Proceedings of the 5th International Conference on Nonlinear Dynamics
ND-KhPI2016
September 27-30, 2016, Kharkov, Ukraine

Dynamics of Multielement Agricultural Aggregates, Taking Into Account Nonholonomic Constraints and Spatial Motion

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Abstract
The paper studies a discrete system of multielement agricultural aggregate composed of a tractor, hopper and seeder. Mechanical model includes these elements, which are considered as rigid bodies and perform spatial motion subject to of the wheels considering elasticity. As the geometric constrains taking into account the flat surface of the earth and the hinges connecting the units. The aggregate is controlled by angle of the steering wheel or the angle between the tractor half-frames. A feature of the model is the account of non-holonomic constraints caused by the rolling of the wheels. This significantly reduces the number of degrees of freedom and also complicates the process of forming the equations of motion. Differential equations are automatically generated by a special system of computer algebra KiDyM based on a general dynamics equation. The gravity force, the driving force and the resistance force applied to the elements of the aggregate defined as force interactions. The studied linear motion, maneuvers with constant and harmonic law change of control angle.

Keywords
Nonholonomic systems, nonlinear dynamics, multibody systems

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Introduction

Although there are many publications on the study of the dynamics of nonholonomic systems, including those which have become classic [1-3], the process of drawing up such systems of equations of motion remains problematic. This is due primarily to the need to carry out non-trivial analytic transformation. The problem becomes even more complex for systems with spatially moving units. The advent of the computer systems of analytical calculations (computer algebra system) makes it easier. However, to simplify the solution of this problem is possible only at the computer implementation of reliable methods during analysis. This approach was developed in NTU "KPI" [4-6] and is implemented in the software package KiDyM. This technique requires a description in the intuitive language of geometric and kinematic (nonholonomic) connections, the inertia parameters of bodies (mass and moments of inertia), the characteristics of the active projections (given) forces and moments, elastic and dissipative forces and moments [7], [8]. Mechanical model – a set of inertial, elastic, and power dissipation elements and structures can be written as formulas in a special file.

Geometric ties are entered as a sequence of displacements and rotations bound at the centers of mass of bodies coordinate systems to each other and relative to the absolute coordinate system. Kinematic relationships are represented by authorized relatively dependent generalized velocities analytical expressions through independent generalized velocities.

All work on the reading of this information, its diagnosis, the conclusion of dynamic equations, their submission in the form of Cauchy and the numerical integration of the analysis unit conducts a special system of computer algebra KiDyM.

The paper studies a discrete system of multielement agricultural aggregate composed of a tractor, hopper and seeder. Mechanical model includes these elements, which are considered as rigid bodies and perform spatial motion subject to of the wheels considering elasticity. As the geometric constrains taking into account the flat surface of the earth and the hinges connecting the units. The aggregate is controlled by angle of the steering wheel or the angle between the tractor half-frames. A feature of the model is the account of non-holonomic constraints caused by the rolling of the wheels. This significantly reduces the number of degrees of freedom and also complicates the process of forming the equations of motion.

1. The algorithm output dynamic equations of nonholonomic systems

The algorithm of nonholonomic systems of equations is presented in the works of one of the authors [4 - 8]. It is based on the method of drawing up equations excluded dependent variations of the generalized coordinates [3] and the principle of d'Alembert-Lagrange. Initial data for it are the kinematic equation (nonholonomic) constraints of the form $\dot{\mathbf{q}}_1 = \mathbf{B}(\mathbf{q},t)\dot{\mathbf{q}}_2 + \boldsymbol{\beta}(\mathbf{q},t)$, expresses the dependence of the generalized speed $\dot{\mathbf{q}}_1$ through independent generalized velocities $\dot{\mathbf{q}}_2$ (pseudo velocity $\boldsymbol{\pi} = \dot{\mathbf{q}}_2$). Also given: position associated with solids relative to the absolute coordinate system in terms of generalized coordinates; inertial characteristics of body – mass and inertia tensor; active forces and moments, including linear elastic and dissipative forces. This information is presented in an analytical form, allows a special computer algebra system KiDyM build equations of motion in the form of [6-7]

