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Investigation of the Parametric Vibrations of Laminated Plates by RFM

Lidiya Kurpa [1*], Olga Mazur[1], Victoria Tkachenko[1]

Abstract
The R-functions theory is applied to study free vibration and dynamic instability of the symmetrically laminated plates subjected to combined static and periodic axial forces. It is assumed that subcritical state of the plate may be inhomogeneous. Theoretical formulation is made on the classical plate theory (CTP). The developed approach is based on combined application of Ritz's method, Galerkin procedure, R-functions theory and Bolotin's method The buckling, instability zones and response curves for laminated plate with different external cutouts are presented and discussed Effects of plate geometrical parameters, parking of layers, mechanical characteristics of the material on buckling, natural frequencies and parametric resonance are also studied

Keywords
Parametric vibrations, R-functions theory, laminated plates

[1] NTU"KhPI" Kharkiv, Ukraine
*** Corresponding author:** l.kurpa@mail.ru

Introduction

Composite laminated plates are widely used in engineering application like mechanical, aerospace, automobile, marine, and civil engineering due to their high strength and flexibility in design.

By the technological requirements many components of the designs have the different cutouts both internal and external. Ways to fix these cutouts may be various and depend on their destination. If construction is subjected to time-dependent external loading then presence of the cutouts may be essentially change the static and dynamic characteristics of the structures. Therefore, a study of the dynamic behavior and parametric vibrations of multi-layer elements of the thin-walled designs structures with cutouts is one of the important problems arising in engineering design.

One of the most effective approaches to investigation of the parametric vibrations of the plates has been proposed by V.V.Bolotin [1].

The vibration responses of laminated composite plates have been extensively studied by a number of researchers. Surveys of the references devoted to analysis of the laminated plates with inhomogeneous subcritical state are given in articles [2-4] and others. From the review it follows that study and analysis of the parametric vibrations of the plates with complex form with cutouts is practically absent.

In this paper the original approach is applied in order to solve this problem. This approach is based on combined application of the R-functions theory [5], variational methods and V.V.Bolotin's approach. The proposed method is applied to investigation of the laminated plates with rectangular and circle external cutouts. The effects of different geometric and mechanical parameters, number of layers, angle of lamina and material properties on the region of dynamic stability/instability, buckling, and response curves are studied.

1. Formulation

Thin laminated plate of the symmetric structure consisting of n layers of the constant thickness is considered. Assume that the plate is subjected to a periodic in-plane load $p = p_0 + p_t \cos\theta t$, where p_0 is a static component, p_t is amplitude of a periodic part, and θ is a frequency of the load. Parametric excitation of plate under periodic load will be investigated in the framework of the classical theory of the thin laminated plates.

The governing differential equations for parametric excitation of laminated composite plates are:

$$N_{11,x} + N_{12,y} = m_1 u,_{tt} \tag{1}$$

$$N_{12,x} + N_{22,y} = m_1 v,_{tt} \tag{2}$$

$$M_{11},_{xx} + 2M_{12},_{xy} + M_{22x},_{yy} + \left(N_{11} w,_{xx} + 2N_{12} w,_{xy} + N_{22} w,_{yy}\right) = m_1 w,_{tt} + \varepsilon_d m_1 w,_t \,. \tag{3}$$

Here u,v and w are the displacements of the plate in directions Ox,Oy and Oz, respectively, ε_d is damping coefficient, and N_{ij} and M_{ij}, $(i,j=1,2)$ are the components of vectors stress resultants $N = \{N_{11}, N_{22}, N_{12}\}$ and moments $M = \{M_{11}, M_{22}, M_{12}\}$ respectively. The stress $N = \{N_{11}, N_{22}, N_{12}\}$ and moments $M = \{M_{11}, M_{22}, M_{12}\}$ resultants may be presented in the following matrix form:

$$N = [C] \cdot \varepsilon \,, \qquad M = [D] \cdot \chi \,, \tag{4}$$

where $\varepsilon = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}\}^T$ and $\chi = \{\chi_{11}, \chi_{22}, \chi_{12}\}^T$ are strain of the middle surface with components:

$$\varepsilon_{11} = u,_x + \frac{1}{2} w,_x^2 \,,\ \varepsilon_{22} = v,_y + \frac{1}{2} w,_y^2 \,,\ \varepsilon_{12} = u,_y + v,_x + w,_x \, w,_y \,,$$

$$\chi_{11} = -w,_{xx} \,, \chi_{22} = -w,_{yy} \,, \chi_{12} = -2w,_{xy} \,.$$

The matrices $[C]$ and $[D]$ in formulas (4) have the following form:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix}, \quad [D] = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}.$$

