are either straight or curved.

Lines intersect in points. The meeting place of two edges is called a point (a vertex). The dot made on paper by a fine pencil point represents a point. No matter how fine the pencil point is, however, the dot is a geometrical solid since it has length, breadth, and thickness, and a point has no length, no breadth, and no thickness. A point indicates but has no size.

Exercise 6.1. Translate the text into Ukrainian

The genesis of the theory of quadratic forms lies in analytic geometry, namely, in the theory of quadratic curves and surfaces. It will be recalled that the equation of a central quadratic curve in a plane, after translating the origin of the rectangular coordinate system to the centre of the curve, is of the form

$$Ax^2 + 2Bxy + Cy^2 = D \qquad (1)$$

It is also possible to perform a rotation of the coordinate axes through an angle α, such that we have the following transformation from the coordinates x, y to the coordinates x', y':

$$x = x' \cos\alpha - y' \sin\alpha$$

$$y = x' \sin\alpha + y' \cos\alpha$$

Then the equation of our curve in the new coordinates will be of “canonical” form:

$$A'x'^2 + C'y'^2 = D \qquad (2)$$

In this equation, the coefficient of the product of unknowns $x'y'$ is, thus, zero. The transformation of coordinates may obviously be interpreted as a linear transformation of the unknowns; the transformation is non-singular since the determinant of its coefficients is equal to unity.

Exercise 6.2. Translate the text into English

Направлений відрізок (або упорядкована пара точок) називається вектором (геометричним). До векторів також належить так званий нульовий вектор, у якого початок і кінець співпадають. Відстань між початком і кінцем вектора називається його довжиною або модулем і позначається $|\vec{a}|$. Модуль нульового вектора дорівнює нулю.

Вектори, що знаходяться на одній прямій або на паралельних прямих, називаються колінеарними.

Вектори називаються компланарними, якщо існує площина, якій вони паралельні.

Два вектори будемо вважати рівними, якщо вони колінеарні, однаково направлені та мають рівну довжину. Проекція вектора $\vec{a}$ на вісь L визначається як $pr_L \vec{a} = |\vec{a}| \cos\varphi$, де φ – кут між вектором і віссю L.

Text 6.2. Read and translate the text into Ukrainian

Essential Vocabulary

straight line – пряма лінія

surface – поверхня

Vertical and Horizontal Lines and Planes

If an object is suspended by a string, the line of the string would pass through the centre of the earth; the line is called a vertical line. Any plane, which contains a vertical line, e.g. the surface of the wall of a room, is called a vertical plane. When a bricklayer is building a wall, he uses a plumb-line[1], which consists of a small lump of lead at the end of a string to test whether the surface of the wall is a vertical plane.

Any straight line, which is perpendicular to a vertical line, is called a horizontal line, and if all the lines that can be drawn in a plane are horizontal, the plane is called a horizontal plane, e.g. the surface of still water in a tank.

A surface is called level if it is a part of a horizontal plane. You can test whether the floor of this room is a horizontal plane by using an instrument called a spirit-level[2], in which the adjustment to the horizontal is shown by the position of a bubble in a glass tube containing alcohol. If a line or plane is neither vertical, nor horizontal, it is called oblique[3]. The words perpendicular and vertical must not be confused.

Two intersecting lines are perpendicular if they form a right-angled corner; a line is vertical if it points to the centre of the earth.

Notes

1) plumb-line – висок, нівелір;
2) spirit-level – спиртовий рівень, ватерпас;
3) oblique – косий, нахилений.

Text 6.3. Read and translate the text into Ukrainian

Essential Vocabulary

angle – кут
bisect – ділити навпіл
circle – коло
circumference – довжина кола
limit – границя
octagon – восьмикутник
perimeter – периметр
polygon – багатокутник
regular – правильний
square – квадрат

Circumference of a Circle

In traditional approaches to mathematics, the circumference of a circle has not always been clearly defined. That is, sometimes the circle itself was called the circumference, and at other times, the measure of the distance around the circle was called the circumference. Here we shall define the circumference as the perimeter of the circle, in other words, the measure of the entire path formed by the circle. This definition is symbolized by the formula $C = \pi d$ or the formula $C = 2\pi r$. There exist more precise definitions of a circumference. To arrive at this more precise definition, it is necessary to introduce the concept of limits. By using the limit concept, the circumference of a circle may be defined as the limit of the perimeter of an inscribed regular polygon. To illustrate this, we can first inscribe a square in a circle. The sum of the sides of the square will be an approximation1 of the circumference of the circle. Then, bisecting the central angles, which are subtended2 by the sides of the square, we can inscribe a regular octagon. The sum of the sides of the octagon will be a closer approximation of the circumference. Next, bisecting the central angles subtended by the sides of the octagon, we can inscribe a regular 16-gon. The sum of the sides of the16-gon will be an even closer approximation of the circumference. By a similar process we can then inscribe a regular 32-gon and 64-gon, and so on. Clearly the sum of *n* sides of an inscribed regular n-gon can be made to approximate the circumference of the circle as closely as desired by choosing n sufficiently large. Thus the circumference of a circle may be defined as the limit of the perimeter of an inscribed regular n-gon as *n* increases.

Notes

1) approximation – приблизне значення;
2) subtended angles – стягнуті кути.

Exercise 6.3. Translate the text into Ukrainian

The number of positive roots of a polynomial *f(x),* a root of multiplicity *m* being counted as *m* roots, is equal to the number of variations in sign in the sequence of coefficients of this polynomial (zero coefficients are not counted) or is less by an even number.

To determine the number of negative roots of the polynomial *f(x)* it is obviously sufficient to apply Descartes theorem to the polynomial *f(-x).* If none of the coefficients of the *f(x)* is zero, then, obviously, changes of sign in the sequence of coefficients of the polynomial *f(–x)* will be associated with preservation of signs in the sequence of coefficients of the polynomial *f(x),* and conversely. Thus, if the polynomial *f(x)* does not have zero coefficients, then the number of its negative roots (counting multiplicities) is equal to the number of preservations of signs in the sequence of coefficients or is less by an even number.

We give another proof of the Descartes theorem that does not rely on the Budan-Fourier theorem.

If $c > 0$, then the number of variations of sign in the sequence of coefficients of the polynomial $f(x)$ is less than the number of variations of sign in the sequence of coefficients of the product $(x - c) f(x)$ by an odd number.