$$\sum_{i=1}^{n}\left\{\tilde{\mathbf{W}}_{C_i}^{u\,T} m_i \vec{a}_{C_i} + \tilde{\mathbf{W}}_{\omega_i}^{u\,T}\left(\left[\vec{J}_i^{(i)}\right]\cdot\vec{\varepsilon}_i^{(i)} + \vec{\omega}_i^{(i)} \times \left[\vec{J}_i^{(i)}\right]\cdot\vec{\omega}_i^{(i)}\right)\right\} - \tilde{\mathbf{W}}_P^T\mathbf{P} = 0$$

Here $\tilde{\mathbf{W}}_{C_i}^{u\,T}$, $\tilde{\mathbf{W}}_{\omega_i}^{u\,T}$, $\tilde{\mathbf{W}}_P^T$ – transposes the so-called structural matrix, respectively, the radius vector of the center of mass of bodies turning angles (angular velocity) of bodies of radius vectors of the active forces of the system, determined by the differentiation of these geometric and kinematic parameters of generalized coordinates with independent variations (for pseudo velocity); m_i, $\left[\vec{J}_i^{(i)}\right]$, $\vec{a}_{C_i}$, $\vec{\omega}_i^{(i)}$, $\vec{\varepsilon}_i^{(i)}$ - mass, inertia tensor, acceleration of the center of mass. angular velocity and angular acceleration of the i-th body mechanical system.

2. Statement of the problem

Obtain equations of motion and to investigate the dynamics of agricultural hitch tractor-seeder-hopper connected by spherical joints. Hitching nonholonomic system seems solid, committing spatial movement on the elastic pneumatics.

Tractor given the dynamic model (Fig.1). It is a tractor as a solid body, whose position is determined by the Cartesian coordinates of the center of mass (p. *O* in Fig.1), three angles Krylov (α, β, γ in Fig.1). The front axle can pivot about a vertical axis passing through its middle. This ensures rotation of the tractor when moving on a plane. Each wheel may be rotated around its axis. The wheels are pneumatic, by which act on the body linear elastic and dissipative forces due to gravity forces. By driving the wheels attached frictional forces and moments, as well as forces of resistance to movement of the tractor. Nonholonomic constraints caused by the lack of wheel slippage at the points of contact.

Hopper given the dynamic model (Fig.2). It is a hopper body as a rigid body, the position of which is determined by the Cartesian coordinates of the center of mass (p. *O* in Fig. 2), Krylov three angles (α, β, γ, see Fig.2). Axis, which is planted on the two wheels, rigidly secured to the housing. Each wheel may be rotated around its axis. The wheels are pneumatic, by which act on the body linear elastic and dissipative forces due to gravity forces. For wheels attached power resistance movement. Nonholonomic constraints caused by the lack of wheel slippage at the points of contact.

Seeder given the dynamic model (Fig.3). It is a planter as a solid body, whose position is determined by the Cartesian coordinates of the center of mass (p. *O* in Fig. 3), Krylov three angles (α, β, γ in Fig. 3). Axis, which is planted on the four wheels, rigidly secured to the housing. Each wheel may be rotated around its axis. The wheels are pneumatic, by which act on the body linear elastic and dissipative forces due to gravity forces. For wheels attached power resistance movement. Nonholonomic constraints caused by the lack of wheel slippage at the points of contact.