The constants C_{ij} and D_{ij} refer to as the reduced stiffness coefficients of the plate are defined by the following formulas:

$$\left(C_{ij}(x,y), D_{ij}(x,y)\right) = \sum_{m=1}^{M} \int_{h_m(x,y)}^{h_{m+1}(x,y)} B_{ij}^{(m)} \left(1, z^2\right) dz, \ (i,j = 1,2,6),$$

$$C_{ij}(x,y) = K_i \sum_{m=1}^{M} \int_{h_m(x,y)}^{h_{m+1}(x,y)} B_{ij}^{(m)} dz, \ (i,j = 4,5),$$

$$(m_1, m_2) = \sum_{m=1}^{M} \int_{h_m(x,y)}^{h_{m+1}(x,y)} (1,z) \rho^{(m)} dz,$$

where $B_{ij}^{(m)}$ are mechanical characteristics and $\rho^{(m)}$ is material density of the m-layer.

The shift factors $K_i (i = 4,5)$ are taken equal to $\frac{5}{6}$. The system (1)-(3) of the motion equations is supplemented by corresponding boundary and initial conditions.

2. Method of solution

The proposed method consists of several stages:

1. Calculation of the prebuckling stress state of the laminated plate.

2. Finding of the buckling load.
3. Solution linear vibration problem of the laminated plate subjected to in plane compressive static load p_0.
4. Solutions of the auxiliary problem like elasticity problem.
5. Solution of the nonlinear vibration problem. Construction of the instability zones and response curves.

Presenting the solution of the system (1)-(3)as truncated series [6] we reduce it to nonlinear ODEs and in the case of one mode approximation to nonlinear ODE [1]:

$$y''(t)+\Omega_L^2((1-2k\cos\theta t)y(t)+\gamma y^3(t))=0,$$

here Ω_L is frequency of loaded plate by static component p_0, , γ is coefficient, which is defined as double integral over considered domain [6]. The first instability region is bounded by curves [1]:

$$\theta_1=2\Omega_L\sqrt{1-k}\,,\quad \theta_2=2\Omega_L\sqrt{1+k}\,.$$

The relation between the frequency ratio and the amplitude of nonlinear vibrations after the loss of stability has the form [1]:

$$A=\frac{2}{\sqrt{3\gamma}}\sqrt{\left(\frac{\theta}{2\Omega_L}\right)^2-1\pm k}\,.$$

Detailed description of stages 1-5 can be found in reference [6].

3. Numerical results

The validation of the proposed method was confirmed by comparison of the obtained results with available ones for rectangular laminated plates [6].

Let us investigate the laminated 3- and 5-layers plate with cutouts (Fig. 2) and the different material properties:

$\frac{G}{E_2}=0.6$, $\nu_1=0.25$, $\frac{E_1}{E_2}=3$ (glass-epoxy); $\frac{E_1}{E_2}=10$ (boron-epoxy); $\frac{E_1}{E_2}=40$ (graphite).

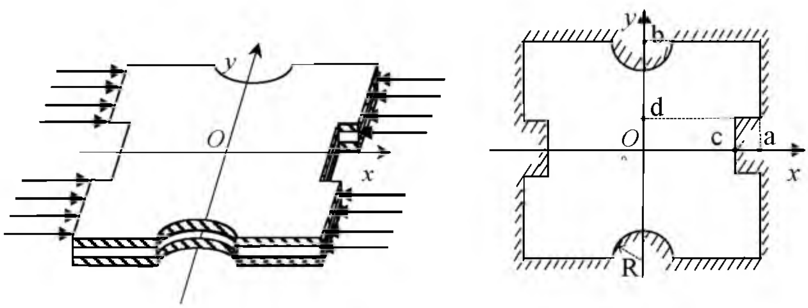

Figure 1. The form of plate

Boundary conditions are taken as:

$$w=w_{,n}=T_n=0\,,\ \forall(x,y)\in\partial\Omega$$

$$N_{11}=-p,\quad \forall(x,y)\in\partial\Omega_1\,(\,\partial\Omega_1:x=\pm\frac{a}{2}\,),$$

$$N_n=0,\quad \forall(x,y)\in\partial\Omega_2=\partial\Omega/\partial\Omega_1\,,$$

where

$$N_n=N_{11}l^2+N_{22}m^2+2N_{12}lm\,,\qquad T_n=N_{12}\left(l^2-m^2\right)+\left(N_{11}-N_{22}\right)lm\,.$$