Unit 7

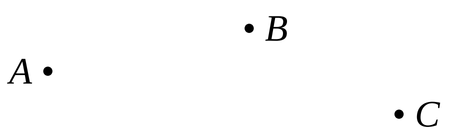
Text 7.1. Read and translate the text into Ukrainian

Essential Vocabulary

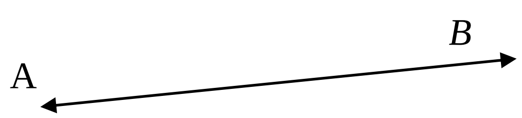
angle – кут
edge – грань; ребро
endpoint – кінцева точка
length – довжина
line – лінія
plane – площина
point – точка
ray – промінь
segment – відрізок
straight line – пряма лінія
subset – підмножина
vertex (pl. vertices) – вершина
width – ширина

Points, Lines, Rays and Angles

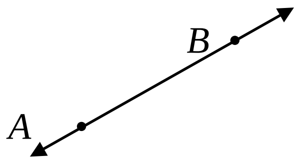
The world around us contains many physical objects from which mathematicians have developed geometric ideas and these objects can serve as models of the geometric figures. The edge of a ruler or an edge of this page is a model of a line. We have agreed to use the word line to mean straight line. A geometric line is the property these models of lines have in common, it has length but no thickness and no width; it is an idea. A participle of dust in the air, or a dot on a piece of paper is a model of a point. A point is an idea about an exact location; it has no dimensions. We usually use letters of the alphabet to name geometric ideas. For example, we speak of the following models of points as point *A*, point *B*, and point *C*.



We speak of the following as line *AB* or line *BA*.



The arrows on the model above indicate that a line $\overleftrightarrow{AB}$ extends indefinitely in both directions Let us agree to use the following figure to name a line. The symbol $\overleftrightarrow{AB}$ means line AB. Can you locate a point C between A and B on the drawing $\overleftrightarrow{AB}$ of above? Could you locate another point between B and C? Could you continue this process indefinitely? Why? Because between any two points on a line there is another point. A line consists of a set of points. Therefore, a piece of the line is a subset of the line. There are many kinds of subsets of a line. The subset of $\overleftrightarrow{AB}$ shown below is called a line segment as you might remember from above.

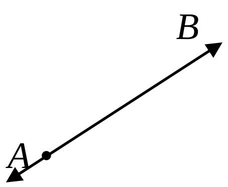


The symbol for line segment AB is marked as follows: $\overline{AB}$ (segment AB). You already know that points A and B are the endpoints of the segment. A line segment is a set of points consisting of the two endpoints and all of the points on the line between them. Notice that the symbol for a line segment ($\overline{AB}$) contains the letters naming the endpoints, that is, only the endpoints need to be given while naming a line segment.

How does a line segment differ from a line? Could one measure the length of a line? Of a line segment? You can judge from the above that a line segment has definite length but a line extends indefinitely in each of its two directions.

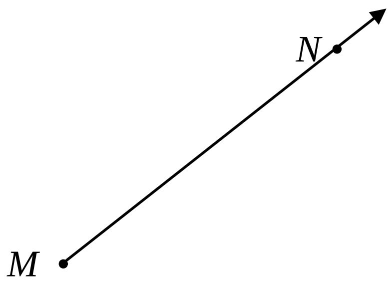
Another important subset of a line is called a ray. It has infinite length and only one endpoint, which is called a vertex. The symbol for ray AB is a one-way arrow over $\overrightarrow{AB}$.

Traditionally, the symbol AB in geometry might represent a line, a line segment, or a ray. We draw the figure that is to be named above the letters ($\overline{AB}$, $\overrightarrow{AB}$, $\overleftrightarrow{AB}$) to eliminate the possible ambiguity.



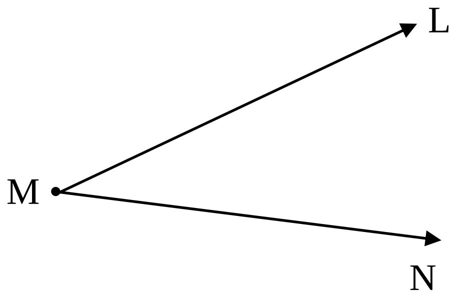
It should be emphasized that in the drawings given above you see pictures of a line, a line segment, and a ray and not the geometric ideas they represent. Let us agree that to draw a geometric figure means to draw its picture. Obviously, if a geometric figure, being formed by a set of points, is an abstract concept, it cannot be seen. Therefore we draw pictures of geometric figures just as we write numerals for numbers.

You certainly remember that by extending a line segment in one direction we obtain a ray. Below is a picture of such an extension.



The arrow indicated that you start at point *M*, go through point *N*, and on without end. This results in what is called ray *MN*, which is denoted by the symbol $\overleftrightarrow{MN}$. Point *M* is the endpoint in this case. Notice that the letter naming the endpoint of a ray is given when first naming the ray.

From what you already know you may deduce that drawing two rays originating from the same endpoint forms an angle. The common point of two rays is the vertex of the angle.



Angles, though open figures, separate the plane into three distinct sets of points: the interior, the exterior, and the angle.

Exercise 7.1. Translate the text into Ukrainian

1. A rectangular plate is vertically immersed in a vessel with fluid. One of its sides with length a = 0.3 *m* lies on a surface of fluid, the length of the vertical side is b = 0.5 m. At what depth is it necessary to divide the rectangle by a horizontal line, so that the force of pressure on the upper and lower parts of the area will be equal among themselves?

2. The circle cylinder, which radius of the base is equal to R and the altitude is H, rotates round the axis with the constant angular rate ω. The density of the material, of which the cylinder is made, is equal to δ. Find the kinetic energy of the cylinder.

3. The dam has a form of an isosceles trapezoid, which bases have the length a = 200 *m* and b = 50 *m* relatively, and the altitude is equal to h = 10 *m*. Calculate the pressure on the dam, if the upper base lies on the level of the free surface of water.

Exercise 7.2. Translate the text into English

Рекомендується додержуватися подальшої схеми:

1. Область *D* проектується на одну з осей, наприклад, O*x*. Цим визначається відрізок [*a*, *b*]: $a \le x \le b$. Величини *a* і *b* будуть відповідно нижньою і верхньою границями зовнішнього інтегралу в формулі, тобто границі у зовнішньому інтегралі завжди є постійними величинами.