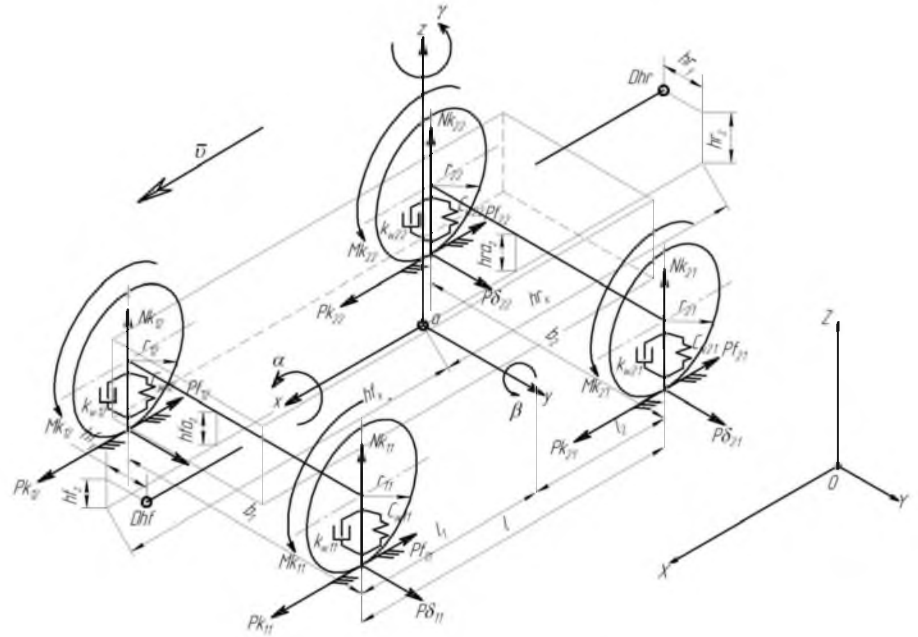

Figure 1. Dynamic model of the tractor

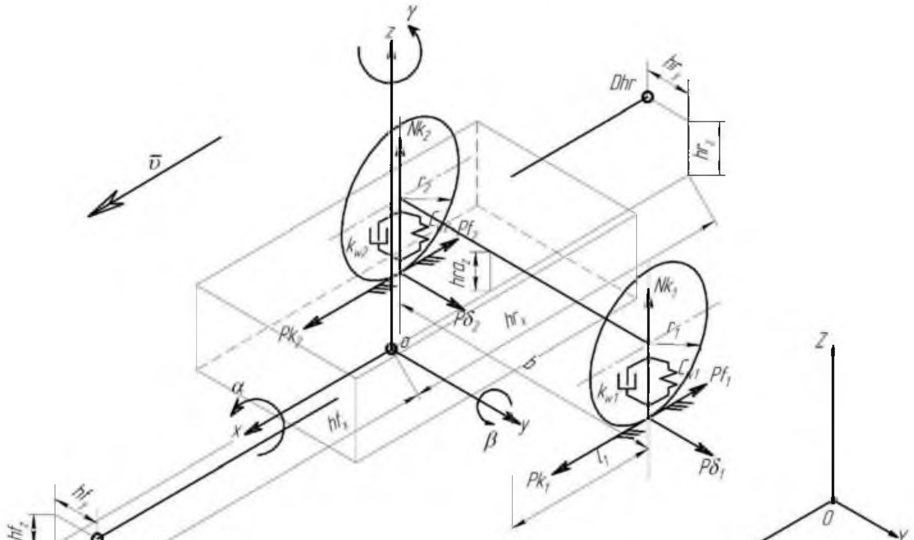

Figure 2. Dynamic model of the hopper

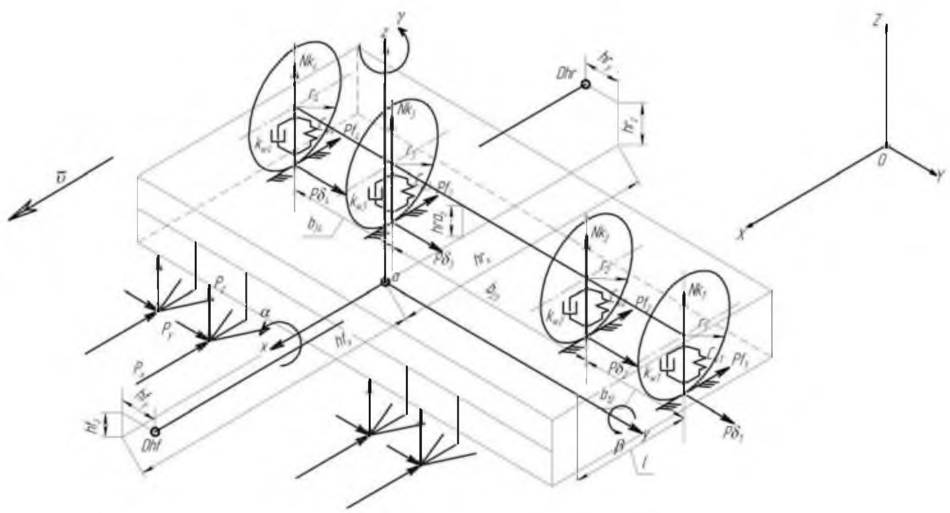

Figure 3. Dynamic model of the seeder

3. Mechanical model coupling

We define the non-holonomic communication in the system according to the method described in [9]. We assume that the elements of the system perform spatial movement. Fig. 4 shows a position of the instantaneous velocity center of the frame; namely, point *P* corresponds to the projection angles and velocities characteristic of the tractor frame points on the plane formed by the axes of the wheels.

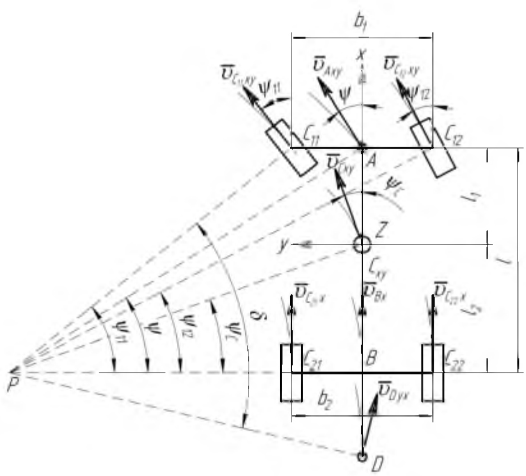

Figure 4. The velocity distribution in the plane of the tractor frame