Here

$$l=\cos\left(\vec{n},Ox\right),\ m=\cos\left(\vec{n},Oy\right)$$

For the given conditions the structure of solution [5] for u, v, w satisfying only the main boundary conditions takes the form:

$$w = \omega^2 \cdot P_1, \ u = P_2, \ v = P_3, \qquad (5)$$

where $\omega(x, y) = 0$ is the equation of the whole boundary domain. The function $\omega(x, y)$ is defined as follows

$$\omega(x, y) = (f_1 \wedge_0 f_2) \wedge_0 (f_3 \wedge_0 f_4) \wedge_0 (f_5 \vee_0 \overline{f_6}).$$

Here the functions $f_i, \left(i = \overline{1,6}\right)$ are defined as

$$f_1 = \frac{1}{a}\left(\left(\frac{a}{2}\right)^2 - x^2\right) \geq 0, \ f_2 = \frac{1}{b}\left(\left(\frac{b}{2}\right)^2 - y^2\right) \geq 0,$$

$$f_3 = \left(x^2 + \left(y - \frac{b}{2}\right)^2 - R^2\right) \geq 0 \ f_4 = \left(x^2 + \left(y + \frac{b}{2}\right)^2 - R^2\right) \geq 0$$

$$f_5 = \frac{1}{2d}\left(d^2 - x^2\right) \geq 0, \ f_6 = \frac{1}{2c}\left(c^2 - x^2\right) \geq 0.$$

The symbols $\wedge_0, \vee_0$ denote R-operations [5]. In (5) $P_i\left(i = \overline{1,3}\right)$ are indefinite components of the structure that are presented as an expansion in a series in a complete system (in this presentation power polynomials are used).

Table 1. Values of the frequency parameter $\Lambda_i = \frac{\lambda_i a^2}{h}\sqrt{\frac{12(1 - \nu_{12}\nu_{21})\rho}{E_2}}$ and the buckling load $p_{cr} = \frac{N_{22}a^2}{E_2 h^3}$.

Material	Ply orientation		p_{cr}	Λ_i			
				$\frac{p_0}{p_{cr}} = 0$	$\frac{p_0}{p_{cr}} = 0.25$	$\frac{p_0}{p_{cr}} = 0.5$	$\frac{p_0}{p_{cr}} = 0.75$
Glass-epoxy	3 layers	0°/90°/0°	28.54	73.60	64.28	52.88	37.33
		90°/0°/90°	21.77	71.35	62.81	52.23	37.41
	5 layers	0°/90°/0°/90°/0°	27.36	73.42	64.17	52.85	37.39
		90°/0°/90°/0°/90°	23.28	72.03	63.23	52.37	27.30
Boron-Epoxy	3 layers	0°/90°/0°	78.66	113.54	98.98	81.14	56.54
		90°/0°/90°	42.33	106.84	96.54	84.07	67.06
	5 layers	0°/90°/0°/90°/0°	75.63	113.78	99.27	81.55	57.33
		90°/0°/90°/0°/90°	57.50	109.76	96.67	80.42	57.42
Graphite	3 layers	0°/90°/0°	254.54	209.48	185.90	156.34	114.64
		90°/0°/90°	93,88	193.73	182.71	169.81	152.43
	5 layers	0°/90°/0°/90°/0°	294.53	212.34	186.38	154.50	110.54
		90°/0°/90°/0°/90°	215.47	201.98	180.69	152.96	113.34

Value of the critical load is increasing if the ratio E_1 / E_2 is increasing. The frequencies behavior is similar. Scheme of the lamination influences on the critical load value greatly. This is especially true for graphite. For example, the critical load for ply orientation 0°/90°/0° is almost three times more than for packing 90°/0°/90°. With the increase of the ratio p_0 / p_{cr} the frequency value decreases. Let us study the dynamic instability and nonlinear vibration for 3-layered graphite plate.