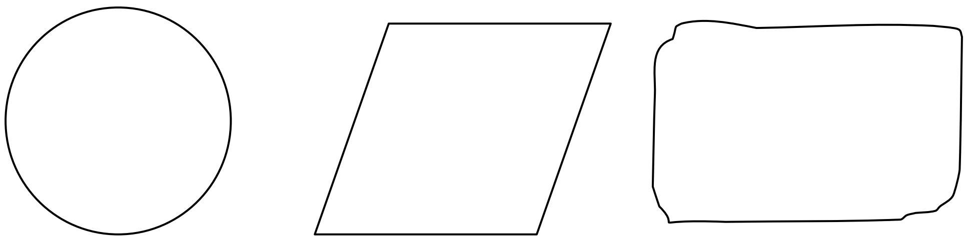
2. Для знаходження границь інтегрування у внутрішньому інтегралі проведемо через будь-яку точку (*x*, 0) є [*a*, *b*] пряму лінію паралельно до осі *Oy*. Ця пряма лінія пересікає границю області *D* у точках M_1 і M_2. Для знаходження координат цих точок необхідно вирішити рівняння нижньої області *ACB* та верхньої області *ADB* границі області *D* для *y*: $y = y_1(x)$ та $y = y_2(x)$ відповідно. При цьому функції $y_1(x)$ і $y_2(x)$ на відрізку [*a*, *b*] є неперервними та однозначними; вони не змінюють своє аналітичне значення. Аналогічно визначаються границі інтеграції у повному інтегралі формули (10.2), де область проектується на осі ординат.

Text 7.2. Read and translate the text into Ukrainian

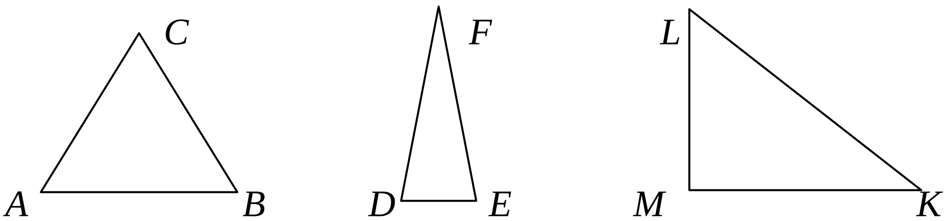
Essential Vocabulary

angle – кут
equilateral triangle – рівносторонній трикутник
hypotenuse – гіпотенуза
intersect – перетинати(ся)
isosceles triangle – рівнобедрений трикутник
leg – катет (трикутника)
plane – площина
polygon – багатокутник
quadrilateral – чотирикутник
right angle – прямий кут
right triangle – прямокутний трикутник
segment – відрізок
side – сторона
triangle – трикутник

A simple closed figure is any figure drawn in a plane in such a way that its boundary never crosses or intersects itself and encloses part of the plane. The following are examples of simple closed figures. Every simple closed figure separates the plane into three distinct sets of points. The interior of the figure is the set of all points in the part of the plane enclosed by the figure. The exterior of the figure is the set of points in the plane which are outside the figure. And finally, the simple closed figure itself is still another set of points.



A simple closed figure formed by line segments is called a polygon. Each of the line segments is called a side of the polygon. Polygons may be classified according to the measures of the angles or the measure of the sides. This is true of triangles – geometric figures having three sides – as well as of quadrilaterals, having four sides.



In the picture above you can see three triangles.

Δ *ABC* is referred to as an equilateral triangle. The sides of such a triangle all have the same linear measure. Δ *DEF* is called an isosceles triangle which means that its two sides have the same measure. You can see it in the drawing above. Δ *LMK* being referred to as a right triangle means that it contains one right angle. In Δ *LMK*, angle *LMK* is the right angle, sides *ML* and *MK* are called the legs, and side *KL* is called the hypotenuse. The hypotenuse refers only to the side opposite to the right angle of a right triangle.

Exercise 7.3. Translate the text into Ukrainian

Many geometrical and physical problems are solved by the theory of extremums. Let us assume we have two values connected by a functional relation, and we have to find the value of one of them (determined on some interval which may be unlimited), at which another one takes the greatest or the least values.

To solve this problem, first of all, we should create an expression, describing relations between the functions (one function expressed through another) and then find the greatest or the least values in the given interval.

Example. Let us denote the radius of the cylinder basis by R and its altitude by H. Then the complete surface is as follows:

$S = 2\pi R^2 + 2\pi RH$

Since the cylinder volume is given, then one of required values, for example H, may be expressed through R by the formula $V = \pi R^2$ H, whence $H = V/\pi R^2$. Substituting this expression in the formula for *S* we obtain:

$S = 2\ (\pi R^2 + V/R)$

i.e. *S* is a function of the single variable *R*, where $R \in (0,\infty)$.

Testing

Test A. Translate the text into Ukrainian

Properties of Functions, Continuous on the Closed Intervals

Properties of the functions continuous on the closed intervals are formulated as the following theorems.

Theorem (Boltsano-Cauchy). If the function $y = f\ (x)$ is continuous on the segment [a, b] and has values of various signs on the ends of this interval then there is at least one point $x = c$ between the points a and b at which the function vanishes: $f\ (c) = 0$, $a < c < b$. This theorem has simple geometric interpretation. The graph of the continuous function $y = f\ (x)$, connecting two points *M1* [a, $f\ (a)$] and *M2* [b, $f\ (b)$], where $f\ (a) < 0$ and $f\ (b) > 0$ (or $f\ (a) > 0$ and $f\ (b) < 0$), intersects the axis *0x* at least at one point.

Theorem. Let the function $y = f\ (x)$ be continuous on the segment [a, b] and $f\ (a) = A$, $f\ (b) = B$, and let C be an arbitrary number located between A and B, i.e. $A < C < B$, then there is such point $x = c$ belonging to the segment [a, b] at which $f\ (c) = C$.

Theorem (the first theorem by Weierstrass). If the function $y = f\ (x)$ is continuous on the segment [a, b], then it is bounded on this segment.

Theorem (the second theorem by Weierstrass). If the function $y = f\ (x)$ is continuous on some segment [a, b], then it achieved its exact upper and lower bounds (i.e. there are such points x_1 and x_2, that $f\ (x_1) = M$, $f\ (x_2) = m$).

Test B. Translate the text into Ukrainian

The Greatest and the Least Values of Functions on the Interval

Let the function $y = f(x)$ be continuous on the interval [a, b]. Then, according the Weierstrass theorem, it achieves the greatest and the least values in this interval. These values can be achieved either on the borders of the interval or at interval point, being the extremums of the function. From this fact we conclude the following plan for defining the greatest and the least values of the function:

1. Define all the critical points, belonging to the given interval [a, b].
2. Calculate the values of the function in the found critical points and on the borders of the interval.