We assume predetermined angle ψ. We express the projection of the velocity characteristic points on the frame associated with it *Cxy* plane and the projection of the angular velocity of the frame on an axis *Cz* perpendicular to it through the projection speed p. *B* on the plane *Cxy*. Obviously, $v_{Bx} = v_{Axy} \cos \psi$. From whence

$$v_{Axy} = \frac{v_{Bx}}{\cos \psi} \tag{1}$$

The angle ψ is sharp, so the formula (1) does not degenerate. Let $\lambda = BC/l = l_2/l$ and find the angle ψ_C for the projection of the center of mass and velocity of projection of the center of mass velocity on the plane *Oxy*:

$$\mathrm{tg}\psi_C = \frac{BC}{PB} = \frac{BC}{l}\frac{l}{PB} = \lambda \mathrm{tg}\psi, \tag{2}$$

$$v_{Cxy} = v_{Bx}\frac{PC}{PB} = v_{Bx}\cos\psi_C. \tag{3}$$

Define the angle ψ_C as

$$\cos\psi_C = \frac{1}{\sqrt{1+\mathrm{tg}^2\psi_C}} = \frac{1}{\sqrt{1+\lambda^2\mathrm{tg}^2\psi}}, \quad \sin\psi_C = \frac{1}{\sqrt{1+\mathrm{ctg}^2\psi_C}} = \frac{\lambda}{\sqrt{\lambda^2+\mathrm{ctg}^2\psi}}.$$