Figures 3, 4 present location of regions of instability for different types of packing layers and values of static load p_0. Note that with increase of the static load zone of the instability occurs at lower values of exciting frequency. At the same time the influence of the load p_0 is more significant for packing layers 0°/90°/0°.

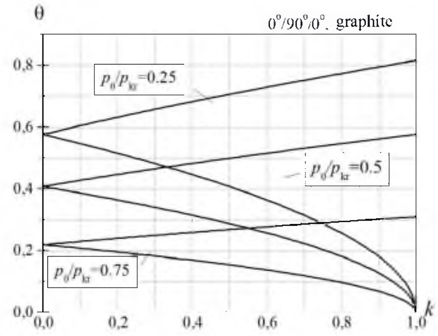

Figure 3. The regions of instability for graphite plate (0°/90°/0°)

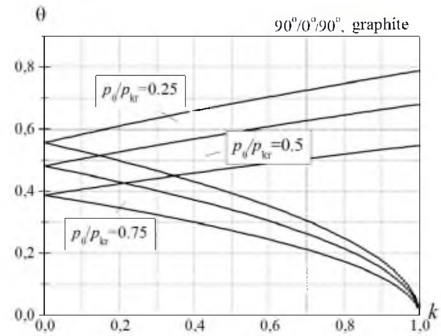

Figure 4. The regions of instability for graphite plate (90°/0°/90°)

Figures 5, 6 show the influence on the amplitude - frequency dependences of the load parameters for steady nonlinear vibrations that occur after the loss of stability. One can see that curves for $p_0 / p_{cr} = 0.25, 0.5$ practically coincide. Increasing of load ratio p_0 / p_{cr} leads to a change in the slope of the curves (fig.5).

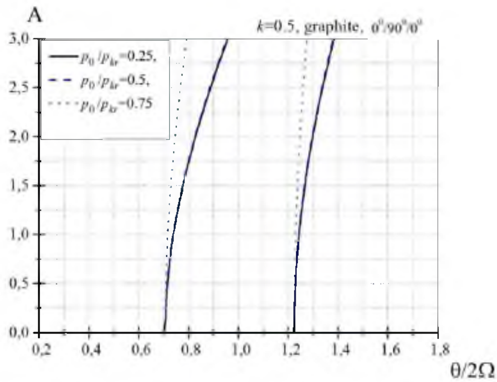

Figure 5. The amplitude - frequency dependences for different ratio p_0 / p_{cr}

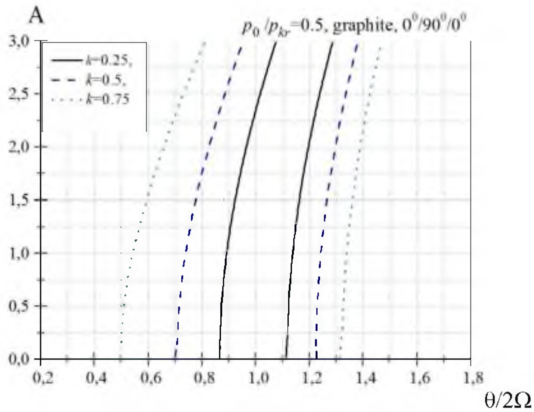

Figure 6. The amplitude - frequency dependences for different k

In Figure 6 the amplitude-frequency dependences are presented for different values of excitation coefficient k , which allows analyzing influence of load parameter p_t on vibrations amplitudes.

Conclusions

Application of the R-functions theory and variational methods to investigations of the parametric vibrations of in-plane loaded laminated plates with a complex form is considered. Mathematical statement of the formulated problems is fulfilled in framework of the classical theory of the laminated plates of the symmetrical structure. The developed algorithm provides calculation of the subcritical state of the laminated plates, finding of the critical load, natural frequencies of the compressed plate, and zones of the instability, and response curves. The proposed approach is realized in system Pole-RL. Numerical results for the laminated plates with rectangular and circle cutouts are carried out. Effects of the different material properties, number of layers and ply-orientations are shown.

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