3. Choose the greatest and the least values from the obtained values. The chosen values are required ones.

Example 1. Find the greatest and the least values of the function $f(x) = x^3 - 3x^2 + 1$ on the interval [-1, 4].

Solution.

$f'(x) = 3x^2 - 6x = 3x(x - 2)$.

$3x(x - 2) = 0$ at $x = 0$ and $x = 2$.

Thus the given function has two stationary points $x_1 = 0$ and $x_2 = 2$ inside the interval [–1, 4]. Let us calculate the function values at these points and on the borders of the interval: $f(0) = 1$; $f(2) = -3$; $f(-1) = -3$; $f(4) = 17$.

As we see, the function takes the greatest value on the right border of the interval while the least value is taken at the interval point $x = 2$ and on the left border of the interval.

Test C. Translate the text into Ukrainian

Differential Equations

The differential equation is an equation connecting an unknown function, its derivatives and independent variables.

If a function of a differential equation depends only on one independent variable, then this equation is called an ordinary differential equation. The highest derivative order of this equation is called an order of the equation. The ordinary differential equation of the n^{th} order is the following:

$F(x, y, y', y'',\dots, y^{(n)}) = 0$.

Any function $y = \varphi(x)$ satisfying a differential equation, i.e. turning it into an identity, is called a solution of this equation. The ratio $\Phi(x, y) = 0$, implicitly presenting a solution of an equation, is called an integral of this equation. The graph of a solution of a differential equation is called its integral curve. The process of finding a solution of a differential equation is called its integration.

Mathematical Signs & Symbols

+	addition, plus, positive	знак додавання або додатної величини
–	subtraction, minus, negative	знак віднімання або від'ємної величини
±	plus or minus	плюс мінус
×, •	multiplication sign, multiplied by, times	знак множення, множення на…
÷, :, /	division, divided by	знак ділення, ділення на…
=	equals, (is) equal to	дорівнює
≠	not equal to	не дорівнює
≈	approximately equals	приблизно дорівнює
→	approaches	наближається
>	greater than	більше ніж
<	less than	менше ніж
≥	greater than or equal to	більше чи дорівнює
≤	less than or equal to	менше чи дорівнює
∞	infinity	нескінченність
$\sqrt{a}$	square root of *a*	квадратний корінь з *a*
$\sqrt[3]{a}$	cube root of *a*	кубічний корінь з *a*
‖	parallel to	паралельно
⊥	perpendicular to	перпендикулярно до
( )	parentheses	круглі дужки
[ ]	brackets	квадратні дужки
{ }	braces	фігурні дужки
M	micron	мікрон (0,001 мм)
°	degree(s)	градус(и)
%	per cent	процент(и)
'	minute(s), foot / feet	хвилини, фути
"	second(s), inch(es)	секунди, дюйми
C_a	C sub *a*	*C* з індексом *a*
C'	C prime	*C* – штрих
C''	C double prime	*C* – два штрихи
dx	differential of *x*	диференціал *x*
∫	integral	інтеграл від...
F(x)	function of *x*	функція від *x*
|x|	absolute value of x	абсолютне значення *x*
&	and	і
:	colon	двокрапка
,	comma	кома

;	semicolon	крапка з комою
‘	apostrophe	апостроф
–	dash	тире
-	hyphen	дефіс
*	asterisk	зірочка
?	interrogation point	знак питання
!	note of exclamation	знак оклику

Greek Letters

α	alpha	альфа
β	beta	бета
γ	gamma	гамма
δ	delta	дельта
ε	epsilon	іпсилон
ζ	zeta	дзета
η	eta	ета
θ	theta	тета
j	iota	йота
κ	kappa	каппа
λ	lambda	лямбда
μ	mu	мі
ν	nu	ні
ξ	xi	ксі
ο	omicron	омікрон
π	pi	пі
ρ	rho	ро
σ	sigma	сигма
τ	tau	тау
υ	upsilon	іпсилон
φ	phi	фі
χ	chi	хі
ψ	psi	псі
ω	omega	омега

Formulae Reading

$a \div b = c$	*a* divided by *b* is equal to *c*
$2 \times 2 = 4$	Twice two is four
$c \times d = b$	*c* multiplied by *d* equals *b*
dx	differential of *x*
$\frac{a+b}{a-b} = \frac{c+d}{c-d}$	*a* plus *b* over *a* minus *b* is equal to *c* plus *d* over *c* minus *d*
$y_{a-b} \cdot x_{b-c} = 0$	*y* sub *a* minus *b* multiplied by *x* sub *b* minus *c* is equal to zero
$\frac{d^2y}{ds^2} + y[1+b(s)] = 0$	the second derivative of *y* with respect to *s* plus *y* times open bracket *one* plus *b* of *s* in parentheses, close bracket is equal to zero
$\int f(x)dx$	the integral of *f(x)* with respect to *x*
$\int_a^b f(x)dx$	the definite integral of *f(x)*with respect to *x* from *a* to *b* (between limits *a* and *b*)
$c(s)= K_{ab}$	*c* of *s* is equal to *K* sub *ab*
$x_{a-b} = c$	*x* sub *a* minus *b* is equal to *c*
$a \propto b$	*a* varies directly as *b*
$a : b :: c : d$; $a : b = c : d$	*a* is to *b* as (equals) *c* is to *d*
$x \times 6 = 42$	*x* times six is forty two; *x* multiplied by six is forty two
$10 \div 2 = 5$	ten divided by two is equal to five; ten over two is five
$\frac{a^2}{c} = b$	*a* squared over *c* equals *b*
$a^5 = c$	*a* raised to the fifth power is *c*; *a* to the fifth degree is equal to *c*
$\frac{a+b}{a-b} = c$	*a* plus *b* over *a* minus *b* is equal to *c*
$a^3 = log_c b$	*a* cubed is equal to the logarithm of *b* to the base *c*
$log_a b = c$	the logarithm of *b* to the base *a* is equal to *c*
$x_{a-b} = c$	*x* sub *a* minus *b* is equal to *c*
$\frac{\partial^2 u}{\partial t^2} = 0$	the second partial derivative of *u* with respect to *t* equals zero