So,

$$v_{Cxy} = v_{Bx}\frac{1}{\sqrt{1+\lambda^2\mathrm{tg}^2\psi}}. \tag{4}$$

Similarly, defining $\mu = 0{,}5b/l$, one obtains

$$v_{C_{21}x} = v_{Bx}\frac{PC_{21}}{PB} = v_{Bx}\frac{PB-0{,}5b}{PB} = v_{Bx}\left(1-0{,}5\frac{b}{l}\frac{l}{PB}\right) = v_{Bx}\left(1-\mu\mathrm{tg}\psi\right), \tag{5}$$

$$v_{C_{22}x} = v_{Bx}\frac{PC_{22}}{PB} = v_{Bx}\frac{PB+0{,}5b}{PB} = v_{Bx}\left(1+0{,}5\frac{b}{l}\frac{l}{PB}\right) = v_{Bx}\left(1+\mu\mathrm{tg}\psi\right), \tag{6}$$

$$v_{C_{11}xy} = \frac{v_{C_{21}x}}{\cos\psi_{11}} = v_{C_{21}xy}\sqrt{1+\mathrm{tg}^2\psi_{11}}. \tag{7}$$

As

$$\mathrm{ctg}\psi_{11} = \frac{PB-0{,}5b}{l} = \frac{PB}{l} - \mu = \mathrm{ctg}\psi - \mu, \ \mathrm{ctg}\psi_{12} = \mathrm{ctg}\psi + \mu,$$

one has $\mathrm{tg}\psi_{11} = \dfrac{\mathrm{tg}\psi}{1-\mu\mathrm{tg}\psi}$, $\mathrm{tg}\psi_{12} = \dfrac{\mathrm{tg}\psi}{1+\mu\mathrm{tg}\psi}$,

and

$$v_{C_{11}xy} = \frac{v_{C_{21}x}}{\cos\psi_{11}} = v_{C_{21}xy}\sqrt{1+\mathrm{tg}^2\psi_{11}} = v_{C_{21}xy}\frac{\sqrt{\mathrm{tg}^2\psi+(1-\mu\mathrm{tg}\psi)^2}}{1\text{-}\mu\mathrm{tg}\psi} = v_{Bx}\sqrt{\mathrm{tg}^2\psi+(1-\mu\mathrm{tg}\psi)^2} \tag{8}$$

$$v_{C_{12}xy} = \frac{v_{C_{22}x}}{\cos\psi_{12}} = v_{C_{22}xy}\sqrt{1+\mathrm{tg}^2\psi_{12}} = v_{C_{22}xy}\frac{\sqrt{\mathrm{tg}^2\psi+(1+\mu\mathrm{tg}\psi)^2}}{1\text{+}\mu\mathrm{tg}\psi} = v_{Bx}\sqrt{\mathrm{tg}^2\psi+(1+\mu\mathrm{tg}\psi)^2} \tag{9}$$

Finally, the angular rotation speed of the tractor in the plane of the frame

$$\omega_z = \frac{v_{Bx}}{PB} = \frac{v_{Bx}}{l}\frac{l}{PB} = \frac{v_{Bx}\mathrm{tg}\psi}{l}. \tag{10}$$

So, let us take as an independent speeds pseudo velocity tractor v_{Bx}, v_{Bz} $\left(v_{By}\equiv 0\right)$, and ω_x, ω_y (ω_z determined from (10)).

We write the expression for the kinetic rate p. *B*,

$$\vec{v}_B^{\text{св}} = \vec{v}_C^{\text{св}} + \vec{\omega}^{\text{св}} \times \overrightarrow{CB^{\text{св}}} = T_{\text{абс}}^{\text{тр}}\begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{bmatrix} + \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \omega_x & \omega_y & \omega_z\\ -l_2 & 0 & -h\end{vmatrix} = T_{\text{абс}}^{\text{тр}}\begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{bmatrix} + \begin{bmatrix}-\omega_y h\\ \omega_x h - \omega_z l_2\\ \omega_z l_2\end{bmatrix}$$

where

$$T_{\text{абс}}^{\text{тр}} = \begin{bmatrix}t_{11} & t_{21} & t_{31}\\ t_{12} & t_{22} & t_{32}\\ t_{13} & t_{32} & t_{33}\end{bmatrix} = \begin{bmatrix}C_\beta C_\gamma + S_\alpha S_\beta S_\gamma & C_\alpha S_\gamma & -S_\beta C_\gamma + S_\alpha C_\beta S_\gamma\\ S_\alpha S_\beta C_\gamma - C_\beta S_\gamma & C_\alpha C_\gamma & S_\alpha C_\beta C_\gamma + S_\beta S_\gamma\\ C_\alpha S_\beta & -S_\alpha & C_\alpha C_\beta\end{bmatrix} \tag{11}$$