$c : d = e : l$	*c* is to *d* as *e* is to *l*
$15 : 3 = 45 : 9$	fifteen is to three as forty five is to nine; the ratio of fifteen to three is equal to the ratio of forty five to nine
$p \approx \sum_{i=0}^{n-1} f(x_i)\Delta X$	*p* is approximately equal to the sum of *x* sub *i* delta *x* sub *i* and it changes from zero to *n* minus one
$\sqrt{a^2+b^2} - \sqrt{a^2+b_1^2} \le \lvert b-b_1 \rvert$	the square root of *a* squared plus *b* squared minus the square root of *a* squared plus *b* sub one squared by absolute value is less or equal to *b* minus *b* sub one by absolute value (by modulus)
$a^z = \lim_{n\to\infty} a^{z_n}$	*a* to the power *z* is less or equal to the limit *a* to the power *z* sub *n* where *n* tends (approaches) the infinity
$\sum_{j=1}^{n} a_j;\ j = 1,2,..n$	the sum of *n* terms *a* sub *j*, where *j* runs from *1* to *n*
$\sqrt[4]{81} = 3$	the fourth root of 81 is equal to three
$c \propto d$	*c* varies directly as *d*
$sin\ \alpha = a$	sine angle α is equal to *a*
$\int \frac{dx}{\sqrt{a^2-x^2}}$	integral of *dx* divided by (over) the square root out of *a* square minus *x* square
$\frac{d}{dy} \int_{x_0}^{x} xdx$	*d* over *dx* of the integral from *x* sub *0* to *x* of capital *xdx*

English-Ukrainian Dictionary

abscissa	абсциса
absolute value of	абсолютне значення
acute angle	гострий кут
add	додавати
addend	доданок (*рус*. слагаемое)
addition	додавання
adjacent	суміжний
algebraic symbol	алгебраїчний символ
altitude	висота
angle	кут
angular	кутовий
approximation	приблизне значення
arabic numbers	арабські цифри
area	площа
arbitrary number	довільне число
arithmetic progression	арифметична прогресія
arithmetic	арифметика
arrangement	розміщення
asymptote	асимптота
axes of coordinates	јсі системи координат
axis	вісь
axis of abscissas	вісь абсцис
axis of ordinates	вісь ординат
base	основа
base field	основне поле
bisect	ділити навпіл
bisector	бісектриса
border	кінець (відрізка)
boundary	границя
broken line	ламана лінія
cardinal number	кількісний числівник
cathetus	катет
characteristic root	характеристичний корінь
chord	хорда
circle	коло
circumference	довжина кола
coefficient	коефіцієнт
collinear	колінеарний
column	стовпець

common divisor	спільний дільник
complanar	компланарний
complementary angles	додаткові кути
complex	комплексний
complex coefficient	комплексний коефіцієнт
complex plane	комплексна площина
cone	конус
constant	постійна величина, константа
continuous	неперервний
contour	контур
convergent	збіжний в одній точці
convergence interval	інтервал збіжності
corner	кут
cosine	косинус
cotangent	котангенс
cube root of	кубічний корінь з
cube	куб
cuboid	кубоїд, прямокутний паралелепіпед
curved line	крива лінія
curved surface	крива поверхня
cyclic group	циклічна група
cylinder	циліндр
cylindrical	циліндричний
decagon	десятикутник
decimal (common) logarithm	десятковий логарифм
decimal fraction	десятковий дріб
definite integral	визначений інтеграл
degree	степінь; градус
denominator	знаменник (*рус.* знаменатель)
derivative	похідна
determinant	визначник
diagonal	діагональ
diameter	діаметр
difference	різниця
differential calculus	диференціальне числення
differential equation	диференціальне рівняння
differential of x	диференціал x
differentiation	диференціювання
digit	цифра
directrix	директриса, напрямна (лінія)
divide (by)	ділити (на)
dividend	ділене (*рус.* делимое)
division	ділення

divisor	дільник (*рус.* делитель)
domain	область
double integral	подвійний інтеграл
edge	грань; ребро
element	елемент
ellipse	еліпс, овал
ellipsoid	еліпсоїд
empty set	пуста множина
endpoint	кінцева точка
equals	дорівнює
equation	рівняння
equilateral	рівносторонній
equilateral triangle	рівносторонній трикутник
even function	парна функція
even number	парне число (*рус.* четное число)
evolution	добування кореня
exponent	показник (степеня)
exponential function	показникова функція
extract	добувати (корінь)
extract the root of	добувати корінь з
extremes	крайні члени (математичної пропорції)
extremum	екстремум
exterior angle	зовнішній кут
factor	множник (*рус.* множитель)
factorial	факторіал
field	поле
finite	кінцевий
formula	формула
fraction	дріб
fractional	дробовий
function of x	функція від x
generatrix (pl. generatrices)	твірна
geometrical progression	геометрична прогресія
height	висота
hexagon	шестикутник
hyperbola	гіпербола
hyperbolic cosine	гіперболічний косинус
hyperbolic function	гіперболічна функція
hypotenuse	гіпотенуза
identity	тотожність
independent variable	незалежна змінна
index (pl. indices)	показник степеня
inequality	нерівність

infinity	нескінченність
integer	ціле число
integral	інтеграл від...
integral calculus	інтегральне числення;
integral curve	інтегральна крива
integration	інтегрування
interior angle	внутрішній кут
intercept	перетинати(ся), перетин
intersect	перетинати(ся),
intersection	точка або лінія перетину
interval	відрізок, числовий інтервал
inverse function	обернена функція
involution	піднесення до степеня
irrational number	ірраціональне число
isosceles triangle	рівнобедрений трикутник
item	доданок (*рус.* слагаемое)
leg	сторона, катет (трикутника)
length	довжина
limit	границя
line	лінія
linear equation	лінійне рівняння
linear function	лінійна функція
linear-fractional function	квадратична функція
linear space	лінійний простір
logarithm	логарифм
logarithmic function	логарифмічна функція
mapping	відображення
mathematics	математика
matrix (pl. matrices)	матриця
means	середнє
median	медіана
minor	мінор
minuend	зменшуване (*рус.* уменьшаемое)
minus	мінус
mixed number	змішане число
module	модуль
multiple root	кратний корінь
multiplicand	множене (*рус.* множимое)
multiplication	множення
multiplicity	кратність
multiplier	множник (*рус.* множитель)
multiply (by)	помножити (на)
natural logarithm	натуральний логарифм