– rotation matrix from an absolute to a tractor-related coordinate system (marked in large letters: C – cosine, S – sine, respectively, of the angles indicated by the subscript). From whence we get 3 equations relating the 7 speed $(\omega_z$ already expressed in (10), a $v_{By} \equiv 0)$

$$t_{11}\dot{X} + t_{21}\dot{Y} + t_{31}\dot{Z} - \omega_y h = v_{Bx}$$
$$t_{12}\dot{X} + t_{22}\dot{Y} + t_{32}\dot{Z} + \omega_x h - \omega_z l_2 = 0$$
$$t_{13}\dot{X} + t_{23}\dot{Y} + t_{33}\dot{Z} - \omega_y l_2 = v_{Bz}$$

Thus, one has

$$v_{Bx} = t_{11}\dot{X} + t_{21}\dot{Y} + t_{31}\dot{Z} - h\omega_y \tag{12}$$

$$\omega_x = \frac{\lambda t_{11}\mathrm{tg}\psi - t_{12}}{h}\dot{X} + \frac{\lambda t_{21}\mathrm{tg}\psi - t_{22}}{h}\dot{Y} + \frac{\lambda t_{31}\mathrm{tg}\psi - t_{32}}{h}\dot{Z} - \lambda\mathrm{tg}\psi\omega_y \tag{13}$$

$$v_{Bz} = t_{13}\dot{X} + t_{23}\dot{Y} + t_{33}\dot{Z} - l_2\omega_y \tag{14}$$

This shows that, excluding non-holonomic constraints frame has a solid 6 degrees of freedom, which means that 6 independent speeds – $\dot{X}, \dot{Y}, \dot{Z}, \omega_x, \omega_y, \omega_z$. Given the fact that the law of variation of the angle ψ must be specified, and the presence of nonholonomic instantaneous velocity center Cxy plane leads to the fact that independent rate becomes 4 – $\dot{X}, \dot{Y}, \dot{Z}, \omega_y$, and the speed of any points and two components of the angular velocity are given by (1)-(10), (12)-(14).

Taking into account the rotation of the wheels the number of degrees of freedom increases further 4. However, their angular velocities can be expressed in terms of the projection of the velocity pp. C_{11}, C_{12}, C_{21}, C_{21} for Cxy plane (see Fig. 5).

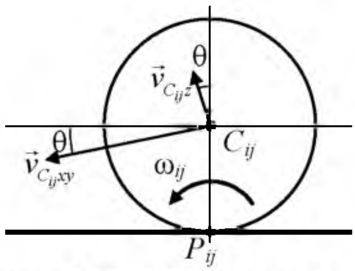

Figure 5. Determining of the wheel speed

First we note that the module perpendicular to the plane of the frame component of velocity of the center wheel $\vec{v}_{C_{ij}z}$ in an actual operation is quite small. At least, it is smaller than the center part of the wheel speed in the plane of the frame $\vec{v}_{C_{ij}xy}$. Also, small angle will θ – the angle between the normal to the plane of the frame and the vertical, i.e. the angle between the horizontal plane and the frame plane. Therefore, we can ignore the effect of this component on the value of the wheel speed. Thus, the angular velocity of the wheel under this assumption will

$$\omega_{ij} = \frac{v_{C_{ij}xy}}{Z_{C_{ij}}\cos\theta} \approx \frac{v_{C_{ij}xy}}{Z_{C_{ij}}}, \tag{15}$$

where $Z_{C_{ij}}$ - applicate wheel center in the absolute coordinate system.