natural number	натуральне число
negative number	від’ємне число (*рус.* отрицательное число)
nonzero	ненульовий
normal form	нормальна форма (рівняння прямої)
not equal to	не дорівнює
numerator	чисельник (*рус.* числитель)
obtuse angle	тупий кут
octagon	восьмикутник
odd function	непарна функція
odd number	непарне число (*рус.* нечетное число)
order	порядок
ordinal number	порядковий числівник
ordinate	ордината
origin of ordinates	початок координат
parabola	парабола
parallel to	паралельно
parallelepiped	паралелепіпед
parallelogram	паралелограм
parameter	параметр
pentagon	п’ятикутник
perimeter	периметр
periodical function	періодична функція
perpendicular lines	перпендикулярні лінії
perpendicular to	перпендикулярно до
plane	площина
plus	плюс
point *A*	точка *A*
polygon	багатокутник
polynomial function	поліном
positive number	додатне число (*рус.* положительное число)
power	степінь
power function	степенева функція
power series	степеневий ряд
prime	просте число
prime power	степінь простого числа
principal diagonal	головна діагональ
prism	призма
product	добуток, (*рус.* произведение)
proportion	пропорція
proportional	пропорційний
pyramid	піраміда

quadratic function	квадратична функція
quadrilateral	чотирикутник
quantity	кількість, величина
quotient	частка (*рус.* частное)
radical sign	знак кореня
radius	радіус
range	область
rank	ранг
raise to power	підносити до степеня
ratio	відношення
rational number	раціональне число
ray	промінь
real coefficient	речовий коефіцієнт
real number	речове число
real root	речовий корінь
reciprocal	обернений
reciprocal number	обернене число
rectangle	прямокутник
rectangular	прямокутний
regular	правильний
remainder	остача (*рус.* остаток)
repeating decimal	періодичний десятковий дріб
repeating function	періодична функція
right angle	прямий кут
right triangle	прямокутний трикутник
Roman numbers	римські цифри
root	корінь
round	коло
scalar	скалярний
scalene triangle	нерівносторонній трикутник
segment	відрізок
sequence	послідовність
series	послідовність
set	множина (*рус.* множество)
side	сторона (квадрата, многокутника)
sign	знак
sine	синус
singular	особливий
solution	розв’язання
solve	розв’язувати
sphere	сфера
square root of	квадратний корінь з
square	квадрат

straight angle	розгорнутий кут
straight line	пряма лінія
subgroup	підгрупа
submatrix (pl. submatrices)	підматриця
subscript	нижній індекс
subset	підмножина
subtract	віднімати
subtraction	віднімання
subtrahend	від'ємник (*рус.* вычитаемое)
succession	послідовність
sum	сума, сумувати
summand	доданок (*рус.* слагаемое)
supplementary angles	суміжні кути
surface	поверхня
tangent	тангенс
term	член, одночлен
terminus	кінець (вектора)
total	підсумок, підводити підсумок
transposition	транспозиція
trapezoid	трапеція
triangle	трикутник
triangular	трикутний
triangular prism	трикутна призма
trigonometrical function	тригонометрична функція
truncated cone	зрізаний конус з непаралельними основами
uniqueness	однозначність
unit	одиниця вимірювання
unity	одиниця
unknown	невідоме
value	величина, значення
variable	змінна
variation	змінювання
vary directly as	змінюватись прямо пропорційно
vary inversely as	змінюватись обернено пропорційно
vector	вектор
vertex (pl. vertices)	вершина
vertical angle	вертикальний кут
vulgar fraction	звичайний дріб
width	ширина
whole number	ціле число
zero	нуль

Ukrainian-English Dictionary

абсолютне значення	absolute value of
абсциса	abscissa
алгебраїчний символ	algebraic symbol
арифметична прогресія	arithmetic progression
асимптота	asymptote
багатокутник	polygon
бісектриса	bisector
вектор	vector
величина	quantity, value
вертикальні кути	vertical angles
вершина	vertex (pl. vertices)
визначник	determinant
висота	height, altitude
від’ємне число	negative number
віднімання	subtraction
віднімати	subtract
відношення	ratio
відображення	mapping
відрізок	segment, interval
вісь абсцис	axis of abscissas
вісь ординат	axis of ordinates
внутрішній кут	interior angle
восьмикутник	octagon
геометрична прогресія	geometrical progression
гіпербола	hyperbola
гіперболічна функція	hyperbolic function
гіперболічний косинус	hyperbolic cosine
гіпотенуза	hypotenuse
гострий кут	acute angle
градус	degree
границя	limit
грань	edge
десятикутник	decagon
десятковий дріб	decimal fraction
десятковий логарифм	decimal (common) logarithm
директриса	directrix
диференціал ікса	differential of *x*
диференційне рівняння	differential equation
диференційне числення	differential calculus
диференціювання	differentiation

діагональ	diagonal
діаметр	diameter
ділене	dividend
ділення	division
ділити (на)	divide (by)
дільник	divisor
диференціювання	differentiation
добування кореня	evolution
добувати корінь з	extract the root of
добуток	product
довжина	length
довжина кола	circumference
довільне число	arbitrary number
додавання	addition
додавати	add
доданок	addend, item, summand
додаткові кути	complementary angles
додатне число	positive number
дріб	fraction
дробовий	fractional
екстремум	extremum
еліпс	ellipse
еліпсоїд	ellipsoid
збіжний в одній точці	convergent
звичайний дріб	vulgar fraction
зменшуване	minuend
змінна	variable
змінювання	variation
змішане число	mixed number
знак кореня	radical sign
знаменник	denominator
значення	value
зовнішній кут	exterior angle
інтеграл від...	integral
інтегральна крива	integral curve
інтегральне числення	integral calculus
інтегрування	integration
інтервал збіжності	convergence interval
ірраціональне число	irrational number
катет	cathetus
квадрат	square
квадратична функція	quadratic function
квадратний корінь з	square root of

кількісний числівник	cardinal number
кількість	quantity
кінець (вектора)	terminus
кінцева точка	endpoint
кінцевий	finite
коефіцієнт	coefficient
колінеарний	collinear
коло	circle, round
компланарний (вектор)	complanar
комплексна площина	complex plane
комплексне число	complex number
комплексний коефіцієнт	complex coefficient
контур	contour
константа	constant
конус	cone
корінь	root
косинус	cosine
котангенс	cotangent
крайні члени (мат. пропорції)	extremes
кратний корінь	multiple root
кратність	multiplicity
крива лінія	curved line
крива поверхня	curved surface
куб	cube
кубічний корінь з	cube root of
кут	angle, corner
кутовий	angular
ламана лінія	broken line
лінійна функція	linear function
лінійне рівняння	linear equation
лінія	line
логарифм	logarithm
логарифмічна функція	logarithmic function
математика	mathematics
матриця	matrix
медіана	median
мінус	minus
множене	multiplicand
множення	multiplication
множина (*рус.* множество)	set
множник	factor, multiplier
модуль	module
напрямна	directrix