So, we have a tractor for generalized coordinates $X_C, Y_C, Z_C, \beta, \alpha, \gamma, \varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}$. As described above, independent of the generalized velocities, and thus independent of variations in the generalized coordinates are respectively, $\dot{X}_C, \dot{Y}_C, \dot{Z}_C, \dot{\beta}$ and $\delta X_C, \delta Y_C, \delta Z_C, \delta\beta$. Dependence of the generalized velocities, and thus dependent variations of the generalized coordinates are respectively, $\dot{\alpha}, \dot{\gamma}, \dot{\varphi}_{11}, \dot{\varphi}_{12}, \dot{\varphi}_{21}, \dot{\varphi}_{22}$ and $\delta\alpha, \delta\gamma, \delta\varphi_{11}, \delta\varphi_{12}, \delta\varphi_{21}, \delta\varphi_{22}$. These dependent variables are expressed through independent using formulas derived from the expressions (10), (13), (15) and kinematic equations Krylov angles $\omega_x = \dot{\alpha}\cos\gamma + \dot{\beta}\cos\alpha\sin\gamma$, $\omega_y = -\dot{\alpha}\sin\gamma + \dot{\beta}\cos\alpha\cos\gamma$ $\omega_z = \dot{\gamma} - \dot{\beta}\sin\alpha$. Thus, we obtain

$$\dot{\gamma} = \frac{v_{Bx}\mathrm{tg}\psi}{l} + \dot{\beta}\sin\alpha,$$

$$\dot{\alpha} = \frac{a\dot{X}_C + b\dot{Y}_C + c\dot{Z}_C - \dot{\beta}\cos\alpha(d\cos\gamma + \sin\gamma)}{\cos\gamma - d\sin\gamma},$$

$$\dot{\varphi}_{11} = \frac{v_{C_{11}xy}}{Z_{C_{11}}}, \quad \dot{\varphi}_{12} = \frac{v_{C_{12}xy}}{Z_{C_{12}}}, \quad \dot{\varphi}_{21} = \frac{v_{C_{21}x}}{Z_{C_{21}}}, \quad \dot{\varphi}_{221} = \frac{v_{C_{22}x}}{Z_{C_{22}}}$$

Find the joint velocity *D* (Figure 4) in absolute CS

$$\vec{v}_D^{\,\text{абс}} = \vec{v}_C^{\,\text{абс}} + \vec{\omega}^{\text{абс}} \times \overline{CD}^{\text{абс}} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} + T_{\text{тр}}^{\text{абс}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_x & \omega_y & \omega_z \\ h_{rx} & h_{ry} & h_{rz} \end{vmatrix}, \quad (16)$$

where $T_{\text{тр}}^{\text{абс}} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}$ - the transposed matrix from (11), h_{rx}, h_{ry}, h_{rz} – distances (see Fig. 4).

Enough is now to translate this vector in of the hopper coordinate system that is attached to the tractor in the hinge p. *D* (Fig.4). Tractor velocity of points on the projection plane of the frame are expressed by one – v_{Bx} by formulas (1), (3)-(10) Similarly, the velocity through p. *D* (16) are expressed of the hopper speed characteristic points, including point coupling and speed a planter. Then, in the same manner it is determined the projection of the velocity characteristic seeder points on its frame through a speed point of the hopper coupling with the drill.

Thus, of the hopper is 2 degrees of freedom – turn around p. *D* on angles $\alpha^{Б}$ and $\beta^{Б}$. Three degree of freedom removes geometric link – general point of the tractor *D* and one – nonholonomic constraint to equation (15). For wheels of the hopper are valid expressions (10) with the substitution indexes $i = 3, j = 1,2$ and velocities from eq. (15).

Thus, the seeder, like the hopper is 2 degrees of freedom – turn around point coupling with a hopper. Three degree of freedom removes geometric link – with common hopper point and one - nonholonomic constraint with the type of eq. (15). For seeder wheels are valid expressions (10) with the substitution indexes i=4, j=1,2,3,4 and velocities from eq. (15).

4. The simulation results

Theoretically investigate mathematical model of the dynamics of the combined tillage sowing, multielement tractor unit for example tractor John Deere 8345R, hopper for seed and fertilizer John Deere 1910 sowing and direct sowing John Deere 1840. Consider the case where traffic tractor unit occurred in the field machine operators influence on the steering wheel. This angle control wheel is set in an analytical form $\psi = 0{,}1\sin 0{,}5t$. Calculate the speed of rotation of the tractor wheels (Fig. 6).

The transition process associated with the deformation of the tire also affected the speed of rotation of the tractor wheels (Fig. 6) but its length. Fluctuation speeds of wheels (all wheel) is 0,275 rpm and period T = 12 s. The front wheels ω_{11}, ω_{12} have lower rotational speed than the rear ω_{21}, ω_{22} on 0,15 rpm.

One calculates the velocity of the center of mass of elements of the unit (Fig. 7). Fluctuation velocities of the centers of mass of elements of the machine axis x (υ_x) according to the tractor, hopper and drill are 0,25 m/s, 0,18 m/s and 0,17 m/s; along the axis y (υ_y) scale fluctuations are 1,156 m/s, 0,87 m/s and 0,86 m/s; on the axis z (υ_z) there is only a transition process at the beginning of the movement.

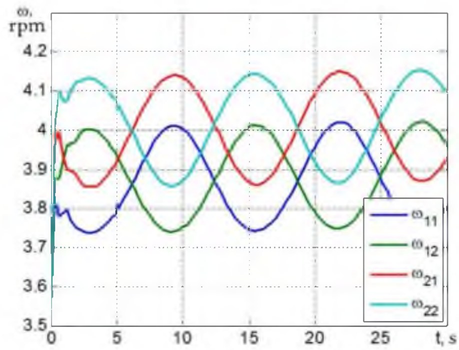

Figure 6. The tractor wheels speed (ω_{11}, ω_{12}, ω_{21}, ω_{22}), respectively of the left and right front, left and right rear

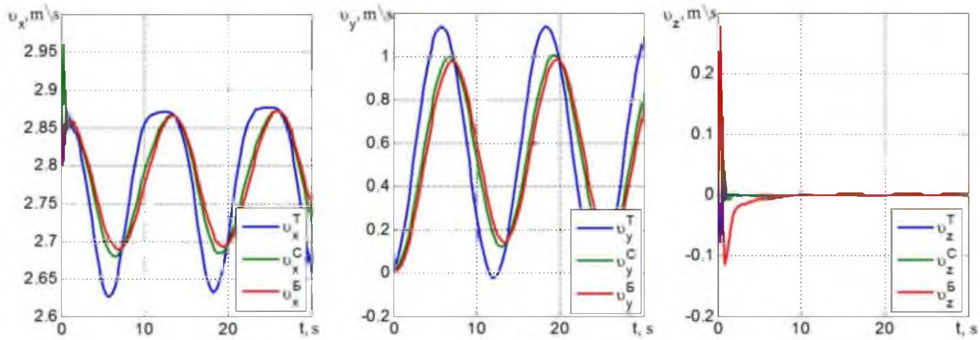

Figure 7. Speed the center of mass of the unit elements (a – υ_x^n, b – υ_y^n, c – υ_z^n)

Conclusions

The proposed method of preparation by eliminating dependent variations of the generalized coordinates of nonholonomic systems of equations and dependent generalized velocities of the general equation of dynamics can automate the formation of equations and the calculation of the dynamics in a special computer algebra system KiDyM. Mathematical model of spatial movement of multi-machines connected via spherical joints, coupling of which is represented nonholonomic system solids committing spatial movement on the elastic pneumatics thus obtained is adequate and properly reflects the dynamic processes taking place.

Research multielement dynamics tractor unit composed classic layout tractor John Deere 8345R and combined-tillage sowing unit as part of John Deere 1910 John Deere sowing and 1840 by the developed method allowed determining the dynamic performance of its operations in the spatial movement elements. An important element speeds centers of mass unit, dynamic radii defined wheel tractor, motion simulation conducted at a constant speed.

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