натуральне число	natural number
натуральний логарифм	natural logarithm
невідоме	unknownd
незалежна змінна	independent variable
ненульовий	nonzero
непарна функція	odd function
непарне число	odd number
нерівність	inequality
нерівносторонній трикутник	scalene triangle
нескінченність	infinity
нижній індекс	subscript
нормальна форма	normal form
нульовий вектор	null vector
обернена функція	inverse function
обернене число	reciprocal number
обернений	reciprocal
область	domain, range
овал	ellipse
одиниця вимірювання	unit
однозначність	uniqueness
одночлен	term
ордината	ordinate
осі системи координат	axes of coordinates
основа	base
остача	remainder
п'ятикутник	pentagon
парабола	parabola
паралелепіпед	parallelepiped
паралелограм	parallelogram
паралельно	parallel to
параметр	parameter
парна функція	even function
парне число	even number
перетинати(ся)	intersect, intercept
периметр	perimeter
періодична функція	periodical function, repeating function
періодичний десятковий дріб	repeating decimal
перпендикулярні лінії	perpendicular lines
перпендикулярно до	perpendicular to
підводити підсумок	total
підгрупа	subgroup
підматриця	submatrix

підмножина	subset
піднесення до степеня	involution
піднімаюче	subtrahend
підносити до степеня	raise to power
підсумок	total
піраміда	pyramid
площа	area
площина	plane
плюс	plus
поверхня	surface
подвійний інтеграл	double integral
показник (степеня	exponent, index (pl. indices)
показникова функція	exponential function
поле	field
поліном	polynomial function
помножити (на)	multiply (by)
порядковий числівник	ordinal number
послідовність	series, sequence, succession
похідна	derivative
початок (вектора)	origin
початок координат	origin of ordinates
правильний	regular
приблизне значення	approximation
призма	prism
промінь	ray
пропорційний	proportional
пропорція	proportion
пряма лінія	straight line
прямий кут	right angle
прямокутний паралелепіпед	cuboid
прямокутний трикутник	right triangle
прямокутний	rectangular
прямокутник	rectangle
пуста множина	empty set
радіус	radius
ранг	rank
раціональне число	rational number
ребро	edge
рівнобедрений трикутник	isosceles triangle
рівносторонній трикутник	equilateral triangle
рівносторонній	equilateral
рівняння	equation
різниця	difference

розв'язання	solution
розв'язувати	solve
розгорнутий кут	straight angle
розміщення	arrangement
середнє	means
синус	sine
скалярний	scalar
спільний дільник	common divisor
степенева функція	power function
степеневий ряд	power series
степінь	degree, power
стовпець	column
сторона	leg (трикутника), side (квадрата, п'ятикутника або шестикутника)
сума	sum
суміжний	adjacent
суміжні кути	supplementary angles
сумувати	sum
сфера	sphere
тангенс	tangent
твірна	generatrix (pl. generatrices)
тотожність	identity
точка *A*	point *A*
точка або лінія перетину	intersection
транспозиція	transposition
трапеція	trapezoid
тригонометрична функція	trigonometrical function
трикутна призма	triangular prism
трикутний	triangular
трикутник	triangle
тупий кут	obtuse angle
факторіал	factorial
формула	formula
функція від *x*	function of *x*
хорда	chord
циліндр	cylinder
цифра	digit
ціле число	integer, whole number
частка	uotient
чисельник	numerator
числовий	інтервал interval
чотирикутник	quadrilateral
шестикутник	hexagon

Keys

Exercise 1.4

The obtained equation defines the parabola. The parabola divides the whole plane into two parts, namely internal and external parts relatively to the parabola. For points belonging to one of its parts the inequality $y^2 < 4 + 4x$ is valid, for the other part $y^2 > 4 + 4x$ (on the parabola $y^2 = 4 + 4$).

In order to understand which of these two parts is the domain of the given function, i.e. satisfies the condition $y^2 < 4 + 4x$, we should check this condition for any point not lying on the parabola. For example, the point of origin O (0,0) lies inside the parabola and satisfies the necessary condition $0 < 4 + 4 \times 0$. Therefore, the studied domain D consists of the points inside the parabola. As the parabola does not belong to the domain D we mark the boundary of the domain on the figure with a dotted line.

Exercise 2.3

Cauchy's theorem for the equation $y' = f(x, y)$ states that, if in some domain the functions $f(x, y)$ and $f''_y(x, y)$ are continuous, then through each point of this domain passes only one integral curve of this equation. Points at which the conditions of Cauchy's theorem are ignored are called the singular points of a differential equation. A curve completely consisting of singular points is called a singular curve of an equation. If a singular curve at the same time is an integral one for some equation then it is called a singular integral curve or a singular solution.

We should mention that it is impossible to say beforehand what will be at the singular point: there can be several or none integral curves passing through it.

Exercise 3.3

If the coefficients and right part of the differential equation

$y^{(n)} + \alpha_1(x)\, y^{(n-1)} + \ldots + \alpha_n(x)\, y = f(x)$

are expanded in a power series on powers $x - \alpha$, convergent in some neighbourhood of the point $x = \alpha$, then the solution of this equation satisfying the initial conditions

$y(\alpha) = y_0, y'(\alpha) = y'_0, \ldots, y^{(n-1)}(\alpha) = y_0^{(n-1)}$,

is expanded in a power series on powers $x - \alpha$, at least, on the smallest convergence interval of the series for the coefficients and right part of the differential equation.

For an approximate solution of a differential equation by means of power series two methods are used: the coefficients comparison and successive differentiation.

Exercise 4.4

The variable u is called a function of independent variables $x_1, x_2, \ldots, x_n$ if for each set of values $(x_1, x_2, \ldots, x_n)$ of these variables from the given domain of their variations one or more values of u are established according to some rule or law. The notation: $u = f(x_1, x_2, \ldots, x_n)$ or $u = u(x_1, x_2, \ldots, x_n)$. In case of a function of three variables the function u is designated by $u = f(x, y, z)$, in case of a function of two variables – $u = f(x, y)$ or $z = f(x, y)$, $z = z(x, y)$.

The set of n values $x_1, x_2, \ldots, x_n$ is called a "point" in the range of their variation and we deal with the value of the function u at this point. If the function is given by an analytical expression (formula) without any additional conditions, then the range of existence of its analytical expression is considered to be its range of definition, i.e. a set of those points, at which the given analytical expression is defined and takes only real and finite values.

Exercise 5.2

If the function $z = f(x, y) \geq 0$, then the double integral of this function is equal to the volume of a cylindrical solid limited by the domain D from below, by the surface $z = f(x, y)$ from above and by the cylindrical surface, which directrix is a boundary of the domain D, and which generatrices are parallel to the axis Oz.

The basis properties of a double integral are similar to the correspondent properties of a definite integral.

Calculation of a double integral is carried out by re-calculation of two definite integrals. Let us assume that any line parallel to the Oy, not more than at two points, intercepts the domain of integration D.

Such a domain will be called a regular or simple domain in the direction of the axis Ox. Let the most left-hand point of the boundary F be A and the most right-hand be B. Let us denote their abscissas by a, b. The points A and B divide the contour F into the lower part ACB, which equation is $y = y_1(x)$, and the upper part ADB, which equation is $y = y_2(x)$.

Exercise 6.2

A direct segment (or an ordered couple of points) is called a vector (geometrical). So-called the null vector whose origin coincides with its terminus also relates to vectors. The distance between an original and terminus of a vector is called its length or module and designated by $|\vec{a}|$. The module of a null vector is equal to zero.

Vectors located on the same or parallel straight lines are called collinear.

Vectors are called complanar if there exists a plane, which they are parallel to.

Two vectors should be considered equal if they are collinear, equally directed and have equal lengths. A projection of a vector $\vec{a}$ on an axis L is defined by $pr_L\,\vec{a} = |\vec{a}\,|\cos\varphi$, where φ is an angle between the vector and the axis L.

Exercise 7.2

It is suggested to follow the given scheme:

1. The domain D is projected on an axis, for example, Ox. This defines the segment $[a, b]$: $a \le x \le b$. The values a and b will be the lower and the upper limits of an external integral in the formula, i.e. the limits in an external integral are always constant values.

2. To find limits of integration in an internal integral we should draw through any point $(x, 0) \in [a, b]$ a straight line parallel to the axis Oy. This line intersects the boundary of the domain D at two points M_1 and M_2. To find the coordinates of these points we have to solve the equations of the lower ACB and the upper ADB domains of the boundary of the domain D for y: $y = y_1(x)$ and $y = y_2(x)$ respectively. The functions $y_1(x)$ and $y_2(x)$ are continuous and single-valued over the interval $[a, b]$; they do not change their analytical expression. Similarly, limits of integration are defined in the repeated integral of the formula, when the domain is projected on the ordinate axis.

Test A

Властивості функцій, неперервних на замкнутих відрізках

Властивості функцій, неперервних на замкнутих відрізках, формулюються нижче у вигляді теорем.

Теорема (Больцано-Коші). Якщо функція $y = f(x)$ неперервна на сегменті $[a, b]$ і на кінцях цього відрізка набуває значення різних знаків, то між точками a і b знайдеться, принаймні, одна точка $x = c$, в якій функція обертається в нуль: $f(c) = 0$, $a < c < b$. Ця теорема має просте геометричне значення. Графік неперервної функції $y = f(x)$, що сполучає точки $M_1\,[a, f(a)]$ і $M_2\,[b, f(b)]$, де $f(a) < 0$ і $f(b) > 0$ (або $f(a) > 0$ і $f(b) < 0$), перетинає вісь $0x$, принаймні, в одній точці.

Теорема. Нехай функція $y = f(x)$ неперервна на сегменті $[a, b]$, причому $f(a) = A$, $f(b) = B$, і нехай C – будь-яке число, поміщене між A і B, тобто $A < C < B$, тоді на сегменті $[a, b]$ знайдеться така точка $x = c$, в якій $f(c) = C$.

Теорема (перша теорема Вейєрштрасса). Якщо функція $y = f(x)$ неперервна на сегменті $[a, b]$, то вона обмежена на цьому сегменті.

Теорема (друга теорема Вейєрштрасса). Якщо функція $y = f(x)$ неперервна на сегменті $[a, b]$, то вона досягає на цьому сегменті своїх точних верхньої і нижньої границь (тобто знайдуться такі точки x_1 і x_2, де $f(x_1) = M$, $f(x_2) = m$).

Test B

Найбільше і найменше значення функцій на відрізку

Нехай функція $y = f(x)$ безперервна на відрізку $[a, b]$. Тоді, за теоремою Вейєрштрасса, на цьому відрізку вона досягає найбільшого і найменшого значень. Ці значення можуть досягатися на кінцях відрізка або у внутрішніх точках, що є точками екстремуму функції. Звідси випливає наступна схема відшукання найбільших і найменших значень:

1. Знайти всі критичні точки функції, що знаходяться усередині відрізка $[a, b]$.

2. Обчислити значення функції в знайдених критичних точках і на кінцях відрізка.

3. Із усіх отриманих значень вибрати найбільше і найменше. Вони і будуть шуканими значеннями.

Приклад 1. Визначити найбільше і найменше значення функції $f(x) = x^3 - 3x^2 + 1$ на відрізку $[-1, 4]$.

Розв'язання.

$f'(x) = 3x^2 - 6x = 3x(x - 2)$.

$3x(x - 2) = 0$ при $x = 0$ і $x = 2$.

Таким чином, дана функція має дві стаціонарні точки $x_1 = 0$ і $x_2 = 2$ усередині відрізка $[-1, 4]$. Обчислимо значення функції в цих точках і на кінцях відрізка: $f(0) = 1$; $f(2) = -3$; $f(-1) = -3$; $f(4) = 17$.

Значить, найбільше значення функція набуває на правому кінці відрізка, найменше – у внутрішніх точках $x = 2$ і на лівому кінці відрізка.

Test C

Диференціальні рівняння

Диференціальним рівнянням називається рівняння, що зв'язує невідому функцію, її похідні і незалежні змінні.

Якщо функція, що входить у рівняння, залежить від однієї незалежної змінної, то рівняння називається звичайним диференціальним рівнянням. Порядок старшої похідної, такої, що входить у дане рівняння, називається порядком рівняння. Звичайне диференціальне рівняння n-го порядку має вигляд: $F(x, y, y', y'', \ldots, y^{(n)}) = 0$.

Всяка функція $y = \varphi(x)$, що задовольняє диференціальному рівнянню, тобто що перетворює його в тотожність, називається розв'язанням цього рівняння. Співвідношення $\Phi(x, y) = 0$, що неявно задає розв'язок рівнянню, називається інтегралом цього рівняння. Графік розв'язання диференціального рівняння називається його інтегральною кривою.

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Навчальне видання

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